Studies on Acoustic Pressure Fluctuations in Thick Walled Hollow Cylinder for Various Industrial Applications

Jithin Ambarayil Joy[a], Muhammad Sajjad [b] , and Dong Won Jung[c*]
Department of Mechanical Engineering, Jeju National University, Jeju, South Korea
Email: [a]jithinjoy30jan@gmail.com, bmsajjadtheone@gmail.com, *cjdwcheju@jejunu.ac.kr

***Abstract*—In this paper the acoustic pressure analysis in a thick-walled hollow cylinder similar to a solid propellant rocket motor with non-uniform port geometry is carried out for meeting the requirements of various types of canister designs for industrial applications. Parametric studies have been carried out with different port geometries with different ranges of frequency (1- 4000 Hz) without impedance and volumetric drag. The effects of pressure fluctuations on the thick walls of the canister at the steady state conditions are discussed. We have shown that the concept of volumetric dilatation in deriving the acoustic pressure governing equations can be replaced by a simple elemental approach.**

***Index Terms*—acoustic pressure, Solid Rocket Motor, thick walled Canister.**

I. Introduction

The mechanical behavior of materials provide information of the deformable bodies such as bars, plates, cylinders, disks and shells etc. The theory of deformable solids can be named as strength of materials or mechanics of deformable solids. Research in the field of solid mechanics is not only for the basic understanding of this field but also for the advancements of engineering techniques in this area [1-8]. The properties of the materials which are used in different industries have been examined thoroughly for the betterment of design of various structures from the stand-points of both safety and economy. This subject is of vital importance in the design of various modern engineering structures such as missiles, rockets, aerospace and surface transportation vehicles, oil refineries, solar and atomic power plants, etc., where the acoustic pressure oscillations are significant. This demand of industry can only be completed if there is a proper understanding of the effect of acoustic pressure fluctuation on stresses, strains and mechanical properties of materials and therefore, having the capability of combining them using the theoretical approach. A designer will concentrate on feasible design of structure which is technologically practicable and economically viable. A designer is also interested in thorough knowledge of stresses at which permanent deformation begins in a material or risk of fracture is to be expected. In addition to this, designer will have to consider the possible deformation in machine parts exposed to high level of stresses. Deformation is an observable and a measurable physical phenomenon and is of prime interest for design engineers.

When a deformable solid material is subjected to some external loading systems such as pressure or temperature, it has been observed that firstly solid deforms elastically. If the loading is continued, then material deforms plastically, and if loading is further continued, it gives rise to time dependent continuous deformation known as creep deformation. It may be possible that number of transition states may occur at the same critical point, so the transition function will have different values at this point. This critical point is known as a multiple point and each branch of it will correspond to a different state. In general, the material from elastic state can change into (i) plastic state, or (ii) creep state, or (iii) first to plastic state and then to creep and vice-versa, depending upon the nature of loading. All these final states are reached through a transition state.

Elastic-plastic and Creep stresses for thick-walled homogeneous and nonhomogeneous cylinders under internal pressure have been analyzed by many authors [9-16]. All these authors considered yield criteria, jump conditions, linear strain measure to calculate the stresses by using concept of infinitesimal strain theory. Transition theory [7] does not require any of the above assumptions and thus solves more general problems using the concept of generalized strain measure [8]. This theory gives the well-known strain measures in addition to the stresses in plasticity and creep problems by determining the asymptotic solution at the transition points of the governing differential equations. It has been widely studied in the literature [1-17]. In the nuclear industry, cylinders made of functionally graded materials under internal and external pressure have become a point of interest due to their wide applications, for example, in steam generator tubes, in which primary coolant flows outside the tubes while secondary water flows inside the tubes. As permissible stress of any material is some proportion of the yield or ultimate stress of the material therefore it incorporates a “safety factor“ which provides a margin against the collapse condition in different types

Manuscript received February 2, 2018; revised April 23, 2018.

doi: 10.18178/joace.6.1.27-30

of thick-walled cylinders due to high pressure. Keeping in mind the various applications of thick-walled cylinders, we have performed the studies on the acoustic pressure fluctuations of these thick-walled cylinders with non-uniform port geometry under internal pressure loadings.

The acoustic pressure is the dynamic pressure which fluctuates. Once the system excited at natural frequency, it will result in resonance. The resonance phenomena occur with all types of vibrations or waves: there is mechanical resonance, acoustic resonance. Electromagnetic resonance, nuclear magnetic resonance (NMR), electron spin resonance (ESR) and resonance of quantum wave functions. Resonant systems can be used to generate vibrations of a specific frequency (e.g., musical instruments), or pick out specific frequencies from a complex vibration containing many frequencies (e.g., filters). At resonant frequencies, small periodic driving forces have the ability to produce large amplitude oscillations, due to the storage of vibrational energy.

Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy and potential energy in the case of a simple pendulum). However, there are some losses from cycle to cycle, called damping. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of unforced vibrations. Some systems have multiple, distinct, resonant frequencies. Resonance occurs widely in nature, and is exploited in many manmade devices. It is the mechanism by which virtually all sinusoidal waves and vibrations are generated. In this paper we have focused on the acoustic pressure fluctuations in thick-walled canister with different port geometries.

II. Modeling and Simulation

Rrelation between Acoustic pressure, Bulk modulus, and Displacement of a particle without using the concept of Volumetric Dilatation

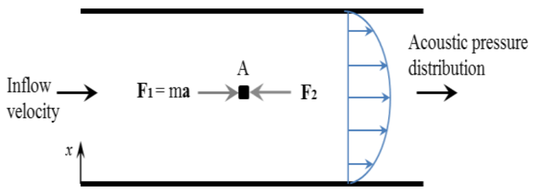

Figure 1 Shows acoustic pressure distribution and forces exerting on a fluid particle

Considering the particle A of mass m which has a force $\mathbf{F_1}$ due to acceleration $\mathbf{a}$ and $\mathbf{F_2}$ is the resistance force acting in a direction opposite to $\mathbf{F_1}$. Since m of the particle is too small, the mass flow rate is assumed as null value. Then the pressure p can be written as follows

$$p = \frac{\mathbf{F_1} - \mathbf{F_2}}{\mathbf{A}} = \frac{\rho V \boldsymbol{a} - \mathbf{F_2}}{\mathbf{A}} \tag{1}$$

where density volume V, and area A

$$p = \frac{\rho V \boldsymbol{a} - \mathbf{F_2}}{\mathbf{A}} \times \frac{V}{V} = \left(\rho \boldsymbol{a} - \frac{\mathbf{F_2}}{V} \right) \times \mathbf{x} \tag{2}$$

where x is the spatial position at which the particle is flowing. The fluid particle velocity is $\mathbf{v}$;

$$\frac{p}{\mathbf{x}} = \rho \boldsymbol{a} - \frac{\mathbf{F_2}}{V} = \rho \boldsymbol{a} - \frac{\mathbf{F_2}}{V} \times \frac{\mathbf{v}}{\mathbf{v}} = \rho \boldsymbol{a} - \gamma \mathbf{v} \tag{3}$$

By the definition of volumetric drag in *Abaqus theory manual*, the term $\mathbf{F_2} / V\mathbf{v}$ is the volumetric drag.

For change in pressure with respect to change in $\mathbf{x}$, Eq. (3) can be written as follows

$$-\frac{dp}{d\mathbf{x}} = \rho(\mathbf{x})\,\mathbf{a} - \gamma(\mathbf{x})\,\mathbf{v} \tag{4}$$

The change in pressure is denoted with minus sign because the pressure should reduce from a point to another because of drag.

With an addition of independent variable θ. Eq. (4) can be written as

$$-\frac{\partial p}{\partial \mathbf{x}} + \gamma(\mathbf{x}, \theta)\,\mathbf{v} - \rho(\mathbf{x}, \theta)\,\mathbf{a} = 0 \tag{5}$$

The constitutive behavior of the fluid is assumed to be inviscid, linear, compressible, and volumetric drag as zero, Eq. (5) can be written as follows

$$\frac{\partial p}{\partial \mathbf{x}} = -\rho(\mathbf{x}, \theta)\,\frac{\partial \mathbf{v}}{\partial t} = -\rho(\mathbf{x}, \theta)\,\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\,\frac{\partial \mathbf{x}}{\partial t} \tag{6}$$

In order to write the above equation for change in pressure with change in time, the term $\frac{\partial p}{\partial \mathbf{x}}$ can be written as $\frac{\partial p}{\partial t}\frac{\partial t}{\partial \mathbf{x}}$. The term $\frac{\partial \mathbf{v}}{\partial t}$ can also be written as $\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial t}$.

$$\frac{\partial p}{\partial t}\frac{\partial t}{\partial \mathbf{x}} = -\rho(\mathbf{x}, \theta)\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\frac{\partial \mathbf{x}}{\partial t} \quad \leftrightarrow \quad \frac{\partial p}{\partial t} = -\rho(\mathbf{x}, \theta)\frac{\partial \mathbf{v}}{\partial \mathbf{x}}\left(\frac{\partial \mathbf{x}}{\partial t}\right)^2 \tag{7}$$

$$\frac{\partial p}{\partial t} = -\rho(\mathbf{x}, \theta)\;\mathbf{v}^2\,\frac{\partial \mathbf{v}}{\partial \mathbf{x}} \tag{8}$$

The term $\frac{\partial \mathbf{v}}{\partial \mathbf{x}}$ is the velocity gradient which can be written as $\frac{\partial}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{u}}{\partial t}$, where $\mathbf{u}$ is displacement of the particle. If $\mathbf{v}$ is the velocity of sound in the medium, then isentropic bulk modulus is $K(\mathbf{x}, \theta) = \mathbf{v}^2 \rho(\mathbf{x}, \theta)$.

$$\frac{\partial p}{\partial t} = -K(\mathbf{x}, \theta)\frac{\partial \mathbf{v}}{\partial \mathbf{x}} = -K(\mathbf{x}, \theta)\frac{\partial}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{u}}{\partial t} \tag{9}$$

$$\int \frac{\partial p}{\partial t}\,dt = -\int K(\mathbf{x}, \theta)\,\frac{\partial}{\partial \mathbf{x}} \cdot \frac{\partial \mathbf{u}}{\partial t}\,dt \tag{10}$$

$$p = -K(\mathbf{x},\theta)\frac{\partial}{\partial \mathbf{x}} \cdot \mathbf{u} \quad (11)$$

Eq. (11) shows the relation between *p*, *K*, and u.

III. Numerical Model

Considering the parameter X_s, the study on the model adopted from Ref.[18], shown in Fig. 2, is carried out with air as the fluid. The density and bulk modulus of the air is taken as 1.2 kg/m^3, and 142 KPa. 4.1 MPa is the pressure loading in inlet P, and 10% of P is given to the throat of SRM. Since the pressure reaches peak value near the entry of nozzle and axial zone of chamber, the pressure loading is imosed on the throat which may include the physics in cold run. The frequency range of 1-4,000 Hz is examined for the steady state dynamics case using a commercial software.

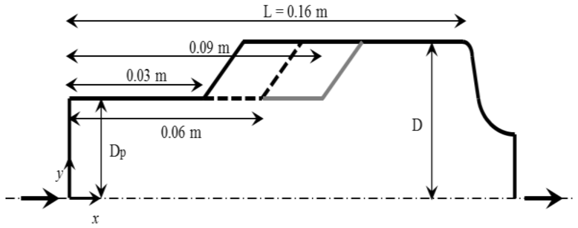

Figure 2 . Shows the 2D axi-symmetrical model, adopted from Ref.[18], with variation in parameter X_s = 0.03 m, 0.06 m, and 0.09 m. The dimensionless parameter, L/D = 4 and Dp/D = 2.

Around 6,500 Acoustic Axisymmetric elements (ACAX3 and ACAX4) is used for the numerical study. The outer profile pressure is set to zero and an axisymmetric boundry condition is given to the domain. The variation in computational data attained with and without the impedance is not significant, hence the analysis were carried out without impedance. Fig. 3 shows the computational doamain meshed for the acoustic analysis without impedance.

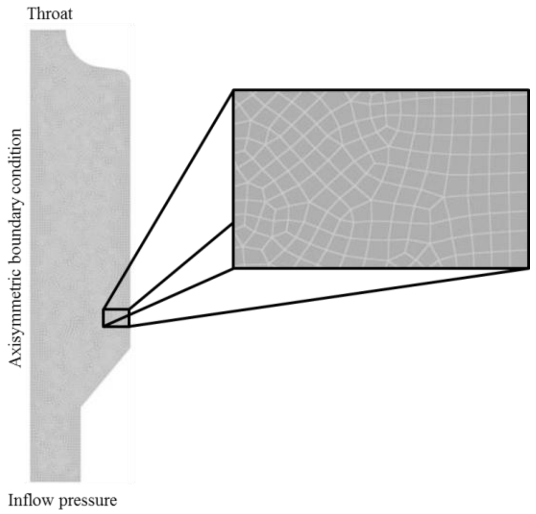

Figure 3 . shows the computational domain meshed for the numerical study

IV. Results and Discussion

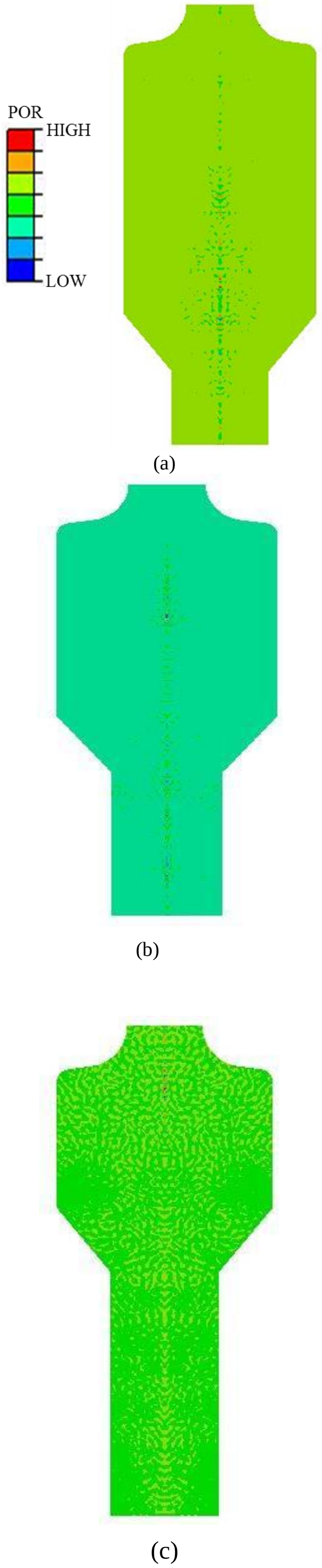

Figure 4 (a), (b), and (c) are the maximum acoustic pressure contour for Xs = 0.03 m, 0.06 m, and 0.09 m excited at the frequency 2938 Hz, 2905 Hz, and 2926 Hz.

Design limit load is typically 1.5 times the endurance limit load chamber wall. The radial and axial loads acts on the chamber wall. These walls have to withstand the chamber pressure, flight accelerations, vibration, thermal stresses, and ignition pressure spike or shock. Zones near the nozzle are critical; heat transfer analysis is usually done [2].

The acoustic pressure oscillations hit resonance near 2900 Hz for all cases. SRM with a divergent location which hits resonance at higher frequency is desirable. Since all the models with different divergent location have almost same resonance frequency in steady state, the choice may be made with considering the efficient fluid flow irrespective of oscillations. Note that the resonance is important parameter for motors oscillates at high frequency, because with the limited limit factor the motor may fail at this condition.

V. Concluding Remarks

The steady state analysis of a thick-walled canister, with different divergent locations at the given conditions, reveals that the resonance frequency is nearly around 2,900 Hz. The detailed derivation for the acoustic pressure without the concept of volumetric dilation is given. We have concluded that the concept of volumetric dilatation in deriving the acoustic pressure governing equations can be replaced by a simple elemental approach.

Acknowledgement

This research was supported by the 2018 scientific promotion program funded by Jeju National University.

References

[1] R. Sharma, "Non-linear stress analysis of thick-walled circular cylinders," Ph.D thesis, Jaypee Institute of Information Technology, India, October 2014.

[2] A. E. Green, "Theoretical elasticity", Oxford University Press, London, 1954.

[3] J. Chakrabarty, "Theory of plasticity," McGraw- Hill, New York, 1987.

[4] B. R. Seth, "Generalized strain measure with applications to physical problems," *Rep. 248, Math. Res. Center, Madison*, 1961.

[5] B. R. Seth, "Transition theory of elastic-plastic deformation, creep and relaxation," *Nature*, vol. 195, (4844), pp. 896-897, 1962.

[6] B. R. Seth, "Asymptotic phenomenon in large rotation," *J. Math. Mech*., vol. 12, pp. 205-212, 1963.

[7] B. R. Seth, "Transition conditions, the yield condition," *Int. J. Non-linear Mechanics*, vol. 5, pp. 279-285, 1970.

[8] B. R. Seth, "Measure-concept in mechanics," *Int. J. Non-linear Mechanics*, vol. 1, pp. 35-40, 1966.

[9] P. G. Hodge Jr., M. Balaban, "Elastic-plastic analysis of a rotating cylinder," *Int. J. Mech. Sci*., Pergamon Press Ltd., vol. 4, pp. 465-476, 1962.

[10] D. Durban, "Large strain solution for pressurized elasto-plastic tube," *J. of Appl. Mech*., *Transactions of the ASME*, vol. 146, pp. 228-230, 1979.

[11] J. Perry, J. Aboudi, "Elasto-plastic stresses in thick walled cylinders," *J. Pressure Vessel Technology, Transactions of the ASME*, vol. 125, pp. 248–252, 2003.

[12] K. M. Liew, S. Kitipornchai, X. Z. Zhang, C. W. Lim, "Analysis of the thermal stress behaviour of functionally graded hollow cylinders", *International Journal of Solids and Structures*, vol. 40, pp. 2355-2380, 2003.

[13] Y. S. Yoo, N. Huh, S. Choi, T. Kim, J. Kim, "Collapse pressure estimates and the application of a partial safety factor to cylinders subjected to external pressure," *Nuclear Engineering and Technology*, vol. 42, no. 4, pp. 450–459, 2010.

[14] A. Parvizi, R. Naghdabadi, J. Arghavani, "Analysis of Al A 359/SiCp functionally graded cylinder subjected to internal pressure and temperature gradient with elastic-plastic deformation," *Journal of Thermal Stresses*, vol. 34, pp. 1054-1070, 2011. Synopsis 15

[15] M. Sadeghian, H. E. Toussi, "Elastic-plastic Axi-symmetric thermal stress analysis of functionally graded cylindrical vessel," *J. Basic Appl. Sci. Res*., vol. 2, no. 10, pp. 10246-10257, 2012.

[16] Y. Bayat, M. Alizadeh, A. Bayat, "Generalized solution of functionally graded hollow cylinder under torsional load," *Journal of Computational and Applied Research in Mechanical Engineering*, vol. 2, no. 2, pp. 23-32, 2013.

[17] S. K. Gupta, R. L. Dharmani, "Creep transition in thick-walled cylinder under internal pressure," *ZAMM*, vol. 59, pp. 517-521, 1979.

[18] V. R. S. Kumar, A. Sameen, H. D. Kim, T. Setoguchi, B. N. Raghunandan, and S. Ragunathan, Studies on Intrinsic Fluid Dynamics Issues Pertinent to High-performance Rockets, *16th International Symposium on Transport Phenomena, Prague 2005*, pp. 1-6..

Ambarayil Joy Jithin was born in Kerala, India, in 1990. He received the B.Tech degree in Mechanical Engineering from the Manav Bharti University, India. In 2017, he joined the Jeju National University, Republic of Korea as a Master student, and at the same year he join Professor Dong-Won Jung lab for his master research.

Muhammad Sajjad was born in Shawa Dir Lower KPK, Pakistan, in 1994. He received his B.Sc degree in Mechanical Engineering from Swedish College of Engineering & Technology Wah Cantt Pakistan, in 2017.

In March 2018, he joined the Jeju National University as a Master student, and at the same time he join Professor Dong-Won Jung lab for his master research.

Professor Dong-Won Jung was born in Seoul, South KOREA on February 23, 1964. He received the Master degree and the Ph.D. degree in mechanical engineering from KAIST, Daejeon, South KOREA, in 1991 and 1995, respectively.

He is currently a Professor in the Department of Mechanical Engineering, Jeju National University, and has been the Director of the Laboratory of Computational Solid Mechanics, Jeju National University, since 1997. From 2006 to 2008, he was the Director of the factory of Jeju National University. Since 2002, he has been the Dean of the Faculty of Mechanical Engineering, Jeju National University. He is a member of KSPE, IJPEM, IJPEM-GT, KSME, KSTP, etc. He has published more than 250 scientific papers in the field of metal forming, FEM analysis, design developments. His research interests include the die development, structure analysis, impact analysis, optimization, polymer and composite materials, etc.