Command Shaped Robust Control of a Flexible Spacecraft

Waseem A. Butt
Department of Electrical Engineering, College of Electrical and Mechanical Engineering, Rawalpindi, Pakistan
Email: butt_waseem@hotmail.com

***Abstract*—In this paper a model of a hypothetical flexible spacecraft that resembles French multi mission satellite SPOT is selected. This model represents a multivariable system that is difficult to control due to strong cross couplings. In order to get a good insight, initially analysis of the model is carried out from the perspective of control design challenge peculiar to this system. Later a systematic multivariable robust controller is designed based on H_∞ Loop Shaping Design Procedure. The designed controller is able to stabilize the overall system but is unable to minimize oscillations in structure that cause fine pointing issues. To remove the problem of small amplitude oscillations for this design, input command shaping technique is applied that is able to effectively minimize oscillations. Finally, robustness and performance of the designed controller is demonstrated through numerical simulations.**

***Index Terms*—Command shaping, flexibility, robust control, spacecraft, loop shaping design**

I. INTRODUCTION

After the successful launch of Russian Sputnik I into the orbit, there has been an increasing demand to launch communication and space exploration satellites. Many countries have now developed the capability of putting satellites into orbit, resulting in great technological advancement in this field. The cost of putting a satellite into orbit is still very high. Satellites have to be designed to operate in harsh, dynamic and uncertain environment. Such operating conditions pose a challenge on control to provide reliable operation, meeting strict design requirements [1].

When a communication satellite is in orbit, it has to stay on its trajectory and point in a fixed direction. This stringent requirement of pointing accuracy is required to maintain reliable communication. As an example of pointing accuracy, the Hubble Space Telescope contains a telescope capable of allowing the separation of stellar objects only 0.1 arc seconds apart [2]. Modern satellites are designed to be as light as possible to decrease payload for launching. This trend has resulted in satellite structures to be very flexible. The flexibility of the structure causes problem in precise pointing due to lightly damped vibration modes being excited. Control systems for flexible space structures must meet tight performance requirements while being robust to model uncertainties.

Conventional control methods have been used very effectively to control a wide variety of systems. These methods are time tested and are presently being used in most of the industries. In order to carry out analysis of multivariable systems that are coupled, conventional control methods are not so easily manageable. Extensive research has been focused on control of flexible multivariable systems. Attitude control of satellites has been tackled by the help of Self Organizing Fuzzy Logic Controllers (FLC), being a suitable candidate, as engineering experience is usually not available for a satellite system for matching performance through fixed fuzzy rules. Such type of controllers are thus more robust than the fixed rule controllers but the outcome is not always optimum [3], [4]. An integrated feedback/feed-forward control system is proposed by [5], for reducing the vibration of flexible structures for large-angle attitude manoeuvre of the spacecraft. Lyapunov based control integrated with the Nil-Mode-Exciting profiler and an extended Input Shaping technique is utilized to effectively improve the transient vibration of the modal response. Vibration suppression is a desirable feature; active vibration suppression for a two-link flexible manipulator is proposed by [6]. This scheme is composed of an input shaper assisted by piezo actuator for active suppression of residual vibrations. This scheme is, however not suitable for a satellite system in which installation of piezo actuators is not practically viable.

Design method that is more systematic is always preferable, as by adopting such a method reliable result is achieved in the minimum possible time. In this paper, a systematic command shaped Loop Shaping design procedure is presented for the design of multivariable attitude controller. Compared with previous work, this design procedure is systematic and practically suitable for ensuring robustness and good pointing accuracy.

II. MODEL

A hypothetical flexible communication spacecraft has been selected for the analysis and design work [3]. The configuration of the spacecraft is as given in Figure 1, direction of flight of the spacecraft is along the Xs axis (Roll axis) and it is assumed to be operating in fine pointing mode with the Yaw axis directed towards the centre of the Earth. A solar array is attached to the centre

Manuscript received February 7, 2018; revised May 21, 2018.

doi: 10.18178/joace.6.1.1-8

of the asymmetric body of the spacecraft by a boom that remains normal to the pitch axis and is rotated about this axis to maintain the optimum exposure to the sun. This solar array moves at a negative pitch axis to follow the sun. The movement of this solar array causes the spacecraft dynamics to change due to inertia coupling in the roll and yaw axis [7], [8].

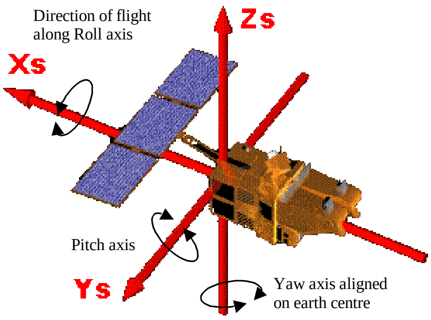

Figure 1: Spacecraft configuration

The spacecraft is assumed to be travelling on its prescribed trajectory (Sun synchronous near polar circular orbit, precessing at the same rate as Earth) [7], [8]. Orbital correction will, therefore, not be considered in this paper.

Precise attitude control of the spacecraft is required to be maintained at all times. Precision in terms of fine pointing is essential for reliable communication, as any oscillation or deviation will result in communication failure. It is achieved by correcting the pitch and roll axis errors. Yaw correction is used to correct the heading orientation. The spacecraft is controlled by the help of three reactions wheels, mounted orthogonally in each axis. The reaction wheels rotate on magnetic bearing to minimise friction and have a linear torque-speed characteristic, which saturates at 0.2 Nm [8].

Three rate-integrating gyroscopes, one on each axis are used to provide the attitude measurement. There is an infrared earth sensor (IRES) on each of the roll and pitch axis, the readings of which are continuously processed with the gyroscope readings. Also there is a sun sensor on the yaw axis that may be used only twice per orbit, due to the orbital configuration assumed [7].

The flexible spacecraft model has a number of modes associated with it. Four modes with the lowest vibrational frequency have been considered, as these modes contribute towards the major structural interaction. For small attitude angle and rate case, the spacecraft dynamics may be represented by following differential equations [3], [7]:

$$\mathbf{T} = \mathbf{E}\dot{\boldsymbol{\omega}} - \sum_{i=1}^{n} \boldsymbol{\lambda}_i \ddot{\boldsymbol{\eta}}_i \tag{1}$$

$$\ddot{\eta}_i + 2\xi_i\omega_{0i}\dot{\eta}_i + \omega_{0i}^2\eta_i = \boldsymbol{\lambda}_i^T \dot{\boldsymbol{\omega}} \quad (\text{i=1, …, n}) \tag{2}$$

where ‘n’ is the number of modes. Equation (1) gives the roll, pitch and yaw axis equations that are coupled by the influence of the bending modes and asymmetry of the spacecraft [7].

Equations (1) & (2) may be expressed in state space form by using partitioned matrix equation as [6]:

$$\left[\begin{array}{c|c} \mathbf{E} & -\boldsymbol{\Lambda} \\ \hline -\boldsymbol{\Lambda}^{\mathbf{T}} & \mathbf{I} \end{array}\right] \begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \ddot{\boldsymbol{\eta}} \end{bmatrix} = \begin{bmatrix} \mathbf{T} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ \dot{\boldsymbol{\eta}}_1 \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\eta}_2 \end{bmatrix} \tag{3}$$

Making use of the relationship:

$$\ddot{\boldsymbol{\varphi}} = \boldsymbol{\Omega}_0 \dot{\boldsymbol{\varphi}} + \dot{\boldsymbol{\omega}} \tag{4}$$

where,

$$\boldsymbol{\Omega}_0 = \begin{bmatrix} 0 & 0 & \omega_0 \\ 0 & 0 & 0 \\ -\omega_0 & 0 & 0 \end{bmatrix} \tag{5}$$

(where ω_0 is the orbital angular velocity), equation (3) becomes [7],

$$\begin{bmatrix} \ddot{\boldsymbol{\varphi}} \\ \ddot{\boldsymbol{\eta}} \end{bmatrix} = \begin{bmatrix} \mathbf{E} & -\boldsymbol{\Lambda} \\ -\boldsymbol{\Lambda}^{\mathbf{T}} & \mathbf{I} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \mathbf{T} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ \dot{\boldsymbol{\eta}}_1 \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\eta}_2 \end{bmatrix} \right\} + \begin{bmatrix} \boldsymbol{\Omega}_0 \dot{\boldsymbol{\varphi}} \\ \mathbf{0} \end{bmatrix} \tag{6}$$

A. *Model Uncertainty*

For a multivariable case each element of the Transfer Function matrix has an uncertainty associated with it. These uncertainty regions have complex shapes at each frequency that require complex mathematics to describe them. This property of the multivariable system automatically introduces structure in the uncertainty representation. We can however, approximate these complex shapes as discs, to give us an additive unstructured uncertainty description involving matrices [9], [10] as,

$$\mathbf{G_p}(\mathbf{s}) = \mathbf{G}(\mathbf{s}) + \boldsymbol{\Delta}_\mathbf{a}(\mathbf{s})$$

in matrix form we have,

$$\mathbf{G_p}(\mathbf{s}) = \begin{bmatrix} \mathbf{g_{11}}(\mathbf{s}) & \mathbf{g_{12}}(\mathbf{s}) & . & \mathbf{g_{1m}}(\mathbf{s}) \\ \mathbf{g_{21}}(\mathbf{s}) & \mathbf{g_{22}}(\mathbf{s}) & . & \mathbf{g_{2m}}(\mathbf{s}) \\ . & . & . & . \\ \mathbf{g_{m1}}(\mathbf{s}) & \mathbf{g_{m2}}(\mathbf{s}) & . & \mathbf{g_{mm}}(\mathbf{s}) \end{bmatrix} + \begin{bmatrix} \delta_{11}(\mathbf{s}) & \delta_{12}(\mathbf{s}) & . & \delta_{1m}(\mathbf{s}) \\ \delta_{21}(\mathbf{s}) & \delta_{22}(\mathbf{s}) & . & \delta_{2m}(\mathbf{s}) \\ . & . & . & . \\ \delta_{m1}(\mathbf{s}) & \delta_{m2}(\mathbf{s}) & . & \delta_{mm}(\mathbf{s}) \end{bmatrix} \tag{7}$$

As a conservative step, we can use the maximum value of gain $\left(\bar{\sigma}(\Delta_a(s)) \le \rho_a(\omega)\right)$ as a single bound on the uncertainty.

The spacecraft model was analyzed to get a single bound of additive uncertainty. To find out the change in the system dynamics due to the solar array rotation, singular value plots were analyzed for the difference of nominal model from the model obtained after rotating the solar array. Singular value plot (function of frequency) gives us the maximum and minimum possible value of gain for any input (direction). Figure 2 shows the singular value plot for the rotation of the solar array in 15 degree steps and also for the maximum value of all the step increments (0~90 degrees).

The solar array is capable of rotating complete 360 degrees, but the situation is same from 90 degrees to 180 degrees. For rotation angles 180 ~ 360 degrees we get similar phenomena as rotation from 0 to 180 degrees due to symmetry. It was observed that worst-case situation occurs for solar array rotation angle of 90 degrees. This analysis was helpful in carrying out the worst-case simulations of the spacecraft.

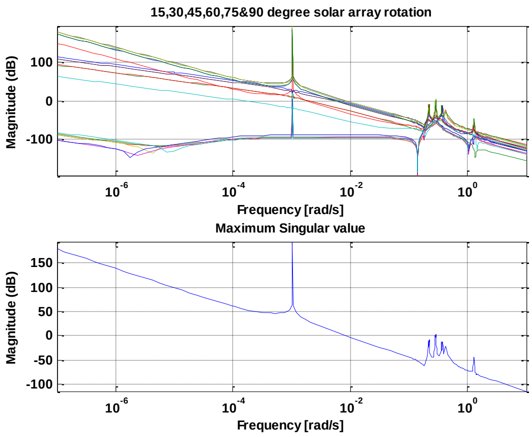

Figure 2. Singular value plots for difference of model from nominal due to solar array rotation and maximum difference of all tested rotations (i.e. 15, 30, 45, 60, 75 & 90 degrees)

III. H_∞ LOOP SHAPING DESIGN PROCEDURE

H_∞ Loop Shaping Design Procedure (LSDP) was selected for the design of multivariable attitude controller of the flexible spacecraft. This design procedure combines the well familiar classical loop shaping and the H_∞ stabilisation technique that provides a design procedure, which is relatively transparent, easy to understand and very effective, [9], [11], [12].

A. Systematic Procedure for LSDP

Following is recommended for a systematic H_∞ loop shaping design [9], 13],

1) Scale the plant inputs and outputs to improve the conditioning of the design problem.
2) Make the plant as diagonal as possible by ordering the inputs.
3) Select pre and post compensators to make the singular values of $\mathbf{G_s} = \mathbf{W_2 G W_1}(\mathbf{s})$ such that there is high gain at low frequencies, low gain at high frequencies and roll off rate of approximately 20 dB/decade at the desired bandwidth.

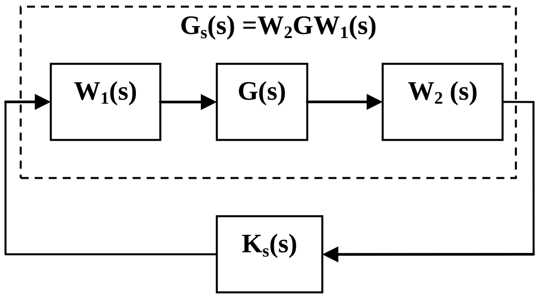

Figure 3. Shaped plant for robust controller synthesis

4) The weight $\mathbf{W_1}$ can be decomposed as $\mathbf{W_1}(\mathbf{s}) = \mathbf{W_p W_a W_g}$. An additional constant weight $\mathbf{W_a}$ may be used with $\mathbf{W_p}$ to align the singular values at the desired bandwidth. Gain matrix $\mathbf{W_g}$ may be cascaded with $\mathbf{W_a}$ to provide control over actuator usage.
5) Find the robustly stabilising controller of the shaped plant $\mathbf{G_s} = \mathbf{W_2 G W_1}(\mathbf{s})$, check the maximum stability margin, if the margin is too small then go back to step 3.
6) If the specifications are not met, further modify the weights.
7) Implement the controller as shown in Figure 4. This configuration is selected so that the references do not excite the dynamics of Ks, which can result in large amount of overshoot (derivative kick).

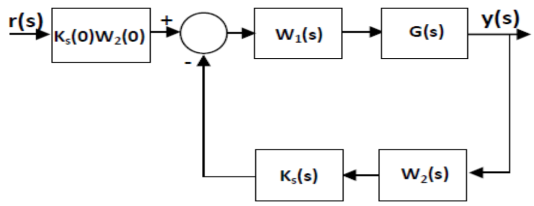

Figure 4. Configuration for practical implementation of the H_∞ loop shaping controller

The advantage of this method is that it is relatively easy to use and is based on classical loop shaping ideas. It also requires no gamma iterations like typical H_∞ optimal design procedure and the procedure is systematic to follow.

B. Flexible Spacecraft Attitude Controller Design

The main objective kept in mind for this design was to get good performance along with satisfactory robustness. Simulink based simulation model (Figure 5) was used to carry out the simulation of the designed controller on the flexible spacecraft structure.

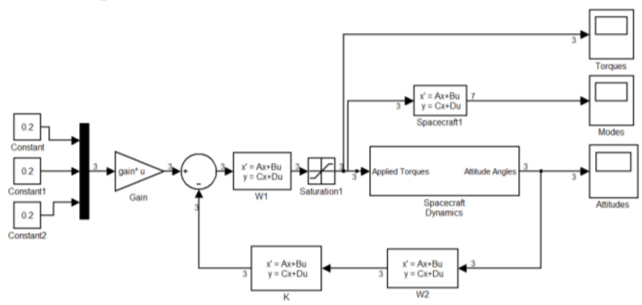

Figure 5. Simulation diagram of flexible spacecraft with H_∞ (LSDP) controller

In order to shape the plant, following weights were used,

$$W_1(s) = \begin{bmatrix} \frac{0.1802s^2+0.02011s+0.00545}{s^2+0.04816s+0.00545} & 0 & 0 \\ 0 & \frac{0.1982s^2+0.02212+0.005995}{s^2+0.04816s+0.00545} & 0 \\ 0 & 0 & \frac{0.3604s^2+0.04023s+0.0109}{s^2+0.04816s+0.00545} \end{bmatrix}$$

$$W_2(s) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

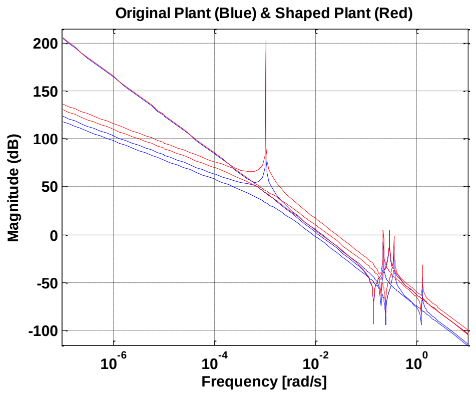

Figure 6. Singular value plot of original and shaped plant

The singular value plot of the shaped plant shows the effect of using a complex pole and a complex zero. In this design, complex pole and zero were selected to get more degree of freedom in shaping the original singular value at the region of interest. It can be seen from the plot that the use of the complex pole makes the singular values plot to dip down. This region was selected because the modal peaks are present in this area and our objective is to avoid the modal peaks at the crossover point. It is similar to adding damping at this point so that the modes are damped out. A complex zero is used at a higher frequency end to cancel the effect of the complex pole. The nature of the problem poses serious problems in flexibility of designs. It can be seen that the natural frequency point of the modes are very close to the cross over point. If we try to decrease the magnitude too much we run into trouble as the slope at crossover becomes greater than -20 dB/decade, thus causing instability. The shaped plant has, as we desired high gain at low frequencies, low gain at high frequencies with a suitable transition between the two.

The next step of using the optimisation algorithm to return the final controller was then carried out, to get a robustly stabilising controller. The problem with this procedure is that the controller order is very high [14].

C. Model Reduction

In order to reduce the order of the controller, curve fitting technique was used by the help of Matlab code. The Matlab code finds and stores the frequency response at various frequencies or frequencies of interest. It then does the fitting of the stored data by using the pre-defined numerator and denominator order, giving relative importance to frequencies and carrying out a number of iterations as defined. If the fitting is not good, the order of the numerator and denominator has to be changed or frequency points redefined. The original controller transfer function matrix returned was a 3x3 matrix. After model reduction, each of the nine transfer functions were reduced to fifth order, which meet the practical memory and computation requirements. Singular value plot (Fig. 7) of the original and the reduced order controller shows precision of the fit,

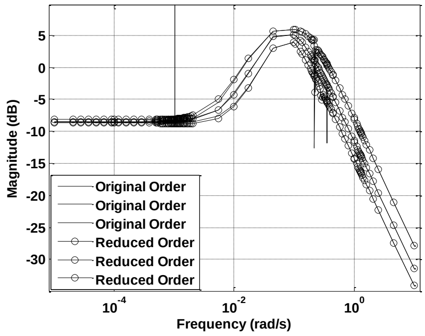

Figure 7. Singular Value plot of Original and Reduced order Controller

D. Simulation Results/Analysis

Following are the plots obtained after carrying out the simulation of the flexible spacecraft (nominal model) with reduced order and original robustly stabilising controller,

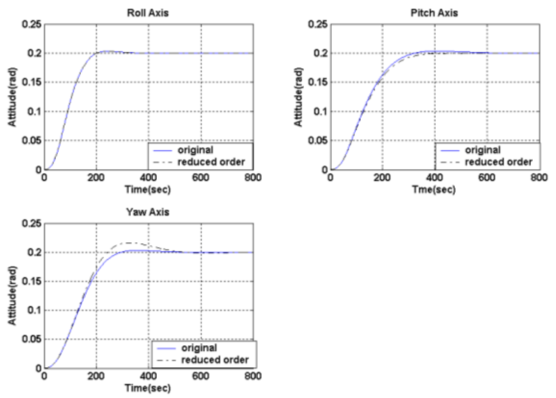

Figure 8. Attitude Response

Attitude responses show that the performance is good and there is very slight difference in results between the reduced order and the full order controller. The results indicate a reasonable selection of weights for shaping the plant.

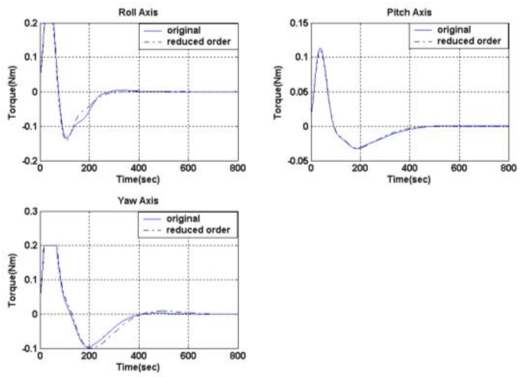

Figure 9: Applied Torques

The torque applied for the control action is reasonable. Initially the Roll and Yaw channel reaction wheels show saturation. This is to quickly remove the effect of the disturbance that has been introduced. The disturbance applied is step, therefore, the controller also responds in a similar fashion.

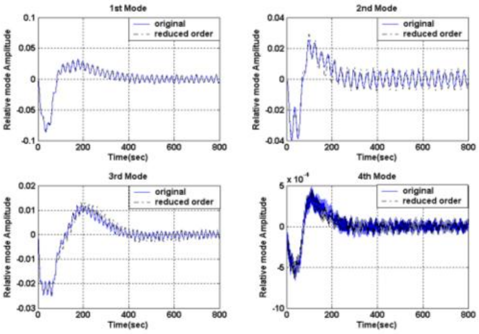

Figure 10. Solar Array Modal Frequencies

Plot of the modal amplitudes for the reduced order and original controller again indicates that appropriate model reduction has been carried out. In order to suppress the vibration modes, care is required to shape the plant so that the modal peaks (see Figure 6) do not cross the zero dB line. It is, however seen that small amplitude vibrations remain for a long time with little damping. The amplitudes of the vibrations are quite less but better performance may be achieved by getting rid of these minor vibrations.

The singular value plots (Figure 11) of Sensitivity and Complementary Sensitivity function indicate good disturbance and noise rejection properties. The singular value of closed loop transfer function shows good reference tracking. Simulations were carried out after introducing disturbance and also after changing the input directions. It was observed that the design is tolerant to disturbance as was predicted by the closed loop relations, for any input direction.

To further analyze the design, robust stability for additive uncertainty [9] was observed. The plot (Figure 12) indicates that the controller is not stable for all frequency values. This does not mean that the design will not work for maximum perturbed plant. The reason for this is that we have been conservative in defining the bound for uncertainty. By being conservative we have catered for some non-existent plants as well. To check the validity of this statement, simulations were carried out after changing the parameters.

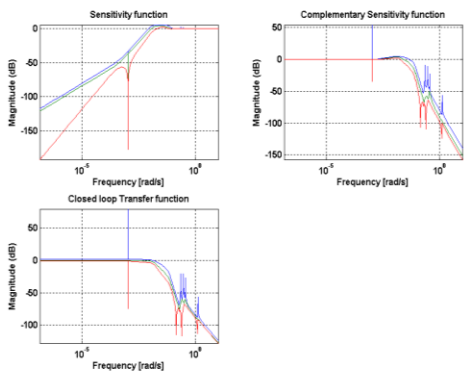

Figure 11. Singular value plots of closed loop relations

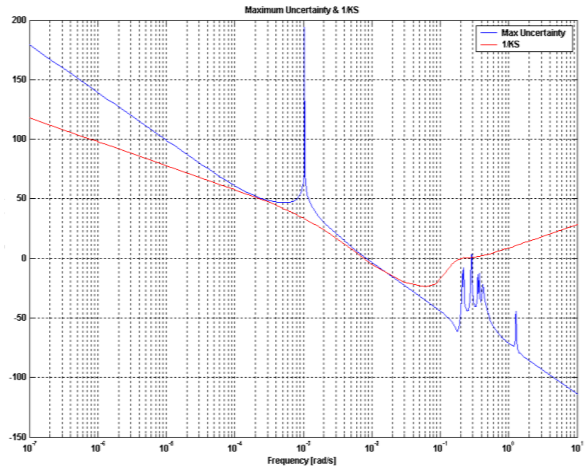

Figure 12. Plot of maximum uncertainty and 1/KS

Robustness as indicated by Figure 12 was verified through extensive simulations. Worst-case condition was determined after which this design remains no good. This condition was reached for solar array rotation of 90 degrees and 55 % reduction in the modal frequency.

IV. Input Command Shaping

Input command shaping method has been selected to take care of the small amplitude modal oscillations that remained after the LSDP design. Input command shaping technique is different from closed loop feedback technique that relies on the system's states to reduce vibrations. This method may be applied to both open loop and closed loop systems with a requisite degree of robustness. There is however, a short move time penalty of the order of one period of the first mode of vibration. By using input command shaping method, the input to the system is shaped in such a way that residual vibrations are minimised or eliminated [15]-[19].

The technique proposed by [15] is based on linear system theory that makes use of impulses of certain amplitude, applied at suitable time intervals to cancel the vibration. The concept of using two impulses to cancel the effect of vibrations induced, for a single mode is given in Figure 13 [15]. When the first impulse is applied the system mode is excited for vibration. On application of the second impulse with a suitable relative amplitude and phase with respect to the first, the vibration imparted by the first impulse is completely removed. If this impulse input sequence is convolved with any arbitrary input, the resultant output also has the same vibration suppression properties. The process can be generalised for more than one vibration mode by convolving the impulse trains designed for each mode. The sequence of impulses act like an input shaper that may be used to shape any input entering the system. The two-impulse input is based on exact knowledge of system's natural frequency and damping ratio. To make the technique more robust, a three-impulse input method was devised as shown in Fig. (14) [15].

The impulse input parameters are given by,

$$K = e^{-\frac{\xi\pi}{\sqrt{1-\zeta^2}}} \tag{21}$$

$$\Delta T = \frac{\pi}{\omega_o\sqrt{1-\zeta^2}} \tag{22}$$

Where,

ζ is the damping ratio of the plant and ω_0 is the un-damped natural frequency.

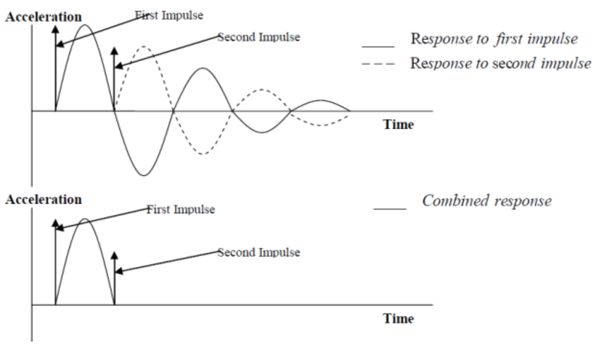

Figure 13. Two impulse input (concept) [15]

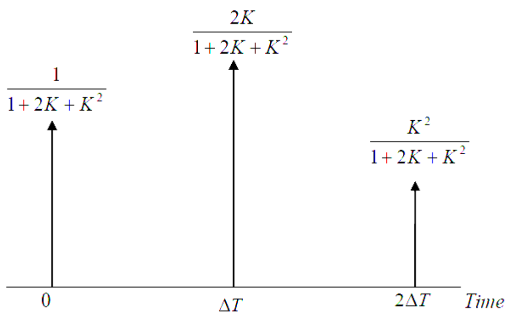

Figure 14.Three impulse input [15]

A. Design of Input Shaper for Flexible Spacecraft

Using equations (21) and (22) the impulse input parameters (for three impulse input method) were calculated for all four modes. The impulse amplitudes and the corresponding timings were convolved to obtain a final sequence of impulses. The input to be shaped was convolved with the final convolved sequence of impulses. The simulation diagram was modified to use the shaped input as given in Figure 15.

To check the efficacy of the design, previously designed reduced order controller that was based on LSDP for good performance was used.

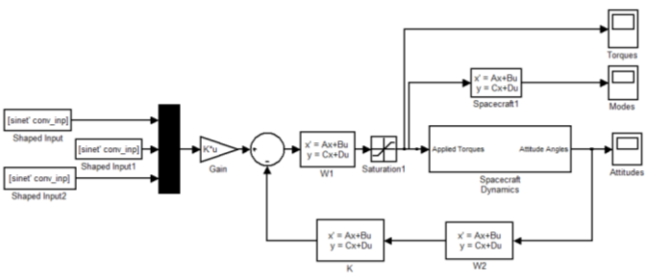

Figure 15. Simulation diagram of flexible spacecraft with H_∞ (LSDP) controller and shaped input

B. Simulation Results/Analysis

For solar array at zero degree position and no change in modal frequency, we have the following results after carrying out the simulation (Nominal model simulation),

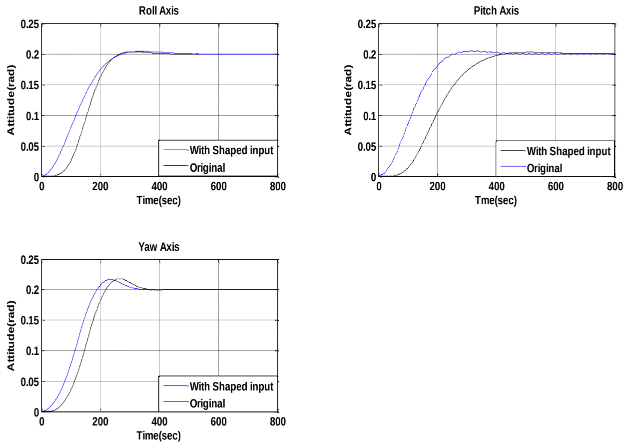

Figure 16. Attitude Response using reduced order controller with shaped input

The attitude angle plots indicate that the performance with the shaped input has become slightly poor, but this was expected due to the design procedure adopted.

The torques applied (Fig. 17) also show that a sudden torque is not applied, that avoids vibrations. This delayed and smooth application of torque results in mitigation of vibrations. A slight saturation of the reaction wheel in the Yaw channel is observed but overall profile of the torque is smooth.

The modal amplitude plots (Fig. 18) show a well behaved response. The high frequency oscillations are almost non-existent now. The design technique has very effectively dealt with the vibrations. Overall the design improvement has resulted in prompt fine pointing accuracy that overweighs a little degradation in performance. Extensive simulations were carried out after varying the spacecraft model parameters. It was observed that the design displays good robustness properties.

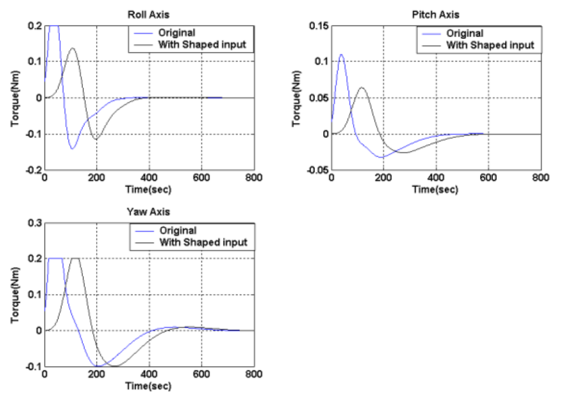

Figure 17. Applied Torques using reduced order controller with shaped input

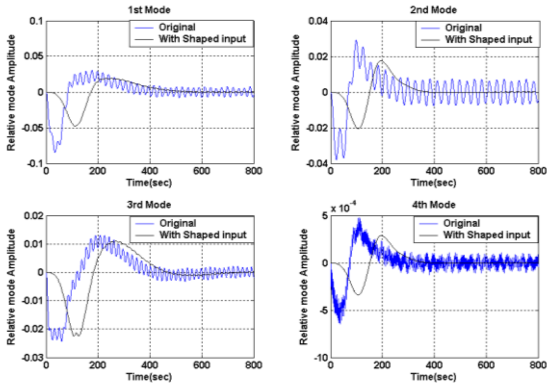

Figure 18. Solar Array Modal Frequencies using reduced order controller with shaped input

V. CONCLUSIONS

In this paper a hypothetical spacecraft model has been presented. Specific features of the model have been explained to address the requirement of control. The model has been analysed for uncertainty by looking at the singular value plots of difference between nominal and changed model due to solar array rotation. The problem has been presented to indicate the need for robust control.

H_∞ LSDP based multivariable attitude controller is designed by following a systematic procedure to realise controllers that are good for any input direction. The order of the designed controller is reduced by using curve fitting technique. The disturbance/noise rejection and reference tracking properties of the design is also indicated to be good. The design, however suffers from small amplitude vibrations of the modes that causes problem of fine pointing.

To further improve the design, command shaping technique has been used. After shaping the input command signal, it is observed that the high frequency oscillations are almost eliminated. The combination of H_∞ LSDP and input command shaping technique, therefore, resulted in a practical solution to the spacecraft attitude correction problem.

APPENDIX A NOTATION

Scalars

n	number of flexure states
ω_0	orbital frequency
ω_{0i}	natural frequency of i th flexure mode
ξ_i	damping coefficient for the i th flexure mode
η_i	vibrational coordinate associated with i th flexure mode
ϕ	roll attitude error
θ	pitch attitude error
ψ	yaw attitude error
$\left.\begin{matrix}\lambda_{\phi i}\\ \lambda_{\theta i}\\ \lambda_{\psi i}\end{matrix}\right\}$	coupling coefficients
$\rho_a(\omega)$	additive uncertainty bound

Vectors

$\mathbf{T}$	external torque acting on spacecraft, $(T_1, T_2, T_3)^T$
$\mathbf{u}$	control signal
$\boldsymbol{\omega}$	body angular velocity
$\mathbf{x}$	state
$\hat{\mathbf{x}}$	state estimate
$\mathbf{y}$	measurement
$\boldsymbol{\eta}$	$(\eta_1, \ldots, \eta_n)^T$
$\dot{\boldsymbol{\eta}}_1$	$\left(2\xi_1\omega_{01}\dot{\eta}_1, \ldots, 2\xi_n\omega_{0n}\dot{\eta}_n\right)^T$
$\boldsymbol{\eta}_2$	$\left(\omega_{01}^2\eta_1, \ldots, \omega_{0n}^2\eta_n\right)^T$
$\boldsymbol{\varphi}$	attitude errors, $(\phi, \theta, \psi)^T$
$\boldsymbol{\lambda}_i$	quantity measuring the coupling of the ith flexure mode with rotations of the centre body about roll, pitch and yaw axes

Matrices

$\left.\begin{matrix}\mathbf{A}\\ \mathbf{B}\\ \mathbf{C}\\ \mathbf{D}\end{matrix}\right\}$	System matrices
$\mathbf{E}$	3x3 inertia matrix, $\begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix}$
$\mathbf{G}(\mathbf{s})$	nominal model
$\mathbf{G}_\mathbf{p}(\mathbf{s})$	plant model

$\mathbf{I}$	identity matrix of compatible order (where used)
$\mathbf{K}(\mathbf{s})$	compensator
$\mathbf{K_s}(\mathbf{s})$	robustly stabilising controller
$\mathbf{0}$	zeroes matrix of compatible order (where used)
$\mathbf{\Delta_a}(\mathbf{s})$	additive perturbation
$\mathbf{\Lambda}$	$\begin{bmatrix} \lambda\phi_1 & . & . & . & \lambda\phi_n \\ \lambda\theta_1 & . & . & . & \lambda\theta_n \\ \lambda\psi_1 & . & . & . & \lambda\psi_n \end{bmatrix}$
$\mathbf{\Omega_0}$	$\begin{bmatrix} 0 & 0 & \omega_0 \\ 0 & 0 & 0 \\ -\omega_0 & 0 & 0 \end{bmatrix}$

REFERENCES

[1] F. Zhang, and G. R. Duan, "Integrated relative position and attitude control of spacecraft in proximity operation missions", *International Journal of Automation and Computing*, vol. 9(4), pp 342-351, August 2012.

[2] P. W. Fortescue, and J. P. W Stark, *Spacecraft Systems Engineering***,** Wiley, 1991.

[3] S. Daley, and K. F. Gill, "Attitude control of a spacecraft using an extended self-organising fuzzy logic controller," in *Proc. Instn Mech. Engrs,* vol 201, no C2, pp. 97-106, 1987.

[4] R. Lasari, et al, "Innovative strategy to improve precision and to save power of a rea-time control process using an online adaptive fuzzy logic controller," *Advances in Fuzzy Systems,* vol. 2013, no 6, May 2013.

[5] H. Okubo and S. Kuwamoto, "Agile attitude control and vibration suppression of flexible spacecraft using control moment gyros," *19th IFAC Symposium on Automatic Control in Aerospace,* September 2-6, 2013, Germany.

[6] Z. Y. Chu and J. Cui, "Experiment on vibration control of a two-link flexible manipulator using an input shaper and adaptive positive position feedback," *Advances in Mechanical Engineering, vol. 7, pp. 1-13, 2015.*

[7] J., Fenton, et al, "Flexible spacecraft attitude measurement and control scheme incorporating state estimation," *Optimal Control Applications and Methods,* vol. 3, pp. 23-40, 1982.

[8] B. Baxter, "Feedback control of a flexible spacecraft with robustness improvement," Mphil thesis, Brunel University, 1991.

[9] S. Skogestad, and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design, Wiley,2nd Ed. 2007.*

[10] J. M. Maciejowski, *Multivariable Feedback Design***,** Addison-Wesley, 1989.

[11] J. Hu, et al, "Systematic H_∞ weighting function selection and its application to the real time control of a vertical take-off aircraft," *Control Engineering Practice*, vol. 8, pp. 241-252, August 1999.

[12] K. Zhou and J. C. Doyle, *Essentials of Robust Control.* Prentice Hall, May 1999.

[13] D. McFarlane and K. Glover, "A loop shaping design procedure using H_∞ synthesis," *IEEE Transactions on Automatic Control,* vol. 37, no 6, June 1992.

[14] C. Valentin-Charbonnel, et al, "Low order robust attitude control of an earth observation satellite," *Control Engineering Practice,* vol. 7, pp. 493-506, January 1999.

[15] N. C. Singer, and W. P. Seering, "Preshaping command inputs to reduce system vibration", *Journal of Dynamic Systems, Measurement and Control*, vol. 20, pp. 291-298, March-April 1990.

[16] W. Singhose, et al, "Comparison of command shaping methods for reducing residual vibration," *Proceedings of the European Control Conference*, 1995.

[17] W. Singhose, R. Eloundou, and J. Lawrence, "Command generation for flexible systems by input shaping and command smoothing," *Journal of Guidance, Control, and Dynamics,* vol. 33, no 6, pp. 1697-1701, November-December 2010

[18] C. Conker, et al, "A review of command shaped techniques for elimination of residual vibrations in flexible-joint manipulators," *Journal of Vibro engineering,* vol. 18, no 5 August 2016.

[19] M. Romano, et al, "Experiments on command shaping control of a manipulator with flexible links," *Journal of Guidance, Control, and Dynamics*, March-April 2002.

Waseem A. Butt received his B.E. degree in Electrical Engineering from the College of Electrical and Mechanical Engineering, Rawalpindi, Pakistan in 1993, his M.Sc. degree in Control Systems with distinction from The University of Sheffield, UK in 2004 and his PhD degree in Control Theory and Control Engineering from Beijing University of Aeronautics and Astronautics (BUAA), Beijing, China in 2012. He is recipient of Laverick-*Webster-Hewitt Prize* and has also been awarded with distinguished foreign student scholarship award of the Beijing Municipal Government and BUAA, China (2011). He is currently affiliated with College of Electrical and Mechanical Engineering, NUST, Rawalpindi, Pakistan. His research interests include robust and adaptive control of flight vehicles.