Optimal Control of Vehicle Active Suspension System

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Abstract—The main aim of Suspension System is to provide the passenger comfort by minimizing the vertical acceleration of the body and to isolate the body irrespective of road profile. A linear model of active suspension system has been developed since it contains all the basic parameters of its performance body deflection, suspension deflection and body acceleration. Two control techniques PID and LQR are used to suppress the vibrations of the system. A comparison between passive and active suspension system using PID and LQR control technique with road disturbance as input has been made. Simulation study has been made on the active suspension system using MATLAB/SIMULINK. Based on the simulation, it has been found that LQR control is better in suppressing the vibrations as compared to PID control as well as Passive suspension system.

Index Terms—Vehicle Active Suspenion system(VASS), Proportional-Integral-Derivative (PID), Linear Quadratic Regulator (LQR)

I. INTRODUCTION

Suspension system of a vehicle is an important part of all automobiles, which basically provides comfort and safety ride to the passengers by isolating the vehicle body from the effect of road irregularities. The passenger comfort is directly proportional to the vertical acceleration of the vehicle body which is transmitted to the passenger. Safety ride is directly belongs to the performance of a vehicle i.e. vehicle handling. Generally we see the trade-off between the two performance criteria of vehicle suspension system i.e. passenger comfort and vehicle handling which were designed traditionally i.e. passive suspension system. Suspension system is basically classified in three categories: passive, semi-active and active suspension system. Passive suspension system consists of spring and damper with fixed spring and damping coefficient, hence through passive suspension system either we achieve better passenger comfort or better vehicle handling. Hence passive suspension system is considered as open loop system.

The problem with the passive suspension is that if someone wants a soft suspension, then it can suppress the body vibration on the effect of road disturbance but reduce the stability of vehicle and if someone wants a hard suspension, then it can reduce the effect of external forces, resulting in reduced suspension but increased body vibration[1]. To overcome this problem we can use either semi-active or active suspension system. Semi-active suspension system contain a variable and controllable damper rather than fixed damper as in case of passive suspension system.

On the other hand, active suspension system capable of adjusting its parameter values itself by using an actuator parallel to the spring and damper to provide better results. Active suspension system is a closed loop system, with a controller in loop to calculate how much energy is dissipated or absorbed by actuator.

A lot has been reported in literature on the control strategies of the active suspension system. Linear quadratic regulator and fuzzy logic controllers are the popular controller used to improve the ride comfort and road handling. A comparison between passive and active suspension system was performed by using different types of road profiles for quarter car model, in which LQR control is found to be better in suppressing the vibrations, than passive system[2][3].LQR and PID control techniques has successfully implemented to the linear active suspension system for a quarter car model. LQR control performs better than PID as far as ride comfort is concerned [4][5][6].A performance comparison between LQR, $\!H_{\!\infty}$ and passive suspension system have been made, in which the results show that LQR control shows a better performance over the $H\infty$ control strategy[7][8][9].

The aim of the paper is to design a linear quadratic regulator controller to improve the passenger ride comfort and road handling for quarter car model. A comparison of body deflection, suspension deflection and body acceleration using PID and LQR control methods with passive suspension has been made using matlab[10].

II. MATHEMATICAL MODELLING

In this research, a quarter vehicle active suspension system with 2 DOF used for modelling of the system. For implementation of two control scheme, we modelled the quarter vehicle system in two different ways: one in time domain and second in frequency domain. In time domain we use a state space modelling for LQR control strategy and frequency domain modelling for PID control strategy

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because PID works on the error signal. Fig. 1 shows the



Figure 1. Quarter Vehicle Model of Active Suspension System

Where,

 m_b = sprung mass (kg) m_w = unsprung mass (kg) k_s = spring constant of car body (N/m) k_t = spring constant of wheel (N/m) b_s = damping coefficient (N-s/m) f_s = Actuator force r = Road disturbance

The parameter of Quarter vehicle model is shown in table 1.

Table 1. Parameters of Quarter Vehicle Model

Parameter	Specification
m_b	300 kg
m_w	60 kg
k _s	16000 N/m
bs	1000 N-s/m
k _t	190000 N/m

The equations of motion for the quarter vehicle suspension system in a vertical plane are written as,

For sprung mass (m_b) -

$$m_b \ddot{x}_b = k_s (x_w - x_b) + b_s (\dot{x}_w - \dot{x}_b) + f_s \tag{1}$$

For unsprung mass (m_w) -

$$m_w \ddot{x}_w = k_t (r - x_w) - k_s (x_w - x_b) - b_s (\dot{x}_w - \dot{x}_b) - f_s$$
(2)

State variables are defined as,

$$x_1 = x_b, \ x_2 = x_w, \ x_3 = \dot{x}_b, \ x_4 = \dot{x}_w$$

Dynamics of the system is described by the following state space model.

quarter vehicle model of active suspension system. State space representation is given by

$$\dot{x} = Ax + Bf_s + Fr \tag{3}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_b} & \frac{k_s}{m_b} & -\frac{b_s}{m_b} & \frac{b_s}{m_b} \\ \frac{k_s}{m_w} & -\frac{(k_s + k_t)}{m_w} & \frac{b_s}{m_w} & -\frac{b_s}{m_w} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_b} \\ -\frac{1}{m_w} \end{bmatrix} [f_s] + \begin{bmatrix} 0 \\ 0 \\ \frac{k_t}{m_w} \end{bmatrix} [r]$$

The frequency domain modelling of suspension system for PID control strategy is also obtained from state space through MATLAB.

III. CONTROLLER DESIGN

In this paper, two types of controller are studied for active suspension system. These are Proportional-Integral-Derivative (PID) and linear quadratic regulator (LQR) controller.

A. Proportional-Integral-Derivative(PID) Controller

PID control is very simple control hence it is widely used in many research and industrial applications. PID basically has a three control i.e. Proportional, Integral and Derivative. A PID is unity feedback controller which calculates error between a desired value called set point (SP) and measured value (MV). The PID aims to minimize the error by manipulating the control variables. For best performance of PID controller, their parameters must be tuned depending upon the nature of the system. The three term of PID controller performs the different control action. P control decreases the rise time of a response, while there is no improvement in offset. I control basically used to eliminate the offset and steady state error but increases the settling time, thus the transient behavior of the system get worse and finally D control action used to get better transient response but stand-alone derivative control introduce a large steady state error. The transfer function of PID controller is,

$$u(t) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{de(t)}{dt}$$
(4)

The obtained tuning parameters of PID are,

For body deflection

 $K_p = 3.15, K_i = 0.8363, K_d = 0.0625$

A) Linear quadratic regulator (LQR) Controller

In case of LQR controller design one must choose the optimal control vector input u(t), so that, the quadratic cost function is minimized. The quadratic cost function is denoted as

$$J_{LQR} = \int_0^\infty (x^T Q x + u^T R u) dt$$
 (5)

Where,

x is state vector and u is control vector

 $Q(n \times n)$ and $R(r \times r)$ are symmetric positive definite weighted matrix $[Q = Q^T \ge 0; R = R^T \ge 0]$

The value of weighted matrix Q(state penalty) and R (control penalty) depends on designer. Designer choose the suitable value of Q and R to find the suitable gain matrix K using MATLAB.

The State variable feedback configuration is shown below in Fig. 2



Figure 2. State variable feedback configuration

A suitable linear full-state feedback control law used as,

$$u(t) = -Kx(t) \tag{6}$$

Where, K is a state feedback gain matrix of LQR defined by

$$K = R^{-1}B^T P \tag{7}$$

Where P satisfies the matrix Algebraic Riccati Equation,

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{8}$$

By taking

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } R = [1]$$

The value of obtained feedback gain matrix K of LQR is given by

$$K = \begin{bmatrix} 0.2846 & -20.4494 & 0.9726 & -0.8260 \end{bmatrix}$$

The Simulink model for LQR control Scheme is shown below in Fig. 3.



Figure 3. The LQR controller based active suspension system

IV. SIMULATION AND RESULTS

For simulation we take two different type of road disturbance. First, the road profile 1 is a single bump as shown in Fig. 4 and represented by eq.9

$$r(t) = \begin{cases} a\{1 - \cos(8 * pi * t)\} & 0.5 \le t \le 0.75 \\ 0 & otherwise \end{cases}$$
(9)

Where, a = 0.05 (road bump height is 10 cm)



Second, the road profile 2 is two bumps as shown in Fig. 5 and represented by eq.10

$$r(t) = \begin{cases} a\{1 - \cos(8 * pi * t)\} & 0.5 \le t \le 0.75 \\ a\{1 - \cos(8 * pi * t)\}/2 & 5.5 \le t \le 5.75 \\ 0 & otherwise \end{cases}$$
(10)

where, a = 0.05 (road bump height is 10 cm and 5 cm)



Figure 5. Road Profile 2

The simulation results are shown in Figs. 6, 7 and 8 for road profile 1.It shows the comparison between Passive, PID and LQR controlled systems for body deflection, suspension deflection and body acceleration respectively with road disturbance.



Figure 7. Suspension Deflection for Road Profile 1



Figure 8. Body Acceleration for Road Profile 1

The simulation results are shown in Figs. 9, 10 and 11 for road profile 2.It shows the comparison between Passive, PID and LQR controlled systems for body deflection, suspension deflection and body acceleration respectively with road disturbance.



Figure 10. Suspension Deflection for Road Profile 2



Figure 11. Body Acceleration for Road Profile 2

Simulation results shows that there is improvement in the ride comfort performance and suppression of vibrations with LQR control as compared to passive and PID based systems in terms of settling time and percentage overshoot in body deflection, suspension deflection and body acceleration.

In comparison to passive and active suspension using PID control scheme, LQR based control scheme gives faster response and lower amplitude in case of body deflection, suspension deflection and body acceleration.

V. CONCLUSIONS

In this paper Proportional-Integral-Derivative (PID) and linear quadratic regulator (LQR) controllers are successfully designed using MATLAB/SIMULINK. Both controllers are capable of stabilizing the suspension system very effectively as compared to passive suspension system. Based on the results discussed in previous section, it accomplished that LQR control scheme gives much better results compared to PID control scheme and passive suspension systems as far as ride comfort and suppression of vibrations are concerned.

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