The Structure of Common Linear Copositive Lyapunov Functions for Continuous time Switched Positive Linear Systems

Zhu Liqin and Wu Zhaorong
School of Mathematical Sciences, University of Jinan, Jinan, China
Email: {ss_zhulq, ss_Wuzr}@ujn.edu.cn

***Abstract*—This paper addressed the structure of common linear copositive Lyapunov function (CLCLFs) for continuoustime switched positive linear systimes (CSPLS). In this note, first of all, for the n×n-dimensinal matrices family, which entries are the Hurwitz and Metzler switch matrices of an n-dimensional CSPLS, we presented, by a straightforward algebraic computation, a procedure for constructing a group n-dimensional vectors, further, we provided a structure theorem of CLCLFs, which shown that all CLCLFs of a CSPLS make a positive open cone generated by these special selected vectors, finally, an numerical example is worked out to illustrate our main results. The results of this technical note are extensions of some recent results in Ref. [1].**

***Index Terms*—positive switched system, common linear copositive Lyapunove function, structure**

I. Introduction

A continuous-time switched positive system (CSPS) is a type of hybrid dynamic system that consists of a family of positive state-space models (Ref. [2], [3]) and a switch-ing signal, which determines the switching between sub-systems. In recent years, CSPSs have been received much attention due to their broad applications in commun-ication systems (Ref. [4]), formation flying (Ref. [5]) and other areas.

In the theory of CSPSs, the stability problem is invest-tigated extensively by many researchers (Ref. [6]-[13]), especially for the stability under arbitrary switching.

We particularly mention a recent paper by Fornasini and Valcher, Ref. [13], where a complete characterization for the existence of a CLCLF for a set of $n\times n$-dimen-sional Metzler matrices $\mathcal{A}_n=\{A_1, A_2, \ldots, A_m\}$ it usually requires to check m^n matrices to be Hurwitz in order to guarantee the existence of a CLCLF for a set of Metzler matrices$\mathcal{A}_n$. Although some necessary and sufficient conditions have been given in Ref. [13], it seems to be difficult to verify those conditions for sufficiently large m and n.

Zhaorong Wu and Yuangong Sun show in Ref. [1] that the existence of a CLCLF for a family of Metzler matrices $\mathcal{A}_n$ can only be determined by *n-1* Metzler Hurwitz matrices, instead of m^n matrices, the existence of a CLCLF for the family $\mathcal{A}_n$ is equivalent to the existence of n-1characteristic constants less than 1, an easily verifiable computation algorithm is provided, which can be carried out by solving the inverse of some matrices rather than checking m^n Metzler matrices to be Hurwitz. Unfortunately, the structure of CLCLFs has not discussed in all known literature. For example, if there exist CLC-LFs for the family $\mathcal{A}_n$, how to structure these CLCLFs? In this note, we will answer this problem.

The main contribution of this note is two folds. We not only shown that all CLCLFs make a positive open cone generated by n special selected vectors, but also gives a method to choose these n vectors.

This note is organized as follows. Problem statements and preliminaries are given in Section II. Section III gives the main results. Finally, an illustrating example is presented in IV.

Notations: For any positive integer m, $\langle m\rangle$ is the set of integers$\{1,2,\ldots,m\}$. I_m is an m-dimensional identity matrix. A diagonal matrix is denoted by diag$\{\ldots\}$. Say A>0($\leq$0, <0) if all entries of matrix A are positive (nonpositive, negative). For a set Ω of n-dimensional vectors, φ is said be a maximal vector if $\varphi\in\Omega$ and $\varphi\leq\psi$ for any $\psi\in\Omega$. A *Metzler matrix* is a real square matrix, whose off-diagonal entries are nonnegative. A *Hurwitz* matrix is a square matrix, if the real part of each eigenvalue of this matrix is negative. The jth column and the (i, j)th entry of a matrix A_k is denoted by col$_j(A_k)$ and $a_{ij}^{(k)}$, respectively. For any $j\in 0$, e_j is an n-dimensional unit vector whose jth entry is 1.

II. Problem Statements

Consider the following continuous-time switched positive linear system (CSPLS)

$$\dot{x}(t)=A_{\sigma(t)}x(t),\quad t\geq 0 \tag{1}$$

where x is the n-dimensional state vector with $n\geq 2$, the piecewise continuous function σ:[0,+∞)→$\langle m\rangle$ is the switching signal. A_k, $k\in\langle m\rangle$, are Metzler and Hurwitz matrices. All these matrices make a family denoted by

Manuscript received Februrary 27, 2013; revised May. 2, 2013

doi: 10.12720/joace.2.2.187-190

$$\mathcal{A}_n = \{A_k: k\in\langle m\rangle\}$$

It is well known that, if $x(t)$, $t \in [0,+\infty)$, is a solution of system (1) satisfying $x(0) \geq 0$, then $x(t) \geq 0$ for any $t \in [0,+\infty)$. System (1) is called to be global asymptotic stability if each solution with the initial condition $x(0) \geq 0$ is asymptotically stable under arbitrary switching.

When considering the stability of system (1) under arbitrary switching, it is usual to find a CLCLF for system (1). By a CLCLF for system (1) (or the family $\mathcal{A}_n$) we mean a function $V(x) = v^T x$ satisfying v 0 and $v^T A_k <0$ for each $k\in\langle m\rangle$. For a family $\mathcal{A}_n$ of $n\times n$-dimensional

Metzler Hurwitz matrices, we have that $v^T A_k< 0$ is equivalent to $v^T(A_kD_k^{-1})<0$, where $D_k =\text{diag}\{a_{11}^{(k)}, a_{22}^{(k)},\ldots, a_{nn}^{(k)}\}$ and v is an n-dimensional vector. Without loss of generality, throughout this note we assume that

(H): $\mathcal{A}_n$ is a family of Metzler Hurwitz matrices, and $a_{ii}^{(k)}= -1$ for $i\in\langle n\rangle$ and $k\in\langle m\rangle$.

The following lemma is one of necessary and sufficient conditions for the existence of a CLCLF for system (1) given

Lemma 1: Given a family $\mathcal{A}_n$ of $n\times n$-dimensional Metzler Hurwitz matrices, there exists a CLCLF for the family $\mathcal{A}_n$ if and only if the following Metzler matrix

$$A_{k_1,\ldots,k_n} = [col_1(A_{k_1}),\ldots,col_n(A_{k_n})] \tag{2}$$

is a Hurwitz matrix for all possible values of $k_1,\ldots, k_n \in\langle n\rangle$.

In the following, for $p\in\langle n\rangle$, let

$$A_k^{(p)} = E_p A_k E_p^T,\quad A_{k_1,\ldots,k_p}^{(p)} = E_p A_{k_1,\ldots,k_p} E_p^T$$

where $k;\ k_1,\ldots, k_p\in\langle m\rangle$, $E_p = [I_p, 0]$ is an $p\times n$-dimensional matrix for $p < n$ and $E_n = I_n$. Denote

$$\mathcal{A}_p = \{A_k^{(p)} : k \in \langle m\rangle\},\ p\in\langle n\rangle$$

Decompose $A_{k_1,\ldots,k_{p+1}}^{(p+1)}$ as the following:

$$A_{k_1,\ldots,k_{p+1}}^{(p+1)} = \begin{pmatrix} A_{k_1,\ldots,k_p}^{(p)} & \xi_{k_{p+1}} \\ \eta_{k_1,\ldots,k_p}^T & -1 \end{pmatrix},\ p\in\langle n-1\rangle \tag{3}$$

where $\eta_{k_1,\ldots,k_p}$ and $\xi_{k_{p+1}}$ are p-dimensional vectors whose i th entries ($p \geq i \geq 1$) are the ($p + 1$, i)th entry and the (i, $p + 1$)th entry of the matrix $A_{k_1,\ldots,k_{p+1}}^{(p+1)}$, respectively.

Denote

$$T_p = \{\theta_{k_1,\ldots,k_p} : k_1,\ldots, k_p\in\langle m\rangle\},\ p\in\langle n-1\rangle \tag{4}$$

where $\theta_{k_1,\ldots,k_p} = (\theta_1^{(k_1,\ldots,k_p)},\ldots,\theta_n^{(k_1,\ldots,k_p)})^T$ satisfy

$$\theta_{k_1,\ldots,k_p}^T A_{k_1,\ldots,k_p}^{(p)} = -\eta_{k_1,\ldots,k_p}^T . \tag{5}$$

Before establishing the main result, we first present the following lemmas given in Ref. [1], which play a key role in proof of the main results.

Lemma 2:(Ref. [1]) For given $p\in\langle n-1\rangle$, assume that there exists a CLCLF for the family $\mathcal{A}_p$. Then, there exists a p-tuple ($k_{1p},\ldots,k_{pp}$) with $k_{ip} \in\langle m\rangle$ for $i\in\langle p\rangle$, such that $\theta_{k_{1p},\ldots,k_{pp}} \in T_p$, and

(i) $\theta_{k_{1p},\ldots,k_{pp}}^T A_{k_1,\ldots,k_p}^{(P)} \leq -\eta_{k_1,\ldots,k_p}^T$, $k_1,\ldots, k_p \in\langle m\rangle$.

(ii) $\theta_{k_1,\ldots,k_p} \leq \theta_{k_{1p},\ldots,k_{pp}}$ for any $k_1,\ldots, k_p \in\langle m\rangle$.

when finding out $\theta_{k_{1p},\ldots,k_{pp}}$, set

$$\lambda_p = \max\{\theta_{k_{1p},\ldots,k_{pp}}^T\ \xi_{k_{(p+1)}} : k_{(p+1)} \in\langle m\rangle\} \tag{5}$$

and $\lambda_p = \theta_{k_{1p},\ldots,k_{pp}}^T\ \xi_{k_{(p+1)p}}$ for some $k_{(p+1)p} \in\langle m\rangle$.

In the following lemma, we see that the characteristic constants λ_p, $p\in\langle n-1\rangle$ play a key role in the existence of a CLCLF.

Lemma 3:(Ref. [1]) There exists a CLCLF for the family $\mathcal{A}_p$ if and only if $\lambda_p <1, p\in\langle n-1\rangle$.

III. Main Results

In this section, we assume that there exists a CLCLF for the family $\mathcal{A}_p$of $n\times n$-dimensional metzler and Hurwitz matrices. Our main task is to find the methods how to construct these CLCLFs.

If $v^{(n)} = (x_1, x_2,\ldots, x_n)^T$ is a CLCLF of the family $\mathcal{A}_p$, set $v_{n-1}^{(n)} = (x_1, x_2,\ldots, x_{n-1})^T$, it is easily to see that $v_{n-1}^{(n)}$ is a CLCLF of the family $\mathcal{A}_{p-1}$. From Lemma 2, for $p = n - 1$, there exists ($n-1$)-tuple ($k_{1(n-1)}$, $k_{2(n-1)}$, …, $k_{(n-1)(n-1)}$) with $k_{i(n-1)} \in\langle m\rangle$, $i\in\langle n-1\rangle$, such that

$$[\theta_{n-1}^{(n)}]^T V^{(n)} = -[\eta^{(n)}]^T \tag{6}$$

where

$$\theta_{n-1}^{(n)} = \theta_{k_{1(n-1)},k_{2(n-1)}\ldots,k_{(n-1)\ n-1}}$$

$$V^{(n)} = V^{(n)} = A_{k_{1(n-1)},k_{2(n-1)}\ldots,k_{(n-1)\ n-1}}^{(n-1)}$$

$$\eta^{(n)} = \eta_{k_{1(n-1)},k_{2(n-1)}\ldots,k_{(n-1)\ n-1}}$$

If the ($n-1$)-dimensional vector

$$\theta_{n-1}^{(n)} = (w_{1n}, w_{2n},\ldots, w_{(n-1)n})^T$$

set the n-dimensional vector

$$\Omega_n = (w_{1n}, w_{2n},\ldots, w_{(n-1)n}, 1)^T$$

From the equation (5), for $p = n-1$, there exists $k_{n(n-1)} \in\langle m\rangle$, such that

$$\max\{[\theta_{n-1}^{(n)}]^T \xi_{k_n} : k_n \in\langle m\rangle\}=[\theta_{n-1}^{(n)}]^T \xi_{k_{n(n-1)}},$$

which denoted by $\lambda_n^{(n)}$, that is

$$\lambda_n^{(n)}=[\theta_{n-1}^{(n)}]^T \xi_{k_{n(n-1)}} \tag{7}$$

By lemma 3, we see $\lambda_n^{(n)}<1$. Set

$$U=A_{k_{1(n-1)},k_{2(n-1)},\dots,k_{(n-1)\,n-1},k_{n(n-1)}} \tag{8}$$

A straightforward computation yields that

$$\Omega_n^T U=-(1-\lambda_n^{(n)})e_n) \tag{9}$$

In essence, Ω_n is the solution of the equation

$$\Omega_n^T U^{(n)}=0$$

where $U^{(n)}$ is the matrix constructed by removing the nth column of the matrix U. In the following, according to the similar way, we will structure a group of n-dimensional vectors $\{\Omega_1,\Omega_2,\dots,\Omega_n\}$, and final we show that all CLCLFs of the family $\mathcal{A}_n$ can be certained by these vectors. To this end, for each $j\in\langle n\rangle$, set $U^{(j)}$ is the $n\times(n-1)$-dimensional matrix constructed by removing the jth column of the matrix U and

$$\Omega_j=(w_{1j},\dots,w_{(j-1)j},1,w_{(j+1)j},\dots w_{nj})^T \tag{10}$$

is the n-dimensional vector satisfying

$$\Omega_j^T U^{(j)}=0 \tag{11}$$

Theorem 1: v_n is a CLCLF of the family $\mathcal{A}_n$ if and only if $v_n=\sum_{j=1}^{n}\upsilon_j\Omega_j$, $\{\upsilon_1,\upsilon_2,\dots,\upsilon_n\}$ is a group of positive constants.

Proof: For $j\in\langle n\rangle$, $k, k_1, k_2,\dots, k_n\in\langle m\rangle$, assume $U_k^{(j)}$ and $U_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)}$ are the $n\times(n-1)$-dimensional matrices constructed by removing the j th column of the matrices A_k and $A_{k_1,k_2,\dots,k_n}$, respectively; $\eta_k^{(j)},\eta^{(j)}$ and $\eta_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)}$ are the j th row of the matrices $U_k^{(j)}$, $U^{(j)}$ and $U_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)}$, respectively; $\xi_k^{(j)}$, $\xi^{(j)}$ are the $(n-1)$-dimensional vectors constructed by removing the j th entry of $\mathrm{col}_j(A_k)$ and $\mathrm{col}_j(U)$, respectively; Further, $V_k^{(j)}$, $V^{(j)}$ and $V_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)}$ are the matrices constructed by removing the j th row of the matrices $U_k^{(j)}$, $U^{(j)}$ and $U_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)}$. We further set

$$V_{n-1}^{(j)}=\{V_1^{(j)},V_2^{(j)},\dots,V_n^{(j)}\}$$

If $v=(x_1,x_2,\dots,x_n)^T$ is a CLCLF of the family $\mathcal{A}_n$, then $v_{n-1}^{(j)}=(x_1,\dots,x_{j-1},x_{j+1},\dots,x_n)^T$ is a CLCLF of $V_{n-1}^{(j)}$. By Lemma 2, there exists $\theta_{n-1}^{(j)}$ such that

$$[\theta_{n-1}^{(j)}]^T V_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)} \le -[\eta_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)}]^T \tag{12}$$

for all $k_1,\dots, k_{j-1}, k_{j+1}, \dots, k_n \in\langle m\rangle$, and if

$$\theta^T V_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)} = -[\eta_{k_1,k_2,\dots,k_{j-1},k_{j+1},\dots k_n}^{(j)}]^T \tag{13}$$

then

$$\theta\le\theta_{n-1}^{(j)} \tag{14}$$

As U is Hurwitz matrix, then for any $j\in\langle n\rangle, V^{(j)}$ is also a Hurwitz matrix, so there exists an unique solution Ω_j for the equation (10) (11). Further from (12) (13) and (14) we get that the vector

$$w_j=(w_{1j},\dots,w_{(j-1)j},w_{(j+1)j},\dots w_{nj})^T$$

satisfying

$$w_j\le\theta_{n-1}^{(j)} \tag{15}$$

A straightforward computation yields that

$$\Omega_j^T U=-(1-\lambda_{n-1}^{(j)})e_j,\ j\in\langle n\rangle \tag{16}$$

where

$$\lambda_{n-1}^{(j)}=w_j^T\xi^{(j)} \tag{17}$$

by Lemma 3 we see

$$\max\{[\theta_{n-1}^{(j)}]^T\xi_k^{(j)}:k\in\langle m\rangle\}<1 \tag{18}$$

To sum up (15) (16) (17) (18) we get that

$$\lambda_{n-1}^{(j)}<1,\quad j\in\langle n\rangle \tag{19}$$

Now we prove the necessity and sufficiency, respectively.

Necessity. If v is a CLCLF of the family $\mathcal{A}_n$, we know from lemma 1 that U is a Hurwitz matrix and $v^T U<0$ .Set

$$v^T U=-(c_1,c_2,\dots,c_n),c_j>0,\ j\in\langle n\rangle \tag{20}$$

and set constants as follows $\upsilon_j=\frac{c_j}{1-\lambda_{n-1}^{(j)}},\ j\in\langle n\rangle$,by (19) we see for any $j\in\langle n\rangle, \upsilon_j>0$. Further by (16)(20) we see $v^T U=-\sum_{j=1}^{n}\upsilon_j(1-\lambda_{n-1}^{(j)})e_j=\sum_{j=1}^{n}\upsilon_j\Omega_j{}^T U$, thus $(v^T-\sum_{j=1}^{n}\upsilon_j\Omega_j{}^T)U=0$. By U is a Hurwitz family, then $v=\sum_{j=1}^{n}\upsilon_j\Omega_j$.

Sufficiency. For any group of positive constants $\{\upsilon_1, \upsilon_2,\dots, \upsilon_n\}$,let $v=\sum_{j=1}^{n}\upsilon_j\Omega_j$,we prove v is a CLCLF of the family $\mathcal{A}_n$. From (12) (13) (14), we get that for any $k\in\langle m\rangle$, $\Omega_j^T A_k=(w_{1j}^{(k)},w_{2j}^{(k)},\cdots,w_{nj}^{(k)})^T, k\in\langle n\rangle$, where

$w_{jj}^{(k)} = w_j^T \xi_k^{(j)} - 1$, and $w_{ij}^{(k)} \le 0, i \in \langle n \rangle$, $i \ne j$, thus $\Omega_j^T A_k \le w_{jj}^{(k)} e_j, j \in \langle n \rangle$. From (15), (18) we get that $w_j^T \xi_k^{(j)} < 1$, so $w_{jj}^{(k)} = w_j^T \xi_k^{(j)} - 1 < 0$. Further from (17) (18)(19), we have that

$$v^T A_k = \sum_{j=1}^{n} \upsilon_j \Omega_j^T A_k \le \sum_{j=1}^{n} \upsilon_j w_{jj}^{(k)} e_j$$

Thus $v^T A_k \prec 0$, namely v is CLCLF of the family $\mathcal{A}_n$.. The proof of Theorem 1 is complete.

IV. Numerrical Example

A numerical example is worked out to illustrate the main results in this section.

Let $\mathcal{A}_3$ ={A_1, A_2}(Ref. [14]), where

$$A_1 = \begin{pmatrix} -1 & 1 & \frac{1}{19} \\ \frac{1}{3} & -1 & \frac{1}{19} \\ 1 & 1 & -1 \end{pmatrix}, A_2 = \begin{pmatrix} -1 & \frac{1}{2} & \frac{1}{20} \\ \frac{2}{3} & -1 & \frac{1}{20} \\ 2 & 2 & -1 \end{pmatrix}$$

From Ref. [14], there exist CLCLFs for the family $\mathcal{A}_3$, and

$$U = A_{2,1,1} = \begin{pmatrix} -1 & 1 & \frac{1}{19} \\ \frac{2}{3} & -1 & \frac{1}{19} \\ 2 & 1 & -1 \end{pmatrix}.$$

Then $U^{(1)}, U^{(2)}, U^{(3)}$ is ,respectively

$$\begin{pmatrix} 1 & \frac{1}{19} \\ -1 & \frac{1}{19} \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} -1 & \frac{1}{19} \\ \frac{2}{3} & \frac{1}{19} \\ 2 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ \frac{2}{3} & -1 \\ 2 & 1 \end{pmatrix}$$

From $\Omega_j^T U^{(j)} = 0$ $j \in \langle 3 \rangle$, we get $\Omega_1^T = (1, \frac{10}{9}, \frac{1}{9})$, $\Omega_2^T = (\frac{44}{51}, 1, \frac{5}{51})$, $\Omega_3^T = (8, 9, 1)$ .Then, by theorem 1, v is a CLCLF of the family $\mathcal{A}_3$, if and only if there is a group of positive constants υ_1 , υ_2 and υ_3 such that $v = \upsilon_1 \Omega_1 + \upsilon_2 \Omega_2 + \upsilon_2 \Omega_3$.

Particularly, set $\upsilon_1 = 2.7$, $\upsilon_2 = 2.55$, $\upsilon_3 = 0.45$, then v = $(8.5, 9.6, 1)^T$ is a CLCLF of the family $\mathcal{A}_3$ which is consistent with Ref. [1].

Acknowledgment

The authors wish to thank the support of the national natural science foundation of China under grant nos 60704039 and 61174217.

References

[1] Z. R. Wu and Y. G. Sun, “On easily verifiable con-ditions for the existence of common linear copositive Lyapunov functions,” *IEEE Transactions on Automatic Control*, 2013.

[2] L. Farina and S. Rinaldi, “Theory and applications, ser. series on pure and applied athematics,” *Positive Linear Systems*, New York: Wiley-Interscience, 2000.

[3] T. Kaczorek, *Positive 1D and 2D Systems*, London, U.K.: Springer Verlag, 2002.

[4] R. Shorten, D. Leith, J. Foy, and R. Kilduff, “Towards an analysis and design framework for congestion control in communication networks,” in *Proc. 12th Yale workshop on adaptive and learning systems*, 2003.

[5] A. Jadbabaie, J. Lin, and A. S. Morse, “Co-ordination of groups of mobile autonomous agents using nearest neighbour rules,” *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988-1001, 2003.

[6] O. Mason and R. N. Shorten, “On the simultaneous diagonal stability of a pair of positive linear systems,” *Linear Algebra Appl.*, vol. 413, pp. 13-23, 2006.

[7] L. Gurvits, R. Shorten, and O. Mason, “On the stability of switched positive linear systems,” *IEEE Trans. Autom. Control*, vol. 52, no. 6, pp. 1099-1103, Jun. 2007.

[8] O. Mason and R. N. Shorten, “On linear copositive Lyapunov functions and the stability of switched positive linear systems,” *IEEE Trans. Autom. Control*, vol. 52, no. 7, pp. 1346-1349, Jul. 2007.

[9] O. Mason and R. N. Shorten, “Quadratic and copostive Lyapunov functions and the stability of positive switched linear systems,” in *Proc. Amer. Control Conf.*, New York, 2007, pp. 657-662.

[10] F. Knorn, O. Mason, and R. N. Shorten, “On linear co-positive Lyapunov functions for sets of linear positive systems,” *Automatica*, vol. 45, no. 8, pp. 1943-1947, Aug. 2009.

[11] L. Fainshil, M. Margaliot, and P. Chigansky, “On the stability of positive linear switched systems under arbitrary switching laws,” *IEEE Trans. Autom. Control*, vol. 54, no. 4, pp. 897-899, Apr. 2009.

[12] S. Bundfuss and M. D¨ur, “Copositive Lyapunov functions for switched systems over cones,” *Syst. Control Lett.*, vol. 58, pp. 342-345, 2009.

[13] E. Fornasini and M. E. Valcher, “Linear copositive Lyapunov functions for continuous-time positive switched systems,” *IEEE Trans. Autom. Control*, vol. 55, no. 8, pp. 1933- 1937, Aug. 2010.

Zhu. Liqin was born in Changyi county, Shandon province, China, on December 7, 1965. She received the bachelor’s degree in Mathematics from school of Mathematical sciences, Qufu normal university, China, in 1986. Since July 1986, she has been a teacher of university of Jinan, China, and taught courses in the areas Mathemetics, e.g., mathe-matical analysis, complex function, ordinary differential equations. Her research interests include ordinary differential equation stability theory, polynomial matrix theory, positive systems and positive switched systems. Associate professor Zhu is a member of Shandong mathematical socity, China.

Wu. Zhaorong was born in Jinan, Shandon province, China, on December 7, 1965. He received the bachelor’s degree in Mathematics from school of Mathematical sciences, Qufu normal university, China, in 1986. Since July 1986, she has been a teacher of university of Jinan, China, and taught courses in the areas Mathemetics, e.g., mathe-matical analysis, real variable function, functional analysis. Her research interests include ordinary differential equation stability theory, delay differential systems, positive systems and positive switched systems. Associate professor Wu is a member of Shandong mathematical socity, China.

