Design Adaptive Fuzzy Sliding Mode Controller for Pantograph Mechanism Apply to Massage Therapy Robot for Healthcare

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Abstract—This paper proposes an adaptive fuzzy sliding mode controller (AFSMC) for pantograph haptic device. It has form mechanism with a 2-DoF redundancy actuated parallel robotic manipulator. An adaptive single input single output (SISO) fuzzy system is applied to calculate each element of the control gain vector in a sliding mode controller for purport to reject phenomenon chattering by the constant value of K and discontinuous function. The adaptive laws are designed based on the Lyapunov stability theory method. The adaptive laws are practiced online by fuzzy controller combine with the sliding mode control (SMC) to control stability the robot manipulator despite external forces disturbance. Many operation situations such as set point control also the trajectory control are simulated also experiment to demonstrate the operation controllers are good working.

Index Terms—sliding mode control, lyapunov stability, adaptive fuzzy control, chattering

I. INTRODUCTION

Over past decades sliding mode control [1]-[4] has become the most popular technique for control of nonlinear systems, especially because of simplicity of the control law, easy implementation and high robustness. Unfortunately, when used with fixed parameter SMC has several drawbacks [5]. The most important are chattering of control input so leading to high moving of mechanical parts and heat losses in electrical power circuits [1] slow convergence and nonzero steady state error. The usual way how to decrease the chattering phenomenon consists in introduction of boundary layer [4]. However attenuation of chattering in this case decreases control performance. To avoid effect different adaptive mechanisms by on-line tuning of SMC surface has been introduced in past decades.

One of the most often used adaptation mechanisms is based on fuzzy logic approach [6], [7]. Different approaches to adaptive fuzzy control has achieved good result for many experimental application [8], [9]. Popularity in last few years has gained especial the self-learning mechanisms based on fuzzy logic since only a very rough model of the controlled plant is sufficient for successful control [10, 11].

In the recent years, AFSMC methods have enjoyed popularity which is supported by many successful applications, automotive industry [12, 13], manipulators[14, 15] or vomechanisms [16, 17].

In [18, 19], fuzzy systems are used to implement the system dynamics as well as the control gain. Both theoretic studies and simulation results have demonstrated that this type of controllers eliminates the chattering on the sliding surface. Sun et al. [20] applied a fuzzy system to approximate the system dynamic. The discontinuous term $sgn(s)$ still exists in the control input. Xu et al. and Gao et al.[15, 21] applied Tagaki - Sugeno type fuzzy systems to estimate the system dynamic. In cases, the first of type fuzzy sliding mode controllers, it is assumed that model the robotic manipulator is totally unknown.

In the second type of fuzzy sliding mode controllers, it is assumed that the model robotic manipulator is known. Tsay et al. [22] propose that the control gain is the product of the inverse of the inertial matrix and gain vector. Each element of the gain vector is decided by an individual fuzzy system based on the value of the sliding surface $s$, and its variation. Chen et al. [23] estimate the control gain by using Tagaki – Sugeno type fuzzy systems. Therefore, it is same problem [21] exists. Choi et al. [24] regard the control gain as individual vector that is computed by fuzzy systems. Bekit et al.[25] propose that control gain $k$ is an n-order vector. Each element $k_i$ is decided by a fuzzy system with two inputs, the sliding surface $s$, and its variation $\dot{s}$. Since the fuzzy systems are non-adaptive, more system information is required to decide the membership functions of the fuzzy system. Both [24, 25] there is no theoretical proof of the stability and convergence of proposed controllers.

This is complex system and uncertainly so the paper focuses the fuzzy sliding mode controller since the model information is exactly unknown with the analysis of physical properties of the robotic manipulator. In the proposed control scheme, the control gain $k$ is considered

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as an individual vector. But, \( k \) is adjusted way an adaptive SISO fuzzy systems which requires less information of the robotic manipulator and therefore simplifies the implementation of controller. Moreover, a theoretical proof of the stability and the convergence of the proposed scheme by the Lyapunov method is provided. After, we compare simulation results with SMC. The paper is organized as follows dynamic modeling is summarized in Section II. In Section III and section IV, the sliding mode controller (SMC) and the adaptive fuzzy sliding mode controller (AFSMC) are presented. Section V show many results of simulation to demonstrate that the controllers are good operation. The results of actual system experience control are showed in section VI. Section VII concludes the paper.

II. DESCRIPTION OF ROBOTIC MANIPULATOR MODEL

The experiment has a 2-DoF redundantly actuated parallel manipulator and device consists of 5 links with length \( l_i \) for \( i = 1,\ldots,5 \). As shown in Fig. 1, a reference frame is established in the workspace of the parallel manipulator. The unit of the frame is a meter. The frame is established in the workspace of the parallel manipulator. The unit of the frame is a meter. The experiment has a 2-DoF redundantly actuated parallel manipulator. The length of links as follows: \( l_0 = 0.2, l_1 = l_4 = 0.26 \) and \( l_2 = l_3 = 0.43 \). The definitions of the joint angles are shown in Fig. 1, \( \alpha_i, \beta_i \) refer to the active joint angles and \( \alpha_2, \beta_3 \) refer to the positive joint angles.

The equation of motion of the pantograph haptic device following form [26, 27]

\[
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} = T + T_h
\]  

(1)

![Figure 1](image)

Figure 1. Coordinate of the 2-DoF redundantly actuated parallel manipulator.

III. SLIDING MODE CONTROLLER

The control objective is to drive the joint position \( q \) to the desired position \( q_d \). Define the tracking error

\[
e = q - q_d
\]  

(2)

Define the sliding surface

\[
s = \dot{e} + \lambda e
\]  

(3)

where \( \lambda = \text{diag}[\lambda_1, \ldots, \lambda_n] \) in which \( \lambda_i \) is a positive constant. The control objective can be achieved by choosing the control input \( \tau \) so that the sliding surface satisfies the sufficient condition [4]

\[
\frac{1}{2} \frac{d}{dt} s_i^2 \leq -\eta_i |s_i|
\]

(4)

where \( \eta_i \) is positive constant. Above equation indicates that the energy of \( s \) should decay as long as \( s \) is not zero. To achieved control \( \tau \), define the reference state.

\[
\dot{s}_r = \dot{\hat{q}} - s = \dot{\hat{q}}_d - \lambda e
\]

(5)

\[
\ddot{s}_r = \ddot{\hat{q}} - \ddot{s} = \ddot{\hat{q}}_d - \lambda \ddot{e}
\]

Choose the control \( \tau \)

\[
\tau = \hat{\tau} - K \text{sgn}(s)
\]

\[
\hat{\tau} = \dot{\hat{M}} \dot{q} + \dot{\hat{C}} q - T_h - A s
\]

(6)

where \( \hat{M} \), \( \hat{C} \) are the estimations of \( M \), \( C \) respectively, \( \Delta M = \text{diag} [K_{n1}, \ldots, K_{nn}], \Delta C = \hat{C} - C \). Assuming \( |\Delta f_i|, |\Delta f_i|_{\text{bound}} \), where \( |\Delta f_i|_{\text{bound}} \) is the boundary of \( |\Delta f_i| \), choose \( K \) such that

\[
K \geq |\Delta f_i|_{\text{bound}}
\]

(8)

To prove the stability of the system, choose the Lyapunov function candidate to be

\[
V = \frac{1}{2} s^T M s
\]

(9)

Since \( M \) is symmetric and positive definite, then for \( s \neq 0 \)

\[
V > 0
\]

(10)

It can be proved that

\[
\dot{V} = s^T [-(C + A)s + \Delta f - K \text{sgn}(s) + Cs]
\]

\[
= \sum_{i=1}^n (s_i [\Delta f_i - K_{ii} \text{sgn}(s_i)]) - s^T A s
\]
When $s_i > 0$, from (8)
$$
\Delta f_i - K_{ii} \text{sgn}(s_i) = \Delta f_i + K_{ii} \leq 0
$$
So that
$$
s_i[\Delta f_i - K_{ii} \text{sgn}(s_i)] \leq 0.
$$
When $s_i < 0$, from (8)
$$
\Delta f_i - K_{ii} \text{sgn}(s_i) = \Delta f_i - K_{ii} \geq 0
$$
So that
$$
s_i[\Delta f_i - K_{ii} \text{sgn}(s_i)] \leq 0
$$
Thus
$$
\sum_{i=1}^{n} (s_i[\Delta f_i - K_{ii} \text{sgn}(s_i)]) \leq 0.
$$
Since $A$ is a positive definite matrix, $-s^TA s \leq 0$.

With these results, it can be proved that
$$
\dot{V} = \sum_{i=1}^{n} (s_i[\Delta f_i - K_{ii} \text{sgn}(s_i)]) - s^T A s \leq 0
$$
Equation (9) can be considered as an indicator of energy of $s$. Thus, (11) guarantees the decay of the energy of $s$ as long as $s \neq 0$. The sufficient condition in (4) is thus satisfied.

where:

- **Desire position block** is coordinate desire of end-effector.
- **Inverse Kinematic block** is calculated from coordinate of $A_3$ deduced angles of $q_1, q_2, q_3, q_4$.
- **Sliding surface block** is calculated as formula (5).
- **Estimate block** is calculated as formula (6).
- **Robotic Manipulator** block is plant of robotic follow formula (1).
- **Sign S block** is Sgn function of Sliding Surface.

There are four basic parts in a fuzzy system. The fuzzification and defuzzification are the interface between the fuzzy systems and crisp systems. The rule base includes a set of “if...then...” rules based on the human experience. Each rule describes a relation between the input space and the output space. For each rule, the inference engine maps the input fuzzy sets to an output fuzzy set according to relation define by the rule. It then combines the fuzzy sets from all the rules in the rule base into the output fuzzy set. This output fuzzy set is translated to a crisp value output $y$ by the defuzzification.

All the four parts can be mathematically formulated. In this paper, by choosing singleton fuzzification, center average defuzzification, Mamdani implication in the rule base and product inference engine, the output of the fuzzy system can be written as
$\sum_{m=1}^{M} \theta^{m} \prod_{i=1}^{n} \mu_{A_{i}}^{m}(x_{i}) = \theta^{T} \Psi(x)$ \hspace{1cm} (12)

where $\theta = [\theta^{1}, \ldots, \theta^{m}, \ldots, \theta^{M}]^{T}$ is the vector of the centers of the membership functions of $y$ , $\Psi(x) = [\Psi^{1}(x), \ldots, \Psi^{m}(x), \ldots, \Psi^{M}(x)]^{T}$ is the vector of the height of the membership function of $y$ in which $\Psi^{m}(x) = \prod_{i=1}^{n} \mu_{A_{i}}^{m}(x_{i}) / \sum_{m=1}^{M} \prod_{i=1}^{n} \mu_{A_{i}}^{m}(x_{i})$, and $M$ is the amount of the rules.

A. Apply Fuzzy Systems to Sliding Mode Control

Rewrite (1)

$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} = T + T_{h}$ \hspace{1cm} (13)

As reminded above, the chattering is caused by constant value of $K$ and discontinuous function $\text{sgn}(s)$.

let the control gain $K \text{sgn}(s)$ be replaced by a fuzzy gain $k$. The new control input is then written as follows

$\tau = \dot{M} \ddot{q}_{f} + \dot{C} \dot{q}_{f} - T_{h} - As - k$ \hspace{1cm} (14)

where $k = [k_{1}, \ldots, k_{i}, \ldots, k_{n}]^{T}$ and each $k_{i}$ is estimated by an individual fuzzy system.

The rule base: to decide the rules for the fuzzy systems, consider (9)

$V = \frac{1}{2} s^{T} M s$

Here, $V$ is regarded as an indicator of energy of $s$. The stability of the system is guaranteed by choosing a control law such that $\dot{V} \leq 0$ and $V = 0$ only when $s = 0$. In the adaptive fuzzy sliding mode control, a fuzzy $k$ is applied to compensate the system uncertainty and reduce the energy of $s$. In this case, (11) can be rewritten as

$\dot{V} = \sum_{i=1}^{n} (s_{i}[\Delta f_{i} - k_{i}]) - s^{T} A s$ \hspace{1cm} (15)

Because of the function $\text{sgn}(s_{i})$ in (11), the control gain has the same sign as $s_{i}$. Therefore, $k_{i}$ should have the same sign as $s_{i}$ . Next, consider the term $(s_{i}[\Delta f_{i} - k_{i}])$ in (15). When $|s_{i}|$ is large, it is expected that $|k_{i}|$ is larger so that $\dot{V}$ has a large negative value. In other word, the energy of $s$ decays fast. When $|s_{i}|$ is very small, $(s_{i}[\Delta f_{i} - k_{i}])$ and has a little effect on the value of $\dot{V}$. Then small $|k_{i}|$ is allowed to avoid chattering.

When $s_{i}$ is zero anyway and therefor $k_{i}$ can be zero. From these analyses, some clues for rule base can be obtained: when $s_{i}$ is large, $k_{i}$ is large; when $s_{i}$ is small, $k_{i}$ is small, $k_{i}$ can be small as long as $|k_{i}| > |\Delta f_{i}|$; when $s_{i}$ is zero, $k_{i}$ is can be zero. This idea is similar to that of applying function $\text{sat}(.)$. In addition, an adaptive law is designed to guarantee that $k_{i}$ can be compensated the system uncertainty. These analyses indicate that the value of $k_{i}$ can be decided by the value of the sliding surface $s_{i}$.

Thus, the fuzzy system for $k_{i}$ should be a SISO system, with $s_{i}$ as the input and $k_{i}$ as the output.

The rules in the rule base are in the following format.

If $s_{i}$ is $A_{i}^{m}$,

Then $k_{i}$ is $B_{i}^{m}$

where $A_{i}^{m}$ and $B_{i}^{m}$ are fuzzy sets. In this paper, it is chosen that both $s_{i}$ and $k_{i}$ have the same kind of membership functions: NB, NM, NS, ZE, PS, PM, PB, where N stands for negative, P positive, B big, M medium, S small and ZE zero. They are all Gaussian membership functions defined as

$\mu_{A}(x_{i}) = \exp \left[ - \left( \frac{x_{i} - \alpha}{\sigma} \right)^{2} \right]$ \hspace{1cm} (16)

where “$A$” represents one of the fuzzy sets NB,...,PB and $x_{i}$ represents $s_{i}$ or $k_{i}$. $\alpha$ is center of ”$A$” and $\sigma$ is the width the same titles, correspondingly, the values of the center and the width of the membership function with a same title for $s_{i}$ and $k_{i}$ are different, respectively. The parameters of the membership functions of $s_{i}$ are pre-defined, while those of $k_{i}$ are updated on-line. Therefore, the controller is an adaptive controller.

Based on the above discussions and definitions of the input and output membership functions, the rule base can be decided as follows:

IF $s_{i}$ is NB, THEN $k_{i}$ is NB
IF $s_{i}$ is NM, THEN $k_{i}$ is NM
IF $s_{i}$ is NS, THEN $k_{i}$ is NS
IF $s_{i}$ is ZE, THEN $k_{i}$ is ZE
IF $s_{i}$ is PS, THEN $k_{i}$ is PS
IF $s_{i}$ is PM, THEN $k_{i}$ is PM
IF $s_{i}$ is PB, THEN $k_{i}$ is PB

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From the knowledge of the fuzzy systems, $k_i$ can be written as

$$k_i = \frac{\sum_{m=1}^{M} \Theta_{mi}^m \mu_{i}^m(s_i)}{\sum_{m=1}^{M} \mu_{i}^m(s_i)} = \Theta_{ki}^T \Psi_{ki}(s_i)$$  \hspace{1cm} (17)

Where

$$\Theta_{ki} = [\Theta_{ki}^1,...,\Theta_{ki}^m,...,\Theta_{ki}^M]^T,$$

$$\Psi_{ki}(s_i) = [\Psi_{ki}^1(s_i),...,\Psi_{ki}^m(s_i),...,\Psi_{ki}^M(s_i)]^T$$

and

$$\Psi_{ki}^m(s_i) = \mu_{i}^m(s_i) / \sum_{m=1}^{M} \mu_{i}^m(s_i).$$

$\Theta_{ki}$ is chosen as the parameter vector. $\Psi_{ki}(s_i)$ is called the function basis vector and can be regarded as the weight of the parameter vector.

**Adaptive fuzzy Sliding Mode Control:** Putting (14) into (13) lead to

$$\dot{M} \dot{s} = (C + A)s + \Delta f - k$$ \hspace{1cm} (18)

where the definition of $\Delta f$ is the same as that in section above and $k_i$ is as in (17).

Define $\hat{\Theta}_{k_i}$ so that $k_i = \Theta_{ki}^T \Psi_{ki}(s_i)$ is the optimal compensation for $\Delta f$. According to [28], there exists $\omega_i > 0$ satisfying

$$|\Delta f - \Theta_{ki}^T \Psi_{ki}(s_i)| \leq \omega_i$$ \hspace{1cm} (19)

where $\omega_i$ can be as small as possible. Define

$$\hat{\Theta}_{ki} = \Theta_{ki} - \hat{\Theta}_{k_i}$$ \hspace{1cm} (20)

Then

$$k_i = \hat{\Theta}_{ki}^T \Psi_{ki}(s_i) + \Theta_{ki}^T \Psi_{ki}(s_i)$$ \hspace{1cm} (21)

Choose the adaptive law as

$$\dot{\hat{\Theta}}_{ki} = s_i \Psi_{ki}(s_i)$$ \hspace{1cm} (22)

Choose a Lyapunov function candidate as

$$V = \frac{1}{2} s^T M s + \frac{1}{2} \sum_{i=1}^{n} (\hat{\Theta}_{ki}^T \dot{\hat{\Theta}}_{ki})$$ \hspace{1cm} (23)

where $M$ is positive symmetric matrix and $\hat{\Theta}_{ki}^T \dot{\hat{\Theta}}_{ki} > 0$ and therefore $V$ is positive the derivation of $V$

$$\dot{V} = \frac{1}{2} [s^T M s + s^T \dot{M} s + s^T M \dot{s}] + \frac{1}{2} \sum_{i=1}^{n} \hat{\Theta}_{ki}^T \dot{\hat{\Theta}}_{ki} + \frac{1}{2} \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

$$= s^T [M s + C \dot{s}] + \frac{1}{2} \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

$$= s^T [- (C + A)s + \Delta f - k + C \dot{s}] + \frac{1}{2} \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

$$= s^T [- (C + A)s + \Delta f - k] + \frac{1}{2} \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

$$= -s^T A s + s^T [\Delta f - k] + \frac{1}{2} \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

$$= -s^T A s + \sum_{i=1}^{n} (s_i [\Delta f - k_i]) + \frac{1}{2} \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

Since $k_i = \hat{\Theta}_{ki}^T \Psi_{ki}(s_i) + \Theta_{ki}^T \Psi_{ki}(s_i)$ then

$$\dot{V} = -s^T A s + \sum_{i=1}^{n} (s_i [\Delta f - \hat{\Theta}_{ki}^T \Psi_{ki}(s_i) + \Theta_{ki}^T \Psi_{ki}(s_i)])$$

$$+ \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

$$\dot{V} = -s^T A s + \sum_{i=1}^{n} (s_i [\Delta f - \Theta_{ki}^T \Psi_{ki}(s_i)])$$

$$+ \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

$$\dot{V} = -s^T A s + \sum_{i=1}^{n} (s_i [\Delta f - \Theta_{ki}^T \Psi_{ki}(s_i)]) + \frac{1}{2} \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

$$\dot{V} = -s^T A s + \sum_{i=1}^{n} (s_i [\Delta f - \Theta_{ki}^T \Psi_{ki}(s_i)]) + \frac{1}{2} \sum_{i=1}^{n} \dot{\hat{\Theta}}_{ki}^T \dot{\hat{\Theta}}_{ki}$$

Since the adaptive law in (22) is chosen as

$$\dot{\hat{\Theta}}_{ki} = s_i \Psi_{ki}(s_i)$$

Then

$$\dot{V} = -s^T A s + \sum_{i=1}^{n} (s_i [\Delta f - \Theta_{ki}^T \Psi_{ki}(s_i)])$$ \hspace{1cm} (24)

From (19), there exists

$$|\Delta f - \Theta_{ki}^T \Psi_{ki}(s_i)| \leq \omega_i$$

And $\omega_i$ can be as small as possible. Assume

$$|\Delta f - \Theta_{ki}^T \Psi_{ki}(s_i)| \leq \omega_i \leq \gamma_i |s_i|$$

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where \(0 < \gamma_i < 1\)

Then the second term at right side of (24) satisfies

\[ s \left[ \Delta f_i - \theta_i^T \Psi_w(s_i) \right] \leq a_i \| s \|^2 = \gamma_i s_i^2 \]  \hspace{1cm} (25)

Therefore

\[ \dot{V} \leq -s^T As + \sum_{i=1}^{n} \gamma_i s_i^2 \]  \hspace{1cm} (26)

The right side of (24) can be written as

\[ \sum_{i=1}^{n} (-a_i s_i^2 + \gamma_i s_i^2) = -s^T (A - \gamma) s \]

where \(\gamma = \text{diag} [\gamma_1,\ldots,\gamma_i,\ldots,\gamma_n]\). Simply choose \(a_i > \gamma_i\) so that \((A - \gamma)\) is a positive definite matrix, therefore \(-s^T (A - \gamma) s \leq 0\) and then

\[ \dot{V} \leq -s^T s \leq 0 \]  \hspace{1cm} (27)

In (27), since \((A - \gamma)\) is positive definite matrix, \(\dot{V} = 0\) only when \(s = 0\). Thus, the overall system with the adaptive law in (22) is asymptotically stable with respect to \(s\). In other words

\[ \lim_{t \to \infty} s = \lim_{t \to \infty} (\dot{e} + \lambda e) = 0 \]  \hspace{1cm} (28)

Or equivalently

\[ \lim_{t \to \infty} q = q_d \quad \text{and} \quad \lim_{t \to \infty} \dot{q} = \dot{q}_d \]  \hspace{1cm} (29)

Therefore, it is proved that, with the adaptive fuzzy sliding mode control input (14), the actual joint position converge to desired.

Similar, in the diagram of SMC (Fig. 2) the Sign S block is replaced by Fuzzy system block which is calculated by (16). AFSMC is indicated in Fig. 3.

\[ \dot{V} \leq \sum_{i=1}^{n} (-a_i s_i^2 + \gamma_i s_i^2) = -s^T (A - \gamma) s \leq 0 \]  \hspace{1cm} (27)

V. SIMULATION RESULTS

The simulation is practiced with parameters in section II with parameters as Table I.

<table>
<thead>
<tr>
<th>Link</th>
<th>Length of link (m)</th>
<th>Mass of link (kg)</th>
<th>Initial Angle (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.26</td>
<td>0.2</td>
<td>(\pi)</td>
</tr>
<tr>
<td>2</td>
<td>0.43</td>
<td>0.4</td>
<td>(\pi/4)</td>
</tr>
<tr>
<td>3</td>
<td>0.43</td>
<td>0.4</td>
<td>(3\pi/4)</td>
</tr>
<tr>
<td>4</td>
<td>0.26</td>
<td>0.2</td>
<td>0</td>
</tr>
</tbody>
</table>

Disturbance is used by pulse function with amplitude 0.006, period = 10s and pulse width = 50% of period.

\[ \lambda = \text{diag}[1 \ 1 \ 1], \ A = \text{diag}[1 \ 1 \ 1], \ K = \text{diag}[1 \ 1 \ 1] \]

The purposes are control end – effector (A3) go to position set point \((0.25 \ 0.25)\) in SMC and AFSMC is applied to control plant despite impact forces external disturbance to plant to demonstrate the controller well working.

SMC without disturbance amplitude 0.006 and coordinate desire position \((0.25 \ 0.25)\).
SMC with disturbance amplitude 0.006 and coordinate desire position (0.25 0.25).

AFSMC without disturbance with coordinate desire position (0.25 0.25).
The simulation results demonstrated AFSMC for responding well than SMC with disturbance impact of end-effector. SMC have overshot and unstable every occur disturbance. Also the chattering phenomenon in SMC clearly indicated in Fig. 4-Fig. 11. Until AFSMC

AFSMC with disturbance amplitude 0.006 and coordinate desire position (0.25 0.25)
showed the chattering phenomenon reject that indicated in Fig. 12-Fig. 19.

VI. ACTUAL SYSTEM EXPERIMENT AND RESULTS

The results of simulation demonstrate that AF-SMC for response better than SMC so authors used AF-SMC algorithm to control experiment real.

As shown in Fig. 20, the actual experiment is a 2-DoF redundantly actuated parallel manipulator designed by Open Lab, Ho Chi Minh University of Education and Technology, Ho Chi Minh City, Vietnam. It is equipped with two servo DC motors with gear drives. The actuated joint angles are measured by absolute optical-electrical encoders. Here, we used DSP STM32F407VGT, tool Waijung 15.04 to support compiler to C/C++ through embedding algorithm control on Matlab/Simulink with sample time 0.01s. With $r_{set}$ is desire position. Data collections are collected through the Terminal software.

The Fig. 21, Fig. 22 showed that result of control end – effector (A3) for good working and stability for desire position. Also the Fig. 23 indicated that the system for stability response.

VII. CONCLUSIONS

In this paper, an adaptive fuzzy sliding control scheme is proposed to control a robotic parallel manipulator. The contribution of paper is design of the adaptive fuzzy sliding controller is to eliminate the chattering and estimate uncertainty parameter of manipulator. The membership functions of the control gain are updated online. So the controller is not only a fuzzy controller but also an adaptive controller. In the adaptive fuzzy sliding mode control, the membership functions of the control gain and thereafter $k$ are updated online to compensate the uncertainty and stability the system is guaranteed without a prior knowledge of the system uncertainty.

The simulation and experiment for good results about control robot manipulator.

APPENDIX DYNAMICS OF ROBOTIC MANIPULATOR

The general form of the robot arm dynamic equation (1) is

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = T + T_h$$

where

$\ddot{q}$, $\dot{q}$ are the joint acceleration and velocity respectively.

$M(q)$ is the inertia matrix, $C(q, \dot{q})\dot{q}$ account for the centrifugal and Coriolis forces and is related to the inertia matrix via

$$(\dot{M}(q) - 2C(q, \dot{q}))\dot{q} = -(\dot{M}(q) - 2C(q, \dot{q}))$$

The control torque input ($T$) and interaction $T_h$ which is attached by end – effector of robot.
\[
M(q) = \begin{bmatrix}
\left(\frac{m_3 + m_2}{3}\right)l_1^2 & m_2l_1 \cos(q_1 - q_2) & 0 & 0 \\
\frac{m_2l_1}{4} \cos(q_1 - q_2) & \frac{m_1l_1}{3} & 0 & 0 \\
0 & 0 & \frac{m_2l_2}{3} & \frac{m_1l_2}{4} \cos(q_1 - q_4) \\
0 & 0 & \frac{m_2l_2}{4} \cos(q_1 - q_4) & \left(\frac{m_3 + m_4}{3}\right)l_2^2
\end{bmatrix}
\]

\[
C(q, \dot{q}) = \begin{bmatrix}
\frac{m_2l_1}{4}q_2s_{12} & -\frac{m_2l_2}{4}(q_1 - q_2)s_{12} & 0 & 0 \\
-\frac{m_2l_2}{4}s_{12} & -\frac{m_2l_2}{4}q_1s_{12} & 0 & 0 \\
0 & 0 & \frac{m_2l_2}{4}q_3s_{34} & -\frac{m_2l_2}{4}(q_3 - q_4)s_{34} \\
0 & 0 & \frac{m_2l_2}{4}(q_3 - q_4)s_{34} & \frac{m_2l_2}{4}q_4s_{34}
\end{bmatrix}
\]

\[
T = \begin{bmatrix} T_1 \\ 0 \\ 0 \\ T_4 \end{bmatrix}
\quad \text{and} \quad
T_h = \begin{bmatrix}
-f_{ox}l_1 \sin q_1 + f_{oy}l_1 \cos q_1 \\
-f_{ox}l_2 \sin q_2 + f_{oy}l_2 \cos q_2 \\
-f_{ox}l_3 \sin q_3 + f_{oy}l_3 \cos q_3 \\
-f_{ox}l_4 \sin q_4 + f_{oy}l_4 \cos q_4
\end{bmatrix}
\]

In the above equations, \( s_{12} = \sin(q_1-q_2) \), the \( f_{ox}, f_{oy} \) represents for external force along the \( x, y \) direction, \( m_1=m_2=0.2\,\text{kg}, \ m_3=m_4=0.4\,\text{kg} \) are the masses of each links, \( l_1=l_2=0.26\,\text{m}, \ l_2=l_3=0.43\,\text{m} \) represent the length of each links.

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REFERENCES


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