Asymptotic Stabilization of a Morphous One-Parameter Chaotic System

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***Abstract*—The asymptotic stabilization of a morphous one-parameter chaotic system using Takagi Sugeno Fuzzy Controllers is reported in this paper. This system is a parametrically modified system realized from the generalized canonical Lorenz system and having only one variable parameter. The system has been proved to be topologically nonequivalent to the classic Lorenz system but generate attractors that morph from Lorenz-like to Chen-like as the parameter is varied positively. A Fuzzy Controller designed via Takagi Sugeno Fuzzy models and stability analysis of the Lyapunov stability type is used to stabilize the system's trajectories in the sense of Lyapunov. Numerical simulation and analysis of fuzzy rules shows that the system converges to equilibrium points with better settling times as the parameter is varied positively. Moreover, the control effort of each fuzzy subsystems formed by the fuzzy rules are over 1/10 less in comparison with stabilization of the classic Lorenz system via the same design principles.**

***Index Terms*—asymptotic stabilization, Chen's system, Lorenz system, lyapunov stability, takagi sugeno fuzzy model**

I. Introduction

Since the discovery of chaotic dynamics in weather systems by Lorenz in 1963 [1] expansive interest by researchers has demonstrated the presence of chaotic dynamics in multitude of natural and man-made systems in almost all sphere of life. From the inception of the systematic study of chaos by scientific community, plethora of literature has appeared on the steady growth in understanding of the phenomenon of chaos, leading to discovery of more chaotic systems and in-depth studies of the classic Lorenz system resulting in its modification to various topologically nonequivalent versions that exhibits the salient qualitative and quantitative properties of chaotic systems, namely that the system must be expansive with at least one positive Lyapunov exponent.

Scientific literature is filled with the discoveries of hybrid chaotic systems inspired from other well-known systems like the Chen's system [2] Lu system [3] unified chaotic system [4] the Sprott's family of chaotic systems [5] and a large body of other new attractors inspired specifically from the classic Lorenz system [6]-[11]. Perhaps, it is safe to say that the Lorenz system is the most versatile and well-studied chaotic system with several and consistently modified versions with topological nonequivalence. In this connection, the Chen's system is worthy of prominent mention as it has served to 'bridge' the gap between the classicality of the Lorenz system and the evolvability of several new hybrid chaotic systems [12]-[15]. Some of the modified systems, although topologically nonequivalent to the canonical Lorenz systems have two nonlinearities and three variable parameters like the original Lorenz system while many others like the Shimizu-Morioka system [16] Rikitake system [17] four-wing attractor system [18] and the eight-wing attractor [19] among others have three or more nonlinear quadratic terms or adjustable parameters.

Controllability and stabilizability of these evolved systems has attracted attention from researchers in recent years. Infact, existentiality (with Lyapunov exponent $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 < 0$ and controllability of a chaotic system are two properties that enhances utilizability of such systems in chaos-based engineering and non-engineering systems. Consequently, different control methods have been used to control the dynamics of chaotic systems. Impulsive control [20], linear feedback controller [21] adaptive fuzzy models [22] Lyapunov function-based fuzzy controllers [23] have been used to drive chaotic regimes to some equilibrium points. In many of the cases involving stability and stabilization, the Lyapunov stability criteria has played a dominant role in stability analysis of designed fuzzy controllers. In this work, attempt is made to asymptotically stabilize a morphous one-parameter chaotic system whose dynamic analysis has been presented in [24]. As a result, this paper dwell on methods of designing a fuzzy controller and the stability analysis methods and then presents the numerical simulation results accordingly without reproducing the results in [24].

II. The Lorenz and Chen's Systems

The classic Lorenz system [1] is an autonomous system represented by a set of three coupled differential equations represented by:

$$\dot{x}_1 = -\sigma x_1 + \sigma x_2$$

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$$\dot{x}_2 = \rho x_1 - x_2 - x_1 x_3$$

$$\dot{x}_3 = x_1 x_2 - \beta x_3 \tag{1}$$

where x_1, x_2 and x_3 are states of the system. For typical values of $\sigma = 10, \rho = 28$ and $\beta = 8/3$, the famous butterfly attractor evolves. On the other hand, the Chen's systems [2] is described by the following set of equations

$$\dot{x}_1 = -ax_1 + ax_2$$

$$\dot{x}_2 = (c - a)x_1 + cx_2 - x_1 x_3$$

$$\dot{x}_3 = x_1 x_2 - \beta x_3 \tag{2}$$

which has a set of chaotic parameters $a = 35, b = 3, c = 28$. The Chen's system is a dual of the Lorenz system but evolves more sophisticated attractors. In the framework of topological classifications given by Vanecek and Celikovsky[25] the linearization of a chaotic system about the origin produces a 3? constant matrix of partial derivatives, $A = \left[a_{ij} \right]_{3?}$ in which the sign of the elemental combination $a_{12}a_{21}$ distinguishes topological nonequivalences. Based on this criterion, the Lorenz system satisfies the condition $a_{12}a_{21} > 0$, while the Chen's system: $a_{12}a_{21} < 0$.

III. The Morphous One-Parameter Chaotic System

The morphous one-parameter chaotic system [24] has an algebraically simple mathematical representation, but nonetheless produces highly complex chaotic attractors that morph from the Lorenz-like system to the Chen's system as a variable parameter r increases positively. The equations governing the system is given by

$$\dot{x}_1 = -x_1 - x_2$$

$$\dot{x}_2 = -x_1 + rx_2 - x_1 x_3$$

$$\dot{x}_3 = -0.1x_3 + x_1 x_2 \tag{3}$$

where r is the variable parameter. When $-1 < r < 1.1$, the system has three equillibria at

$$S_0(0,0,0), S_1\left(\sqrt{\frac{1+r}{10}}, -\sqrt{\frac{1+r}{10}}, -1-r\right),$$

$$S_2\left(-\sqrt{\frac{1+r}{10}}, \sqrt{\frac{1+r}{10}}, -1-r\right)$$

[24]. Other dynamic properties of the system in (3) as it morphs for four cases of $r \in [0.05, 1]$ were also investigated in [24].

IV. Takagi Sugeno Fuzzy Modelling and Control

Since its inception in [26] the TS fuzzy models have played a dominant role in the modeling of complex nonlinear systems that are deficient of exact descriptions and have mathematically intractable dynamics. The TS fuzzy model has fuzzy sets in its antecedent part and a linear function of the input-output variables or singleton in the consequent part. A simple form of the TS fuzzy model is given as follows:

$$\text{IF } x \text{ is } M_1 \text{ THEN } y = Ax + B \tag{4}$$

where x is the input variable, M_1 is a linguistic variable, y is the output, A and B are constants.

A. Design of the Takagi-Sugeno (T-S) Fuzzy Controllers

Given an autonomous nonlinear dynamic system comprising a plant and a Fuzzy Controller described by

$$\dot{x} = f(x) + b(x)u, x(t_0) = x_0 \tag{5}$$

where $x = [x_1, x_2, \ldots x_n]^T$ is a state vector, $f(x) = [f_1(x), f_2(x) \ldots f_n(x)]^T$ and $b(x) = [b_1(x), b_2(x) \ldots b_3(x)]^T$ are function vectors describing the dynamics of the plant, u is a control signal generated by the FLC. The FLC consists of p rules. The overall control signal applying to the plant is a function of u_i and ξ_i [23] where u_i is the control signal generated by each fuzzy subsystem formed by the fuzzy rules. The i-th fuzzy rule of the Fuzzy Controller is of the following form:

Rule i: IF x_1 is X_{i1} AND is x_2 is X_{i2} AND ...
AND x_n is X_{in}
THEN $u = u_i(x)$ (6)

where X_{i1}, X_{i2} ... X_{in} are input fuzzy labels, and $u = u_i(x)$ is the control output. Moreover, each fuzzy rule therefore generates a degree of fulfillment $\xi_i(x)$ given by:

$$\xi_i = \min(\xi_{i1}, \xi_{i2} \cdots \xi_{in}) \tag{7}$$

$$\xi_i \in [0,1], i = 1, 2, \ldots p$$

Definition [27] A fuzzy subsystem associated with fuzzy rule i is a system with a plant of (5) controlled by only u_i, which is the output of fuzzy rule i in the form of (6). By using the singleton fuzzifier in conjunction with min-max inference and the weighted sum defuzzification method, the overall Fuzzy Controller output control signal is given by [27]

$$U = \frac{\sum_{i=1}^{p} \xi_i(x) u_i(x)}{\sum_{i=1}^{p} \xi_i(x)} \tag{8}$$

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The control objective is to drive the trajectories of the system from their chaotic regimes to some stable points in the sense of Lyapunov.

B. Stabilization Controller Synthesis

The Lyapunov stability criterion [30][31]was employed to analyze the local stability of each fuzzy rule and prove the global asymptotic stability of the Controller using the approach outlined in [23][27] and proved in [28].

Theorem [29] Let $x = 0$ be an equilibrium point of the system in eq (4) and $D \subset R^n$ be a domain containing $x = 0$. Let $V : D \to R$ be a continuously differentiable function such that

$$V(0) = 0 \text{ and } V(x) > 0 \text{ in } D - \{0\} \tag{9}$$

$$\dot{V}(x) \le 0 \quad \text{in } D - \{0\} \tag{10}$$

Then $x = 0$ is stable. Moreover, if

$$\dot{V}(x) < 0 \quad \text{in } D - \{0\} \tag{11}$$

Then $x = 0$ is asymptotically stable.

The positive definite function $V(x)$ satisfying (9) and (10) is called Lyapunov function candidate whose existence is a sufficient condition for stability. The Lyapunov function [28] was chosen:

$$V^j(x_1, x_2, x_3) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \tag{12}$$

$$j = 1, 2...p, p = 9$$

The partial derivative of (12) yields:

$$\dot{V}^j(x_1, x_2, x_3) = x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 \tag{13}$$

Customarily, a control input term u is added to (3) and transformed into the following:

$$\begin{aligned} \dot{x}_1 &= -x_1 - x_2 + u \\ \dot{x}_2 &= -x_1 + rx_2 - x_1x_3 \\ \dot{x}_3 &= -0.1x_3 + x_1x_2 \end{aligned} \tag{14}$$

Inserting (14) into (13) and factorizing yields

$$\begin{aligned} \dot{V}^j(x_1, x_2, x_3) = -x_1^2 - 2x_1x_2 + rx_2^2 + \\ x_1u - x_1x_3 - 0.1x_3^2 \end{aligned} \tag{15}$$

Assuming $x_3 = 0$;

$$\dot{V}^j(x_1, x_2, x_3) = -x_1^2 - 2x_1x_2 + rx_2^2 + x_1u \tag{16}$$

The stability analysis algorithm requires the transformation of (15) into a form in (5) and then solving for *u*.

$$-x_1^2 - 2x_1x_2 + rx_2^2 + x_1u = 0$$

$$-x_1^2 - 2x_1x_2 + rx_2^2 = -x_1u$$

$$u = x_1 + 2x_2 - \frac{rx_2^2}{x_1} \tag{17}$$

After manipulating (17), the control signal is derived as

$$u = x_1 + 2x_2 - \frac{rx_2^2}{x_1} \tag{18}$$

The relationship (18) is the computational relationship for the derivative of control input for each of the subsystems formed by the Fuzzy rules. Setting $u = u_i$, for $i = 1, 2...p$, and beginning with $r = 0.06$, each fuzzy rule was analyzed for local stability. Solving for u and inserting each control output value $u = u_i$ in (15) proves the local stability of each fuzzy subsystem formed by each fuzzy rule. The partial derivative in (13) and the global asymptotic stability of the system in the sense of Lyapunov. Moreover, the aggregated fuzzy controller output signal applied to the chaotic was uniquely small. The fuzzy control scheme, membership function and the rule base are given in Fig. 1 and Fig. 2 and Table I respectively.

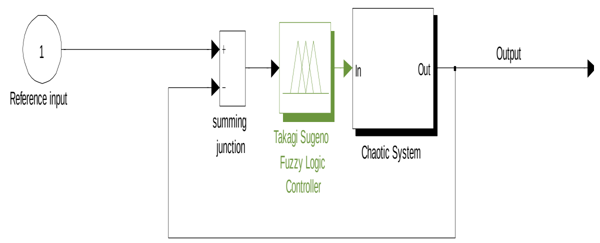

Figure 1. Fuzzy logic control scheme

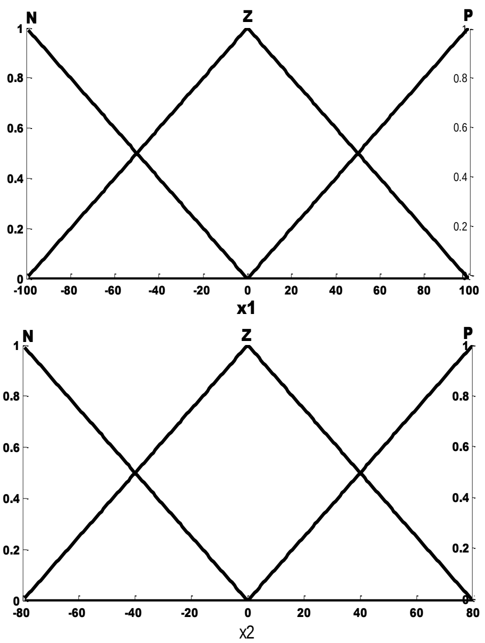

Figure 2. Triangular membership function for x_1, x_2 ;

$x_1 \in [-100, 100]; x_2 \in [-80, 80]; x_3 \in \varnothing$

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TABLE I: FUZZY CONTROLLER RULE BASE

Rule	Antecedent		Consequent
	x_1	x_2	u_i
1	**P**	**P**	$\boldsymbol{u_1}$
2	N	N	u_2
3	P	N	u_3
4	N	P	u_4
5	P	Z	u_5
6	N	Z	u_6
7	Z	P	u_7
8	Z	N	u_8
9	Z	Z	u_9

V. DISCUSSION

Various values of r were simulated for fixed initial conditions. It was observed that the chaotic system is highly sensitive to changes in initial conditions than the classic Lorenz system. Specifically, as the value of $r \in [0.05, 1]$ increases for an initial condition, the trajectories stabilizes faster with less settling time. It was also observed that as the value of r exceeds unity, i.e. $r = 1.5$, the phase portrait of the system in the x-y plane is torus-like and is also stabilizable with the designed Fuzzy Controller. The partial derivative of the Lyapunov function candidate was negative semi-definite for all fuzzy subsystems, $\dot{V}^j(x_1, x_2, x_3) \le 0$ for all $j = 1, 2 \ldots p, p = 9$, consequently, the closed loop system is globally asymptotically stable in the sense of Lyapunov. The simulation results show that the method of designing the Fuzzy Controller is effective and efficient as it can robustly stabilize the systems from several initial conditions and different values of *r*.

VI. SIMULATION RESULTS

A. *Open Loop Simulations*

The open loop system was simulated using MATLAB software for two values of $r = 0.06$ and $r = 0.6$ for the same initial conditions $[x_1(0), x_2(0), x_3(0)] = [10, 60, 50]$. The results are given in the following Figures.

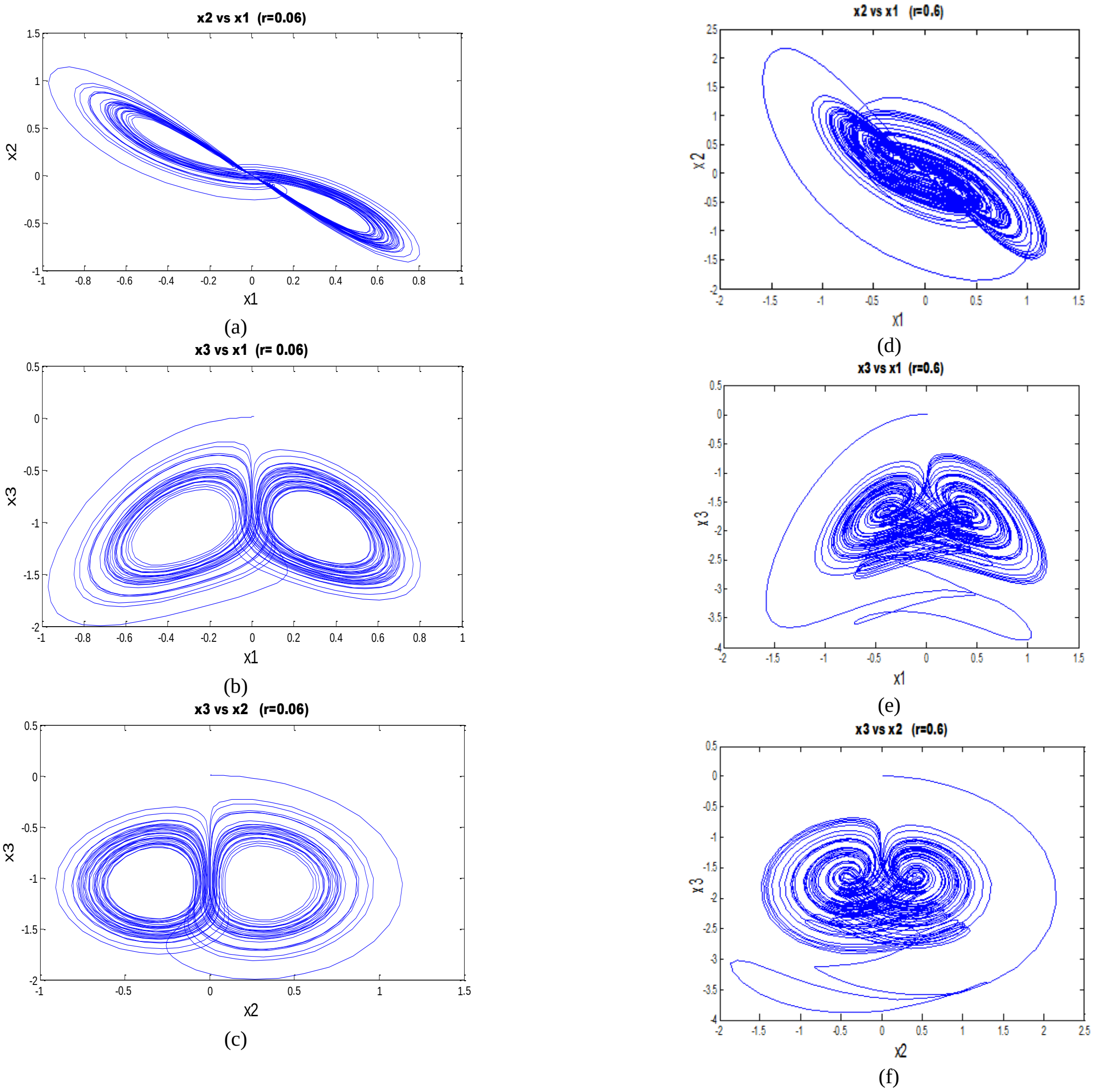

Figure 3. Phase portraits of the one-parameter chaotic system as it morphs from (i) $r = 0.06$ - (a) x_2 - x_1 (b) x_3 - x_1 (c) x_3 - x_2 planes to $r = 0.6$ - (d) x_2 - x_1 (e) x_3 - x_1 (f) x_3 - x_2 planes

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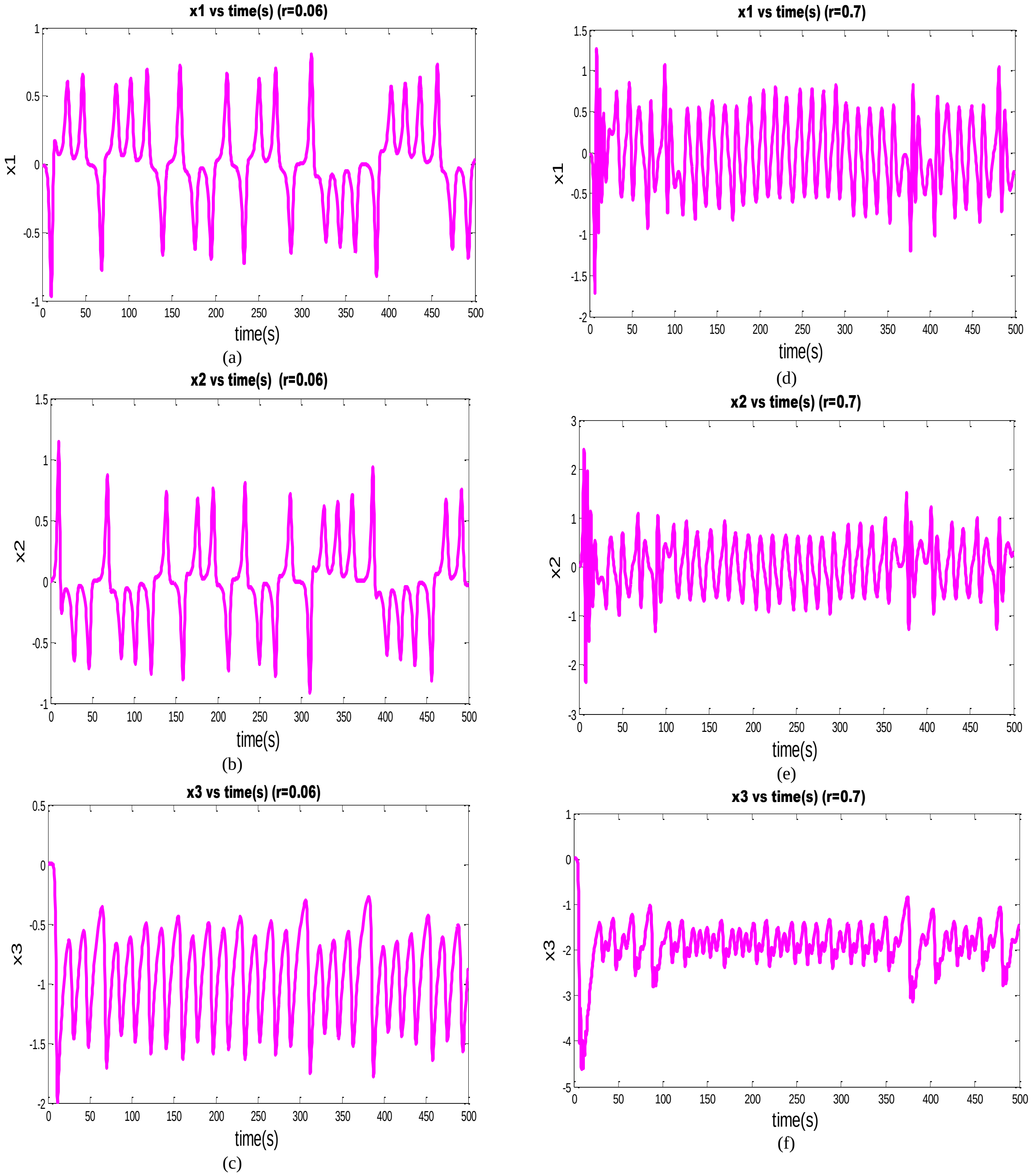
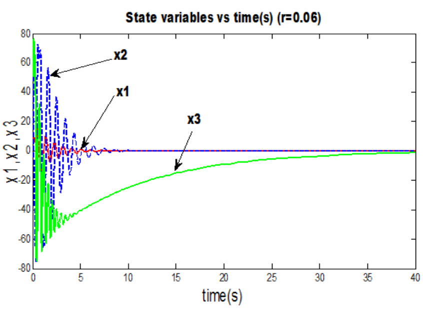


Figure 4: Evolution of system's state trajectories as it morphs from (i) Lorenz-like system (a) (a)$r = 0.06$; (b); $r = 0.1$; (c); $r = 0.33$; (d); $r = 0.7$(e); $r = 0.74$(f); $r = 1.0$ vs times(s) (b) x_2 vs time (s) (c) x_3 vs time (s) to (ii) Chen-like system (d) x_1 vs times(s) (e) x_2 vs time (s) (f) x_3 vs time (s)

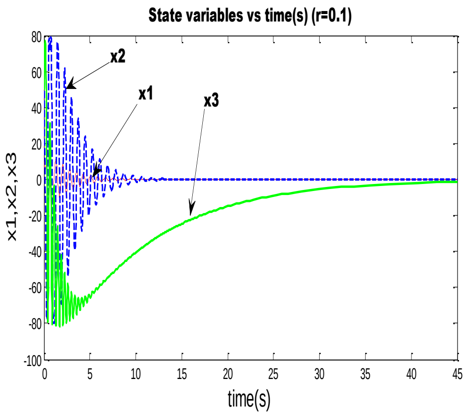
The evolution of the open loop system's trajectories as it morphs from Lorenz-like to Chen's system are given in the following Figures.

B. Closed Loop Simulation

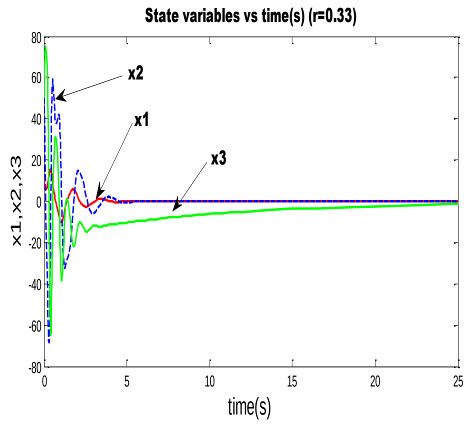
The Closed loop system was simulated using MATLAB software for six values of $r = 0.06, 0.1, 0.33, 0.7, 0.74, 1.0$ FOR the same initial conditions $[x_1(0), x_2(0), x_3(0)] = [10, 60, 50]$. The results are given in the following Figures:



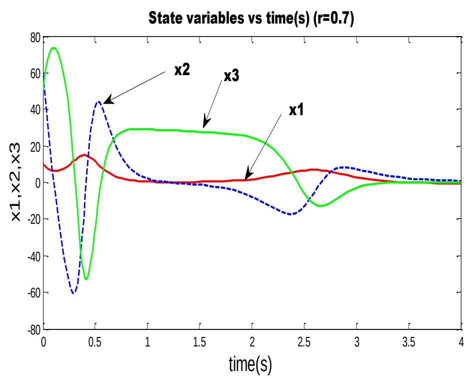
(a)



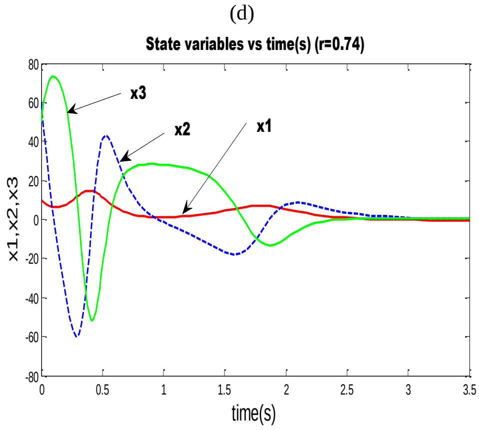
(b)



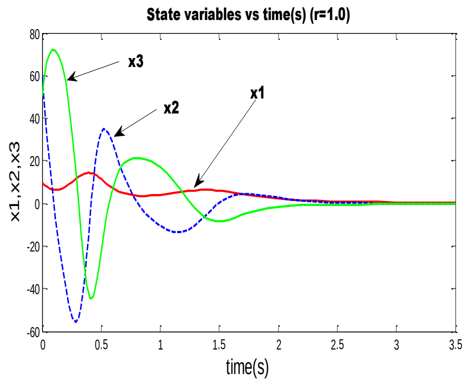
(c)



(d)



(e)



(f)

Figure 5. Stabilized system's state trajectories for various values of (a) $r = 0.06$; (b); $r = 0.1$; (c); $r = 0.33$; (d); $r = 0.7$(e); $r = 0.74$(f); $r = 1.0$

VII. Conclusion

This paper reported the stabilization of a morphous one-parameter chaotic system which is topologically nonequivalent to the Lorenz and Chen's system but nevertheless morphs from Lorenz-like system to the Chen's system as the parameter r is increased between 0.05 and 1.0. The Fuzzy Controller asymptotically stabilizes the highly aperiodic trajectories of all the fuzzy subsystems in the sense of Lyapunov, with a uniquely small control effort.

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