Design Adaptive Fuzzy Sliding Mode Controller for Pantograph Mechanism Apply to Massage Therapy Robot for Healthcare

Phan Thanh Phuc[1], Nguyen Dao Xuan Hai[2], and Nguyen Truong Thinh[1]
[1]Ho Chi Minh City University of Technology and Education, Ho Chi Minh City, Vietnam
[2]Open Laboratory, Department of Mechatronic Engineering, Ho Chi Minh City, Vietnam
Email: phanthanhphuc@gmail.com, ndxuanhai@gmail.com, thinhnt@hcmute.edu.vn

***Abstract*—This paper proposes an adaptive fuzzy sliding mode controller (AFSMC) for pantograph haptic device. It has form mechanism with a 2-DoF redundancy actuated parallel robotic manipulator. An adaptive single input single output (SISO) fuzzy system is applied to calculate each element of the control gain vector in a sliding mode controller for purport to reject phenomenon chattering by the constant value of K and discontinuous function. The adaptive laws are designed based on the Lyapunov stability theory method. The adaptive laws are practiced online by fuzzy controller combine with the sliding mode control (SMC) to control stability the robot manipulator despite external forces disturbance. Many operation situations such as set point control also the trajectory control are simulated also experiment to demonstrate the operation controllers are good working.**

***Index Terms*—sliding mode control, lyapunov stability, adaptive fuzzy control, chattering**

I. Introduction

Over past decades sliding mode control [1]-[4] has become the most popular technique for control of nonlinear systems, especially because of simplicity of the control law, easy implementation and high robustness. Unfortunately, when used with fixed parameter SMC has several drawbacks [5]. The most important are chattering of control input so leading to high moving of mechanical parts and heat losses in electrical power circuits [1] slow convergence and nonzero steady state error. The usual way how to decrease the chattering phenomenon consists in introduction of boundary layer [4]. However attenuation of chattering in this case decreases control performance. To avoid effect different adaptive mechanisms by on-line tuning of SMC surface has been introduced in past decades.

One of the most often used adaptation mechanisms is based on fuzzy logic approach [6], [7]. Different approaches to adaptive fuzzy control has achieved good result for many experimental application [8], [9]. Popularity in last few years has gained especial the self-learning mechanisms based on fuzzy logic since only a very rough model of the controlled plant is sufficient for successful control [10, 11].

In the recent years, AFSMC methods have enjoyed popularity which is supported by many successful applications, automotive industry [12, 13], manipulators[14, 15] or vomechanisms [16, 17].

In [18, 19], fuzzy systems are used to implement the system dynamics as well as the control gain. Both theoretic studies and simulation results have demonstrated that this type of controllers eliminates the chattering on the sliding surface. Sun *et al.* [20] applied a fuzzy system to approximate the system dynamic. The discontinuous term $sgn(s)$ still exists in the control input. Xu *et al*. and Gao *et al*.[15, 21] applied Tagaki - Sugeno type fuzzy systems to estimate the system dynamic. In cases, the first of type fuzzy sliding mode controllers, it is assumed that model the robotic manipulator is totally unknown.

In the second type of fuzzy sliding mode controllers, it is assumed that the model robotic manipulator is known. Tsay *et al.* [22] propose that the control gain is the product of the inverse of the inertial matrix and gain vector. Each element of the gain vector is decided by an individual fuzzy system based on the value of the sliding surface $\boldsymbol{s}_i$ and its variation. Chen *et al.* [23] estimate the control gain by using Tagaki – Sugeno type fuzzy systems. Therefore, it is same problem [21] exists. Choi *et al.* [24] regard the control gain as individual vector that is computed by fuzzy systems. Bekit *et al*.[25] propose that control gain $\boldsymbol{k}$ is an n-order vector. Each element k_i is decided by a fuzzy system with two inputs, the sliding surface $\boldsymbol{s}_i$ and its variation $\dot{\boldsymbol{s}}_i$. Since the fuzzy systems are non-adaptive, more system information is required to decide the membership functions of the fuzzy system. Both [24, 25] there is no theoretical proof of the stability and convergence of proposed controllers.

This is complex system and uncertainly so the paper focuses the fuzzy sliding mode controller since the model information is exactly unknown with the analysis of physical properties of the robotic manipulator. In the proposed control scheme, the control gain $\boldsymbol{k}$ is considered

Manuscript received March 15, 2019; accepted June 10, 2019.

doi: 10.18178/joace.7.1.13-23

as an individual vector. But, $\boldsymbol{k}$ is adjusted way an adaptive SISO fuzzy systems which requires less information of the robotic manipulator and therefore simplifies the implementation of controller. Moreover, a theoretical proof of the stability and the convergence of the proposed scheme by the Lyapunov method is provided. After, we compare simulation results with SMC. The paper is organized as follows dynamic modeling is summarized in Section II. In Section III and section IV, the sliding mode controller (SMC) and the adaptive fuzzy sliding mode controller (AFSMC) are presented. Section V show many results of simulation to demonstrate that the controllers are good operation. The results of actual system experience control are showed in section VI. Section VII concludes the paper.

II. Description of Robotic Manipulator Model

The experiment has a 2-DoF redundantly actuated parallel manipulator and device consists of 5 links with length l_i for $i = 1,...,5$. As shown in Fig. 1, a reference frame is established in the workspace of the parallel manipulator. The unit of the frame is a meter. The parallel manipulator is actuated by two DC servo motors located at the bases **A1** and **A5**, and the end – effector is mounted at the common joint **A3**. Coordinates of the two bases are **A1** (0 0), **A5** (0.2 0). The length of links as follows: $l_0 = 0.2$, $l_1 = l_4 = 0.26$ and $l_2 = l_3 = 0.43$. The definitions of the joint angles are shown in Fig. 1, q_1, q_4 refer to the active joint angles and q_2, q_3 refer to the positive joint angles.

The equation of motion of the pantograph haptic device following form [26, 27]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = T + T_h \tag{1}$$

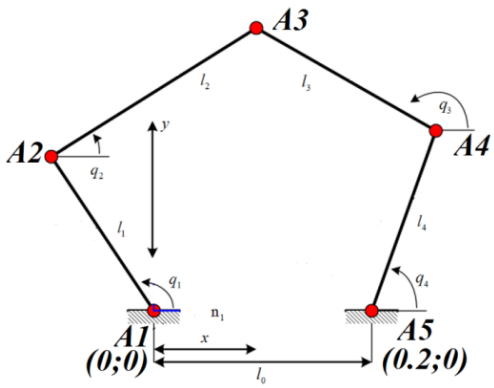

Figure 1. Coordinate of the 2-DoF redundantly actuated parallel manipulator.

III. Sliding Mode Controller

The control objective is to drive the joint position q to the desired position q_d . Define the tracking error

$$e = q - q_d \tag{2}$$

Define the sliding surface

$$s = \dot{e} + \lambda e \tag{3}$$

where $\lambda = diag[\lambda_1,..,\lambda_i,...\lambda_n]$ in which λ_i is a positive constant. The control objective can be achieved by choosing the control input $\boldsymbol{\tau}$ so that the sliding surface satisfies the sufficient condition [4]

$$\frac{1}{2}\frac{d}{dt}s_i^2 \le -\eta_i \,|\, s_i \,| \tag{4}$$

where " η_i " is positive constant. Above equation indicates that the energy of $\boldsymbol{s}$ should decay as long as $\boldsymbol{s}$ is not zero. To achieved control $\boldsymbol{\tau}$, define the reference state.

$$\begin{aligned} \dot{q}_r &= \dot{q} - s = \dot{q}_d - \lambda e \\ \ddot{q}_r &= \ddot{q} - \dot{s} = \ddot{q}_d - \lambda \dot{e} \end{aligned} \tag{5}$$

Choose the control $\boldsymbol{\tau}$

$$\boldsymbol{\tau} = \hat{\boldsymbol{\tau}} - \boldsymbol{K}\,\mathrm{sgn}(s)$$

$$\hat{\boldsymbol{\tau}} = \hat{\boldsymbol{M}}\ddot{q}_r + \hat{\boldsymbol{C}}\dot{q}_r - T_h - \boldsymbol{A}\boldsymbol{s} \tag{6}$$

where $\hat{\boldsymbol{M}}$,$\hat{\boldsymbol{C}}$ are the estimations of $\boldsymbol{M}$, $\boldsymbol{C}$ respectively, $K = diag[K_{11},...,K_{ii},...,K_{nn}]$ is a diagonal positive define matrix in which K_{ii} is a positive constant and $A = diag[a_1,...,a_i,...,a_n]$ is also a diagonal positive define matrix in which a_i is a positive constant.

Putting (6) to (1) lead to

$$\boldsymbol{M}\dot{\boldsymbol{s}} + (\boldsymbol{C} + \boldsymbol{A})\boldsymbol{s} = \Delta\boldsymbol{f} - \boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s}) \tag{7}$$

where $\Delta\boldsymbol{f} = \Delta\boldsymbol{M}\ddot{q}_r + \Delta\boldsymbol{C}\dot{q}_r$, $\Delta\boldsymbol{M} = \hat{\boldsymbol{M}} - \boldsymbol{M}$, $\Delta\boldsymbol{C} = \hat{\boldsymbol{C}} - \boldsymbol{C}$. Assuming $|\,\Delta\boldsymbol{f}_i\,| < |\,\Delta\boldsymbol{f}_i\,|_{bound}$, where $|\,\Delta\boldsymbol{f}_i\,|_{bound}$,is the boundary of $|\,\Delta\boldsymbol{f}_i\,|$, choose $\boldsymbol{K}$ such that

$$\boldsymbol{K} \ge |\,\Delta\boldsymbol{f}_i\,|_{bound} \tag{8}$$

To prove the stability of the system, choose the Lyapunov function candidate to be

$$V = \frac{1}{2}s^T\boldsymbol{M}\boldsymbol{s} \tag{9}$$

Since $\boldsymbol{M}$ is symmetric and positive define, then for $s \ne 0$

$$V > 0 \tag{10}$$

It can be proved that

$$\dot{V} = s^T[-(\boldsymbol{C} + A)s + \Delta\boldsymbol{f} - \boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s}) + \boldsymbol{C}\boldsymbol{s}]$$

$$= \sum_{i=1}^{n}(\boldsymbol{s}_i[\Delta\boldsymbol{f}_i - \boldsymbol{K}_{ii}\,\mathrm{sgn}(\boldsymbol{s}_i)]) - s^T\boldsymbol{A}\boldsymbol{s}$$

When $\boldsymbol{s}_i > 0$, from (8)

$$\Delta \boldsymbol{f}_i - \boldsymbol{K}_{ii}\,\mathrm{sgn}(\boldsymbol{s}_i) = \Delta \boldsymbol{f}_i + \boldsymbol{K}_{ii} \le 0$$

So that

$$\boldsymbol{s}_i[\Delta \boldsymbol{f}_i - \boldsymbol{K}_{ii}\,\mathrm{sgn}(\boldsymbol{s}_i)] \le 0 .$$

When $\boldsymbol{s}_i < 0$, from (8)

$$\Delta \boldsymbol{f}_i - \boldsymbol{K}_{ii}\,\mathrm{sgn}(\boldsymbol{s}_i) = \Delta \boldsymbol{f}_i - \boldsymbol{K}_{ii} \ge 0$$

So that

$$\boldsymbol{s}_i[\Delta \boldsymbol{f}_i - \boldsymbol{K}_{ii}\,\mathrm{sgn}(\boldsymbol{s}_i)] \le 0$$

Thus

$$\sum_{i=1}^{n} (\boldsymbol{s}_i[\Delta \boldsymbol{f}_i - \boldsymbol{K}_{ii}\,\mathrm{sgn}(\boldsymbol{s}_i)]) \le 0 .$$

Since $\boldsymbol{A}$ is a positive definite matrix, $-s^T \boldsymbol{A}\boldsymbol{s} \le 0$. With these results, it can be proved that

$$\dot{V} = \sum_{i=1}^{n} (\boldsymbol{s}_i[\Delta \boldsymbol{f}_i - \boldsymbol{K}_{ii}\,\mathrm{sgn}(\boldsymbol{s}_i)]) - s^T \boldsymbol{A}\boldsymbol{s} \le -s^T \boldsymbol{A}\boldsymbol{s} \le 0 \tag{11}$$

Equation (9) can be considered as an indicator of energy of $\boldsymbol{s}$. Thus, (11) guarantees the decay of the energy of $\boldsymbol{s}$ as long as $\boldsymbol{s} \ne \boldsymbol{0}$. The sufficient condition in (4) is thus satisfied.

where:

Desire position block *is coordinate desire of end-effector.*

Inverse Kinematic block *is calculated from coordinate of* ***A3*** *deduced angles of* $q_1\, q_2\, q_3\, q_4$.

Sliding surface block *is calculated as formula* (5).

$\hat{\boldsymbol{\tau}}$ ***block*** *is calculated as formula* (6).

Estimate block *to estimate approximate system robotic manipulator.*

Robotic Manipulator *block is plant of robotic follow formula* (1).

Sign S block *is Sgn function of Sliding Surface.*

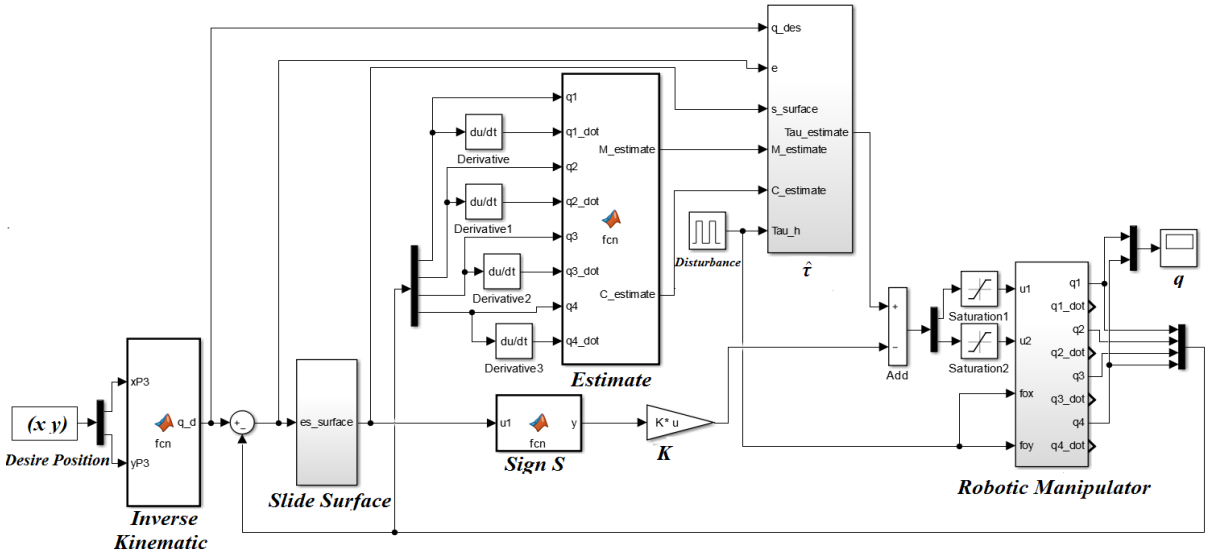

Figure 2. Diagram of SMC.

IV. Adaptive Fuzzy Sliding Mode Controller

Since sliding mode control occurs chattering on the sliding surface in the classical sliding mode control. In other words, although the system is stable, there is no guarantee that $\lim_{t\to\infty} s = 0$. The chattering is caused by the constant value of $\boldsymbol{K}$ and discontinuous function $sgn(s)$ by and saturation function $sat(s/\phi)$. In this section, a fuzzy control gain $\boldsymbol{k}$ is applied to construct an adaptive fuzzy sliding mode controller.

Normal, a fuzzy system has one or more inputs and a single output. A multiple-output system can be considered by combine of more one single output systems [4].

There are four basic parts in a fuzzy system. The fuzzification and defuzzification are the interface between the fuzzy systems and crisp systems. The rule base includes a set of "if…then…" rules based on the human experience. Each rule describes a relation between the input space and the output space. For each rule, the inference engine maps the input fuzzy sets to an output fuzzy set according to relation define by the rule. It then combines the fuzzy sets from all the rules in the rule base into the output fuzzy set. This output fuzzy set is translated to a crisp value output y by the defuzzication.

All the four parts can be mathematically formulated. In this paper, by choosing singleton fuzzification, center average defuzzification, Mamdani implication in the rule base and product inference engine, the output of the fuzzy system can be written as

$$y=\frac{\sum_{m=1}^{M}\theta^m\prod_{i=1}^{n}\mu_{A_i^m}(x_i^*)}{\sum_{m=1}^{M}\prod_{i=1}^{n}\mu_{A_i^m}(x_i^*)}=\boldsymbol{\theta}^T\Psi(x) \tag{12}$$

where $\boldsymbol{\theta}=\left[\theta^1,...,\theta^m,...\theta^M\right]^T$ is the vector of the centers of the membership functions of y, $\Psi(x)=\left[\Psi^1(x),...,\Psi^m(x),...,\Psi^M(x)\right]^T$ is the vector of the height of the membership function of y in which $\Psi^m(x)=\prod_{i=1}^{n}\mu_{A_i^m}(x_i^*)/\sum_{m=1}^{M}\prod_{i=1}^{n}\mu_{A_i^m}(x_i^*)$, and M is the amount of the rules.

A. Apply Fuzzy Systems to Sliding Mode Control

Rewrite (1)

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}=T+T_h \tag{13}$$

As reminded above, the chattering is caused by constant value of $\boldsymbol{K}$ and discontinuous function $sgn(s)$, let the control gain $\boldsymbol{K}\,\mathrm{sgn}(\boldsymbol{s})$ be replaced by a fuzzy gain $\boldsymbol{k}$. The new control input is then written as follows

$$\boldsymbol{\tau}=\hat{\boldsymbol{M}}\ddot{q}_r+\hat{\boldsymbol{C}}\dot{q}_r-T_h-\boldsymbol{As}-\boldsymbol{k} \tag{14}$$

where $k=[k_1,...,k_i,...,k_n]^T$ and each k_i is estimated by an individual fuzzy system.

The rule base: to decide the rules for the fuzzy systems, consider (9)

$$V=\frac{1}{2}s^T\boldsymbol{M}s$$

Here, V is regarded as an indicator of energy of $\boldsymbol{s}$. The stability of the system is guaranteed by choosing a control law such that $\dot{V}\le 0$ and $\dot{V}=0$ only when $\boldsymbol{s}=0$. In the adaptive fuzzy sliding mode control, a fuzzy $\boldsymbol{k}$ is applied to compensate the system uncertainty and reduce the energy of $\boldsymbol{s}$. In this case, (11) can be rewritten as

$$\dot{V}=\sum_{i=1}^{n}(\boldsymbol{s}_i[\Delta\boldsymbol{f}_i-k_i])-s^TAs \tag{15}$$

Because of the function $\mathrm{sgn}(\boldsymbol{s}_i)$ in (11), the control gain has the same sign as $\boldsymbol{s}$. Therefore, k_i should have the same sign as $\boldsymbol{s}_i$. Next, consider the term $(\boldsymbol{s}_i[\Delta\boldsymbol{f}_i-k_i])$ in (15). When $|\boldsymbol{s}_i|$ is large, it is expected that $|k_i|$ is larger so that $\dot{V}$ has a large negative value. In other word, the energy of $\boldsymbol{s}$ decays fast. When $|\boldsymbol{s}_i|$ is very small, $(\boldsymbol{s}_i[\Delta\boldsymbol{f}_i-k_i])$ and has a little effect on the value of $\dot{V}$. Then small $|k_i|$ is allowed to avoid chattering. When $\boldsymbol{s}_i$ is zero anyway and therefor k_i can be zero. From these analyses, some clues for rule base can be obtained: when $\boldsymbol{s}_i$ is large, k_i is large; when $\boldsymbol{s}_i$ is small, k_i is small, k_i can be small as long as $|k_i|>|\Delta\boldsymbol{f}_i|$; when $\boldsymbol{s}_i$ is zero, k_i is can be zero. This idea is similar to that of applying function $sat(.)$. In addition, an adaptive law is designed to guarantee that k_i can be compensated the system uncertainty. These analyses indicate that the value of k_i can be decided by the value of the sliding surface $\boldsymbol{s}_i$. Thus, the fuzzy system for k_i should be a SISO system, with $\boldsymbol{s}_i$ as the input and k_i as the output.

The rules in the rule base are in the following format.

If $\boldsymbol{s}_i$ is A_i^m,

Then k_i is B_i^m

where A_i^m and B_i^m are fuzzy sets. In this paper, it is chosen that both $\boldsymbol{s}_i$ and k_i have the same kind of membership functions: NB, NM, NS, ZE, PS, PM, PB, where N stands for negative, P postive, B big, M medium, S small and ZE zero. They are all Gaussian membership functions defined as

$$\mu_A(x_i)=\exp\left[-\left(\frac{x_i-\alpha}{\sigma}\right)^2\right]$$

where "A" represents one of the fuzzy sets NB,...,PB and x_i represents $\boldsymbol{s}_i$ or k_i. α is center of "A" and σ is the width the same titles, correspondingly, the values of the center and the width of the membership function withh a same title for $\boldsymbol{s}_i$ and k_i are different, respectively. The parameters of the membership functions of $\boldsymbol{s}_i$ are pre-defined, while those of k_i are updated on-line. Therefore, the controller is an adaptive controller.

Based on the above discussions and definitions of the input and output membership functions, the rule base can be decided as follows:

IF $\boldsymbol{s}_i$ is NB, THEN k_i is NB

IF $\boldsymbol{s}_i$ is NM, THEN k_i is NM

IF $\boldsymbol{s}_i$ is NS, THEN k_i is NS

IF $\boldsymbol{s}_i$ is ZE, THEN k_i is ZE

IF $\boldsymbol{s}_i$ is PS, THEN k_i is PS

IF $\boldsymbol{s}_i$ is PM, THEN k_i is PM

IF $\boldsymbol{s}_i$ is PB, THEN k_i is PB (16)

From the knowledge of the fuzzy systems, k_i can be written as

$$k_i = \frac{\sum_{m=1}^{M} \theta_{ki}^m \mu_A^m(\boldsymbol{s}_i)}{\sum_{m=1}^{M} \mu_A^m(\boldsymbol{s}_i)} = \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i) \tag{17}$$

Where $\theta_{ki} = [\theta_{ki}^1, ..., \theta_{ki}^m, ..., \theta_{ki}^M]^T$, $\Psi_{ki}(\boldsymbol{s}_i) = [\Psi_k^1(\boldsymbol{s}_i), ..., \Psi_{ki}^m(\boldsymbol{s}_i), ..., \Psi_{ki}^M(\boldsymbol{s}_i)]^T$ and $\Psi_{ki}^m(\boldsymbol{s}_i) = \mu_A^m(\boldsymbol{s}_i) / \sum_{m=1}^{M} \mu_A^m(\boldsymbol{s}_i)$. θ_{ki} is chosen as the parameter vector. $\Psi_{ki}(\boldsymbol{s}_i)$ is called the function basis vector and can be regarded as the weight of the parameter vector.

Adaptive fuzzy Sliding Mode Control: Putting (14) into (13) lead to

$$\boldsymbol{M}\dot{\boldsymbol{s}} = (\boldsymbol{C} + \boldsymbol{A})\boldsymbol{s} + \Delta\boldsymbol{f} - \boldsymbol{k} \tag{18}$$

where the definition of $\Delta\boldsymbol{f}$ is the same as that in section above and k_i is as in (17)

Define θ_{kid} so that $k_i = \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)$ is the optimal compensation for $\Delta\boldsymbol{f}_i$. According to [28], there exists $\omega_i > 0$ satisfying

$$\left|\Delta\boldsymbol{f}_i - \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)\right| \le \omega_i \tag{19}$$

where ω_i can be as small as possible. Define

$$\tilde{\theta}_{ki} = \theta_{ki} - \theta_{kid} \tag{20}$$

Then

$$k_i = \tilde{\theta}_{k_i}{}^T \Psi_{k_i}(\boldsymbol{s}_i) + \theta_{k_i}{}^T \Psi_{k_i}(\boldsymbol{s}_i) \tag{21}$$

Choose the adaptive law as

$$\dot{\tilde{\theta}}_{k_i} = s_i \Psi_{k_i}(\boldsymbol{s}_i) \tag{22}$$

Choose a Lyapunov function candidate as

$$V = \frac{1}{2} s^T \boldsymbol{M} s + \frac{1}{2} \sum_{i=1}^{n} (\tilde{\theta}_{ki}{}^T \tilde{\theta}_{ki}) \tag{23}$$

where $\boldsymbol{M}$ is positive symmetric matrix and $\tilde{\theta}_{ki}{}^T \tilde{\theta}_{ki} > 0$ and therefore V is positive the derivation of V

$$\dot{V} = \frac{1}{2}[\dot{s}^T \boldsymbol{M} s + s^T \dot{\boldsymbol{M}} s + s^T \boldsymbol{M} \dot{s}] + \frac{1}{2} \sum_{i=1}^{n} \dot{\tilde{\theta}}_{ki}{}^T \tilde{\theta}_{ki} + \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

$$= \frac{1}{2}[s^T \dot{\boldsymbol{M}} s + 2 s^T \boldsymbol{M} \dot{s}] + \frac{1}{2} \sum_{i=1}^{n} 2 \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

$$= s^T [\dot{\boldsymbol{M}} s + C s] + \sum_{i=1}^{n} \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

$$= s^T [-(\boldsymbol{C} + A) s + \Delta\boldsymbol{f} - \boldsymbol{k} + \boldsymbol{C}\boldsymbol{s}] + \sum_{i=1}^{n} \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

$$= s^T [-A s + \Delta\boldsymbol{f} - k] + \sum_{i=1}^{n} \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

$$= -s^T A s + s^T [\Delta\boldsymbol{f} - k] + \sum_{i=1}^{n} \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

$$= -s^T A s + \sum_{i=1}^{n} (\boldsymbol{s}_i [\Delta\boldsymbol{f} - k]) + \sum_{i=1}^{n} \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

Since $k_i = \tilde{\theta}_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i) + \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)$ then

$$\dot{V} = -s^T A s + \sum_{i=1}^{n} (\boldsymbol{s}_i [\Delta\boldsymbol{f} - (\tilde{\theta}_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i) + \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i))])$$
$$+ \sum_{i=1}^{n} \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

$$= -s^T A s + \sum_{i=1}^{n} \boldsymbol{s}_i [\Delta\boldsymbol{f} - \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)] +$$
$$\sum_{i=1}^{n} (-\boldsymbol{s}_i \tilde{\theta}_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)) + \sum_{i=1}^{n} \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}$$

$$= -s^T A s + \sum_{i=1}^{n} \boldsymbol{s}_i [\Delta\boldsymbol{f} - \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)] +$$
$$\sum_{i=1}^{n} [-\boldsymbol{s}_i \tilde{\theta}_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i) + \tilde{\theta}_{ki}{}^T \dot{\tilde{\theta}}_{ki}]$$

$$= -s^T A s + \sum_{i=1}^{n} \boldsymbol{s}_i [\Delta\boldsymbol{f} - \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)] +$$
$$\sum_{i=1}^{n} \tilde{\theta}_{ki}{}^T [-\boldsymbol{s}_i \Psi_{ki}(\boldsymbol{s}_i) + \dot{\tilde{\theta}}_{ki}]$$

Since the adaptive law in (22) is chosen as

$$\dot{\tilde{\theta}}_{ki} = \boldsymbol{s}_i \Psi_{ki}(\boldsymbol{s}_i)$$

Then

$$\dot{V} = -s^T A s + \sum_{i=1}^{n} \boldsymbol{s}_i [\Delta\boldsymbol{f}_i - \theta_{kid}{}^T \Psi_{ki}(\boldsymbol{s}_i)] \tag{24}$$

From (19), there exits

$$\left|\Delta\boldsymbol{f}_i - \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)\right| \le \omega_i$$

And ω_i can be as small as possible. Assume

$$\left|\Delta\boldsymbol{f}_i - \theta_{ki}{}^T \Psi_{ki}(\boldsymbol{s}_i)\right| \le \omega_i \le \gamma_i \left|\boldsymbol{s}_i\right|$$

where $0 < \gamma_i < 1$

Then the second term at right side of (24) satisfies

$$s_i\left[\Delta \boldsymbol{f}_i - \theta_{ki}{}^T \Psi_{ki}(s_i)\right] \le \omega_i \left|s_i\right|^2 = \gamma_i s_i{}^2 \quad (25)$$

Therefore

$$\dot{V} \le -s^T A s + \sum_{i=1}^{n} \gamma_i s_i{}^2 \quad (26)$$

The right side of (24) can be written as

$$\sum_{i=1}^{n} (-a_i s_i{}^2 + \gamma_i s_i{}^2) = -s^T (A-\gamma) s$$

where $\gamma = diag[\gamma_1, ..., \gamma_i, ..., \gamma_n]$. Simply choose $a_i > \gamma_i$ so that $(A-\gamma)$ is a positive definite matrix, therefore $-s^T (A-\gamma) s \le 0$ and then

$$\dot{V} \le \sum_{i=1}^{n} (-a_i s_i{}^2 + \gamma_i s_i{}^2) = -s^T (A-\gamma) s \le 0 \quad (27)$$

In (27), since $(A-\gamma)$ is positive definite matrix, $\dot{V} = 0$ only when $s = 0$. Thus, the overall system with the adaptive law in (22) is asymptotically stable with respect to s. In other words

$$\lim_{t\to\infty} s = \lim_{x\to\infty} (\dot{e} + \lambda e) = 0 \quad (28)$$

Or equivalently

$$\lim_{t\to\infty} q = q_d \text{ and } \lim_{t\to\infty} \dot{q} = \dot{q}_d \quad (29)$$

Therefore, it is proved that, with the adaptive fuzzy sliding mode control input (14), the actual joint position converge to desired.

where

Similar, in the diagram of SMC (Fig. 2) the **Sign S block** is replaced by **Fuzzy system block** which is calculated by (16). AFSMC is indicated in Fig. 3.

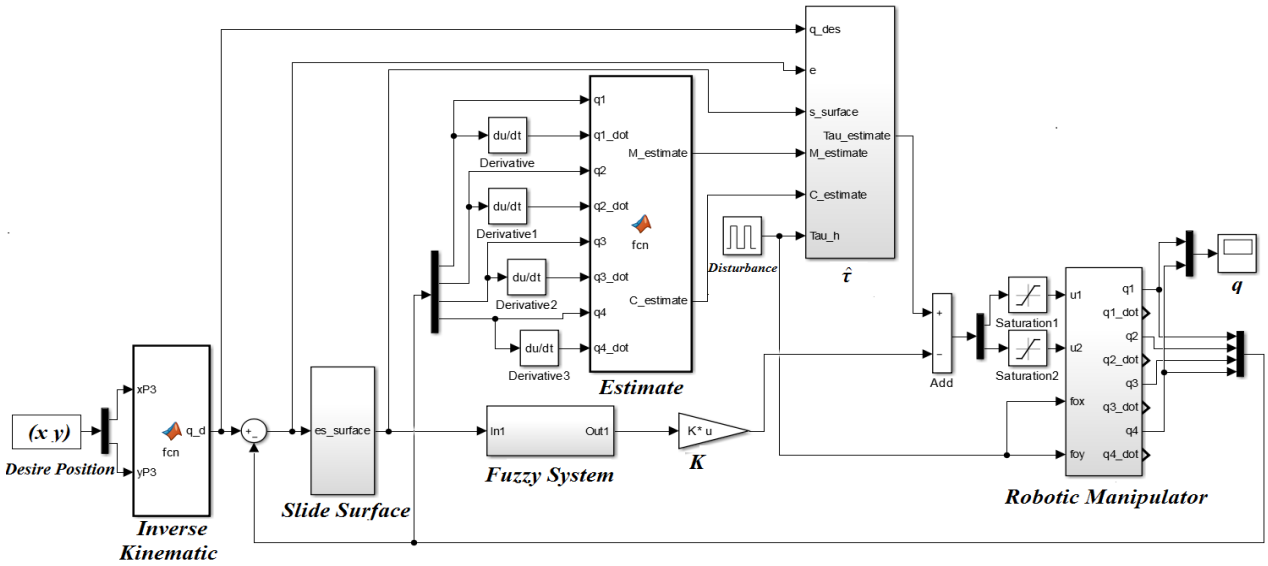

Figure 3. Diagram of AFSMC.

V. Simulation Results

The simulation is practiced with parameters in section II with parameters as Table I.

TABLE I. Prameters of Robottic Manipulator

Link	Length of link *(m)*	Mass of link *(kg)*	Initial Angle *(rad)*
1	0.26	0.2	pi
2	0.43	0.4	pi/4
3	0.43	0.4	3*pi/4
4	0.26	0.2	0

Disturbance is used by pulse function with amplitude 0.006, period = 10s and pulse width = 50% of period.

$$\lambda = diag[1 \quad 1 \quad 1 \quad 1], \; \boldsymbol{A} = diag[1 \quad 1 \quad 1 \quad 1],$$
$$\boldsymbol{K} = diag[1 \quad 1 \quad 1 \quad 1]$$

The purposes are control end – effector (**A3**) go to position set point (0.25 0.25) in SMC and AFSMC is applied to control plant despite impact forces external disturbance to plant to demonstrate the controller well working.

SMC without disturbance amplitude 0.006 and coordinate desire position (0.25 0.25).

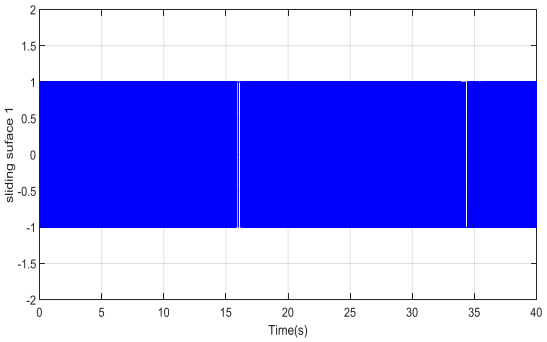

Figure 4. Sliding surface of joint 1 in SMC for set point without disturbance.

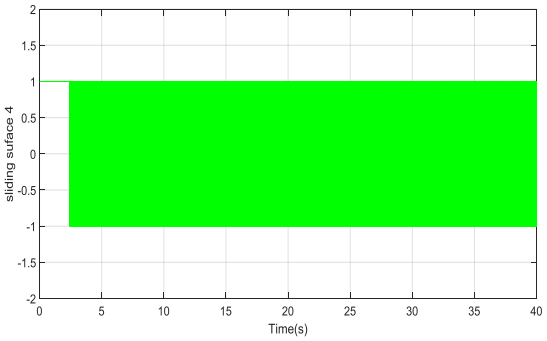

Figure 5. Sliding surface of joint 4 in SMC for set point without disturbance.

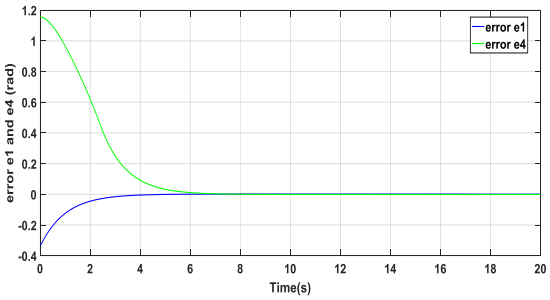

Figure 6. Tracking errors e_1 and e_4 in SMC.

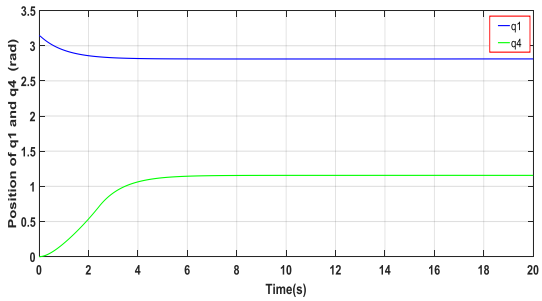

Figure 7. Response of angles q_1 and q_4 in SMC.

SMC with disturbance amplitude 0.006 and coordinate desire position (0.25 0.25).

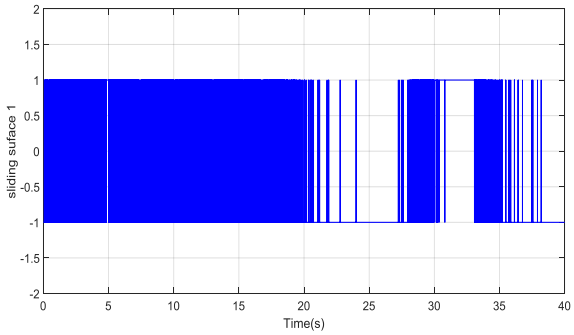

Figure 8. Sliding surface of joint 1 in SMC for set point with disturbance.

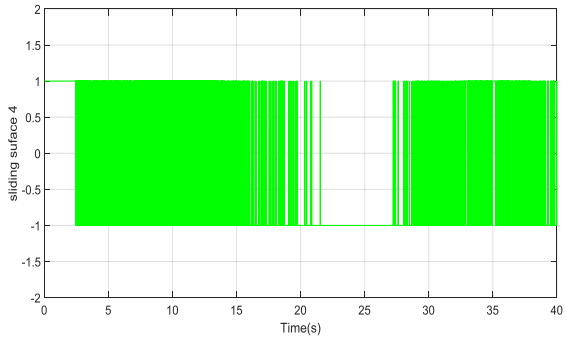

Figure 9. Sliding surface of joint 4 in SMC for set point with disturbance.

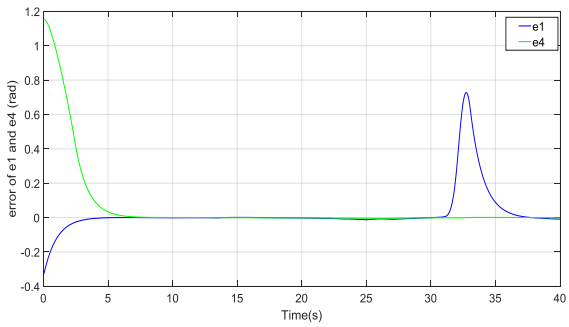

Figure 10. Tracking errors e_1 and e_4 in SMC.

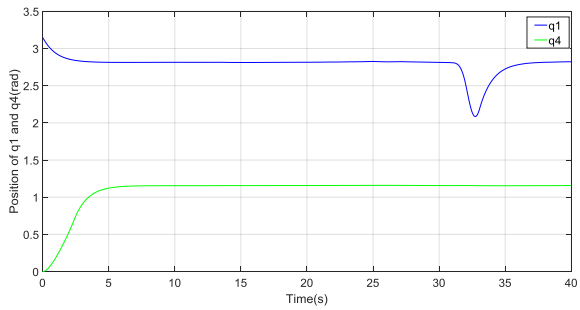

Figure 11. Position of angles q_1 and q_4 in SMC for set point with disturbance.

AFSMC without disturbance with coordinate desire position (0.25 0.25).

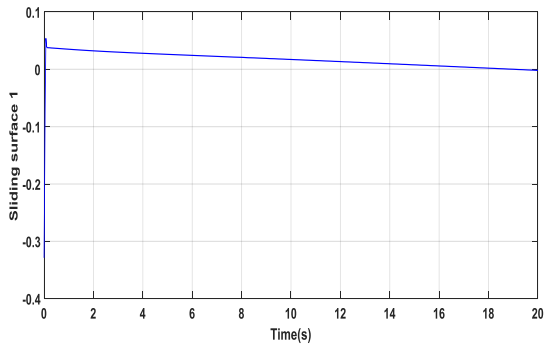

Figure 12. Control gain k_1 in AFSMC for set point without disturbance.

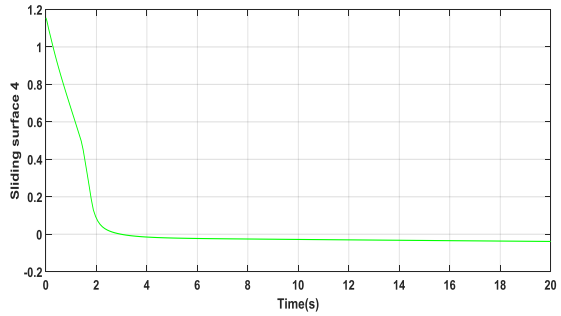

Figure 13. Control gain k_4 in AFSMC for set point without disturbance.

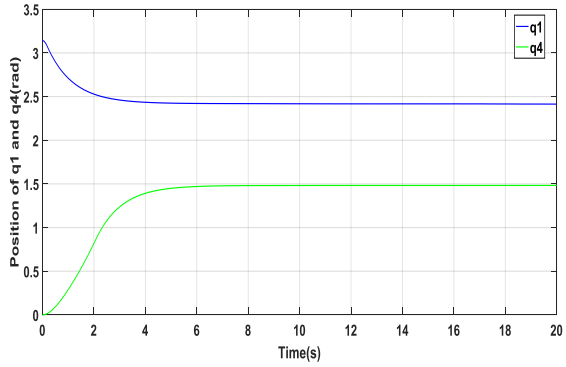

Figure 14. Response of angles q_1 and q_4 in AFSMC for set point without disturbance.

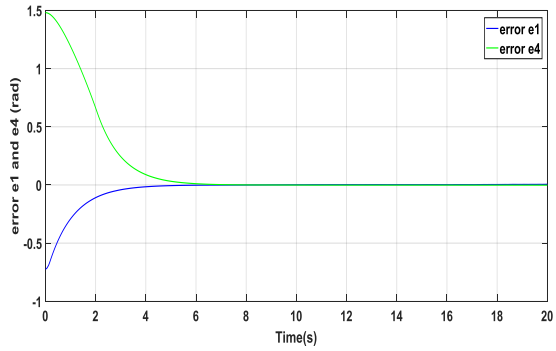

Figure 15. Tracking errors e_1 and e_4 in AFSMC for set point without disturbance.

AFSMC with disturbance amplitude 0.006 and coordinate desire position (0.25 0.25)

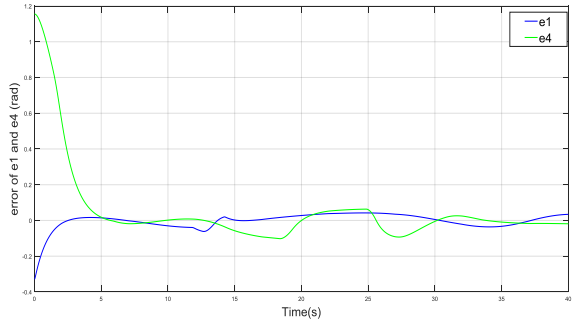

Figure 16. Tracking errors e_1 and e_4 in AFSMC for set point with disturbance.

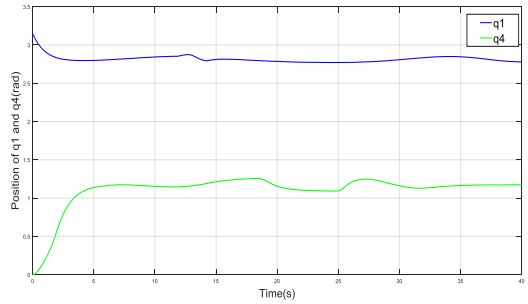

Figure 17. Response of angles q_1 and q_4 in AFSMC for set point with disturbance.

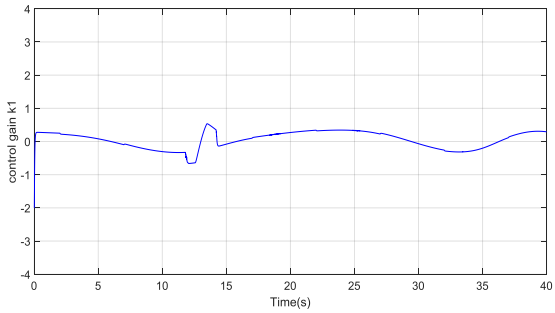

Figure 18. Control gain k_1 in AFSMC for set point with disturbance.

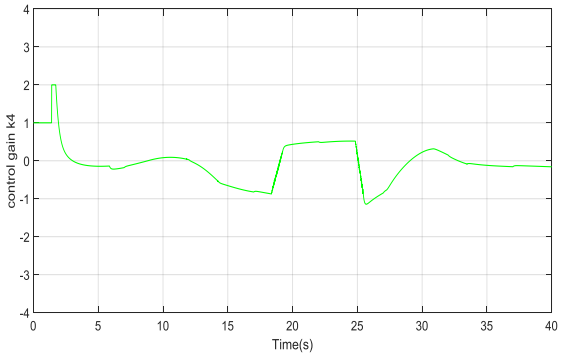

Figure 19. Control gain k_4 in AFSMC for set point with disturbance.

The simulation results demonstrated AFSMC for responding well than SMC with disturbance impact of end – effector. SMC have overshot and unstable every occur disturbance. Also the chattering phenomenon in SMC clearly indicated in Fig. 4-Fig. 11. Until AFSMC

showed the chattering phenomenon reject that indicated in Fig. 12-Fig. 19.

VI. Actual System Experiment and Results

The results of simulation demonstrate that AFSMC for response better than SMC so authors used AFSMC algorithm to control experiment real.

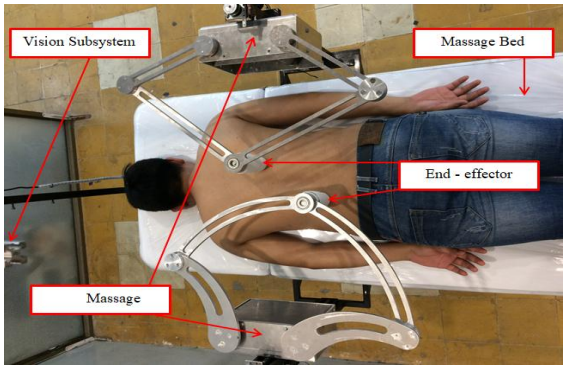

Figure 20. Experience of massage therapy robot.

As shown in Fig. 20, the actual experiment is a 2-DoF redundantly actuated parallel manipulator designed by Open Lab, Ho Chi Minh University of Education and Technology, Ho Chi Minh City, Vietnam. It is equipped with two servo DC motors with gear drives. The actuated joint angles are measured by absolute optical-electrical encoders. Here, we used DSP STM32F407VGT, tool Waijung 15.04 to support compiler to C/C++ through embedding algorithm control on Matlab/Simulink with sample time *0.01s*. With *r_set* is desire position. Data collections are collected through the Terminal software.

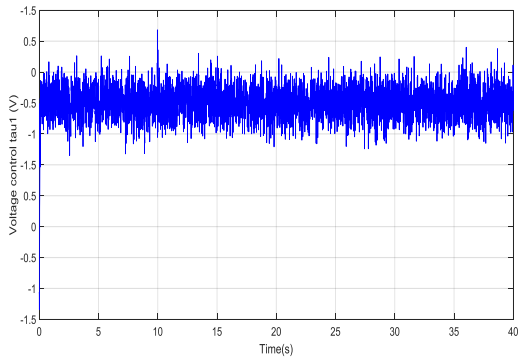

Figure 21. Control Voltage of τ_1.

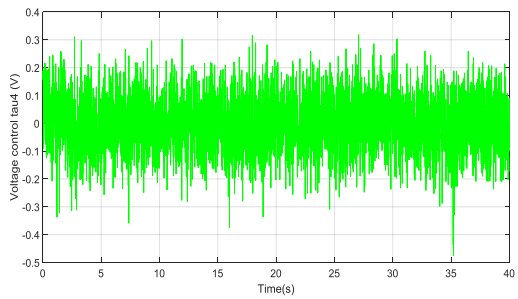

Figure 22. Control Voltage of τ_4.

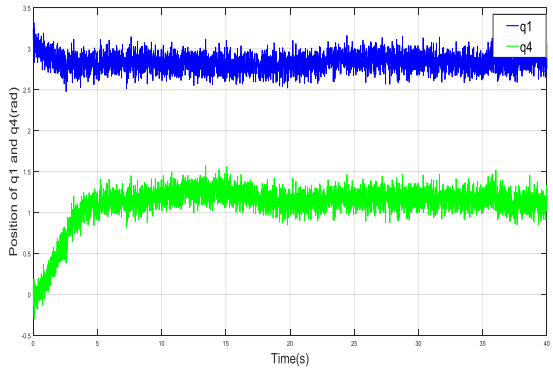

Figure 23. Response of angles q_1 and q_4.

The Fig. 21, Fig. 22 showed that result of control end – effector (**A3**) for good working and stability for desire position. Also the Fig. 23 indicated that the system for stability response.

VII. Conclusions

In this paper, an adaptive fuzzy sliding control scheme is proposed to control a robotic parallel manipulator. The contribution of paper is design of the adaptive fuzzy sliding controller is to eliminate the chattering and estimate uncertainty parameter of manipulator. The membership functions of the control gain are updated on-line. So the controller is not only a fuzzy controller but also an adaptive controller. In the adaptive fuzzy sliding mode control, the membership functions of the control gain and therefor ***k*** are updated online to compensate the uncertainty and stability the system is guaranteed without a prior knowledge of the system uncertainty.

The simulation and experiment for good results about control robot manipulator.

Appendix Dynamics of Robotic Manipulator

The general form of the robot arm dynamic equation (1) is

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}=T+T_h$$

where

$\ddot{q}$, $\dot{q}$ are the joint acceleration and velocity respectively.

$M(q)$ is the inertia matrix, $C(q,\dot{q})\dot{q}$ account for the centrifugal and Coriolis forces and is related to the inertia matric via

$$(\dot{M}(q)-2C(q,\dot{q}))^T=-(\dot{M}(q)-2C(q,\dot{q}))$$

The control torque input (T) and interaction T_h which is attached by end – effector of robot.

$$M(q)=\begin{bmatrix} \left(\frac{m_1}{3}+m_2\right)l_1^2 & m_B l_1 l_2\cos(q_1-q_2) & 0 & 0 \\ \frac{m_2 l_1 l_2}{4}\cos(q_1-q_2) & \frac{m_2 l_2^2}{3} & 0 & 0 \\ 0 & 0 & \frac{m_3 l_3^2}{3} & \frac{m_3 l_3 l_4}{4}\cos(q_3-q_4) \\ 0 & 0 & \frac{m_3 l_3 l_4}{4}\cos(q_3-q_4) & (\frac{m_4}{3}+m_3)l_4^2 \end{bmatrix}$$

$$C(q,\dot{q})=\begin{bmatrix} \frac{m_2 l_1 l_2}{4}\dot{q}_2 s_{12} & -\frac{m_2 l_1 l_2}{4}(\dot{q}_1-\dot{q}_2)s_{12} & 0 & 0 \\ -\frac{m_2 l_1 l_2}{4}(\dot{q}_1-\dot{q}_2)s_{12} & -\frac{m_2 l_1 l_2}{4}\dot{q}_1 s_{12} & 0 & 0 \\ 0 & 0 & \frac{m_3 l_3 l_4}{4}\dot{q}_4 s_{34} & -\frac{m_3 l_3 l_4}{4}(\dot{q}_3-\dot{q}_4)s_{34} \\ 0 & 0 & -\frac{m_3 l_3 l_4}{4}(\dot{q}_3-\dot{q}_4)s_{34} & \frac{m_3 l_3 l_4}{4}\dot{q}_3 s_{34} \end{bmatrix}$$

$$T=\begin{bmatrix} T_1 \\ 0 \\ 0 \\ T_4 \end{bmatrix} \text{ and } T_h=\begin{bmatrix} -f_{ox}l_1\sin q_1+f_{oy}l_1\cos q_1 \\ -f_{ox}l_2\sin q_2+f_{oy}l_2\cos q_2 \\ -f_{ox}l_3\sin q_3+f_{oy}l_3\cos q_3 \\ -f_{ox}l_4\sin q_4+f_{oy}l_4\cos q_4 \end{bmatrix}$$

In the above equations, s_{12} is sin $(q_1\text{-}q_2)$, the f_{ox}, f_{oy} represents for external force along the x y direction, m_1=m_4=0.2kg, m_2=m_3=0.4kg are the masses of each links, l_1=l_4=0.26m, l_2=l_3=0.43m represent the length of each links.

ACKNOWLEDGMENT

This working was supported by Associate Prof Nguyen Truong Thinh at Open Lab of Faculty Mechanical Engineer, Ho Chi Minh University of Technology and Education, Ho Chi Minh City, Vietnam.

REFERENCES

[1] V. I. Utkin, *Sliding Modes in Control and Optimization*, Springer Science & Business Media, 2013.

[2] J. J. Slotine and S. S. Sastry, "Tracking control of non-linear systems using sliding surfaces, with application to robot manipulators," *International journal of control*, vol. 38, pp. 465-492, 1983.

[3] J. J. E. Slotine, "Sliding controller design for non-linear systems," *International Journal of control*, vol. 40, pp. 421-434, 1984.

[4] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*, vol. 199, Prentice hall Englewood Cliffs, NJ, 1991.

[5] P. Huŝek, "Adaptive fuzzy sliding mode control for uncertain nonlinear systems," *IFAC Proceedings Volumes*, vol. 47, pp. 540-545, 2014.

[6] Y. H. Chang, C. W. Chang, C. W. Tao, H. W. Lin, and J. S. Taur, "Fuzzy sliding-mode control for ball and beam system with fuzzy ant colony optimization," *Expert Systems with Applications*, vol. 39, pp. 3624-3633, 2012.

[7] S. Elangovan and P. Y. Woo, "Adaptive fuzzy sliding control for a three-link passive robotic manipulator," *Robotica*, vol. 23, pp. 635-644, 2005.

[8] L. C. Hung, H. P. Lin, and H. Y. Chung, "Design of self-tuning fuzzy sliding mode control for TORA system," *Expert Systems with Applications*, vol. 32, pp. 201-212, 2007.

[9] P. Husek, "Stability margin of linear systems with parameters described by fuzzy numbers," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 41, pp. 1174-1182, 2011.

[10] P. Hušek and O. Cerman, "Fuzzy model reference control with adaptation of input fuzzy sets," *Knowledge-Based Systems*, vol. 49, pp. 116-122, 2013.

[11] D. Ling, "Fuzzy model reference learning control for a nonlinear model of hydrogenerator unit," in *Proc. 2009 International Workshop on Intelligent Systems and Applications*, 2009, pp. 1-4.

[12] S. J. Huang and H. Y. Chen, "Adaptive sliding controller with self-tuning fuzzy compensation for vehicle suspension control," *Mechatronics*, vol. 16, pp. 607-622, 2006.

[13] S. J. Huang and W. C. Lin, "Adaptive fuzzy controller with sliding surface for vehicle suspension control," *IEEE Transactions on Fuzzy Systems*, vol. 11, pp. 550-559, 2003.

[14] Y. Guo and P. Y. Woo, "An adaptive fuzzy sliding mode controller for robotic manipulators," *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, vol. 33, pp. 149-159, 2003.

[15] D. Gao, Z. Sun, and W. Wang, "Adaptive fuzzy sliding mode control for robotic manipulators," in Proc. *2010 8th World Congress on Intelligent Control and Automation*, 2010, pp. 4811-4816.

[16] R. J. Wai, C. M. Lin, and C. F. Hsu, "Adaptive fuzzy sliding-mode control for electrical servo drive," *Fuzzy Sets and Systems*, vol. 143, pp. 295-310, 2004.

[17] R. J. Wai and K. H. Su, "Adaptive enhanced fuzzy sliding-mode control for electrical servo drive," *IEEE Transactions on Industrial Electronics*, vol. 53, pp. 569-580, 2006.

[18] M. R. Emami, A. A. Goldenberg, and I. B. Turksen, "A robust model-based fuzzy-logic controller for robot manipulators," in *Proc. 1998 IEEE International Conference on Robotics and Automation (Cat. No. 98CH36146)*, 1998, pp. 2500-2505.

[19] F. Y. Hsu and L. C. Fu, "Nonlinear control of robot manipulators using adaptive fuzzy sliding mode control," in *Proc. 1995 IEEE/RSJ International Conference on Intelligent Robots and Systems. Human Robot Interaction and Cooperative Robots*, 1995, pp. 156-161.

[20] F. Sun, Z. Sun, and G. Feng, "An adaptive fuzzy controller based on sliding mode for robot manipulators," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 29, pp. 661-667, 1999.

[21] H. Xu, F. Sun, and Z. Sun, "The adaptive sliding mode control based on a fuzzy neural network for manipulators," in *1996 IEEE International Conference on Systems, Man and Cybernetics.*

Information Intelligence and Systems (Cat. No. 96CH35929), 1996, pp. 1942-1946.

[22] T. Tsay and J. H. Huang, "Robust nonlinear control of robot manipulators," in *Proc. the 1994 IEEE International Conference on Robotics and Automation*, 1994, pp. 2083-2088.

[23] J. S. Chen, C. S. Liu, and Y. W. Wang, "Control of robot manipulator using a fuzzy model-based sliding mode control scheme," in *Proc. 1994 33rd IEEE Conference on Decision and Control*, 1994, pp. 3506-3511.

[24] S. B. Choi and J. Kim, "A fuzzy-sliding mode controller for robust tracking of robotic manipulators," *Mechatronics*, vol. 7, pp. 199-216, 1997.

[25] B. Bekit, J. F. Whidborne, and L. D. Seneviratne, "Fuzzy sliding mode control for a robot manipulator," in *Proc. 1997 IEEE International Symposium on Computational Intelligence in Robotics and Automation CIRA'97.'Towards New Computational Principles for Robotics and Automation'*, 1997, pp. 320-325.

[26] I. S. Khalil, E. Globovic, and A. Sabanovic, "High precision motion control of parallel robots with imperfections and manufacturing tolerances," in *2011 IEEE International Conference on Mechatronics*, 2011, pp. 39-44.

[27] D. R. Swiss, "Springback Force Considerations in Compliant Haptic Interfaces," 2015.

[28] L. X. Wang and L. X. Wang, *A Course in Fuzzy Systems and Control*, vol. 203: Prentice Hall PTR Upper Saddle River, NJ, 1997.

Phan Thanh Phuc received the B.S degree in Mechatronic Engineering from Industrial University of Ho Chi Minh, Ho Chi Minh, Vietnam, in 2015 and the M.E degree in Mechatronic Engineering from Ho Chi Minh University of Technology and Education, Ho Chi Minh, Vietnam, in 2018.

Currently, he is working at Central Transportation 6 College, Ho Chi Minh City, Vietnam.

Nguyen Dao Xuan Hai, he received his B.S (2019) degree from the department of Machinery engineering at Ho Chi Minh University of Technology and Education (HCMUTE), Vietnam. He was a researcher in Robot Mechanism, injection molding, Python programming.

Currently, he works in Open Laboratory, Department of Mechatronic Engineering, Vietnam. His research interests are service robot, injection molding and applicant Python in Machine Learning

Nguyen Truong Thinh, he is Associate Professor of Mechatronics at Ho Chi Minh City University of Technology and Education. He received his Ph.D in Mechanical Engineering at Chonnnam National University (Korea) in 2010 and obtained a positive evaluation as Associate Professor in 2012. His main research interests are Industrial Robotics, Service robotics, Mechatronics, Industrial Automation.