Optimal Tuning of a LQR Controller for an Inverted Pendulum Using the Bees Algorithm

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***Abstract*—The paper presents the design of a Linear Quadratic Regulator (LQR) controller for an Inverted Pendulum (IP) system using The Bees Algorithm (BA) to provide optimal parameters of LQR. Inverted Pendulum is a typical highly nonlinear and unstable system and widely used as a benchmark for testing different control techniques in control theory. LQR is an optimal control method that can achieve the closed loop control of multivariable dynamical systems with minimum control effort. In LQR controller design, state (Q) and control (R) weighting matrices are main design parameters which are defined by designer using trial and error method in general. Automatic tuning of the weighting matrices with an optimization algorithm ensure expected efficiency from LQR controller. Also the technique consider to design of time domain specifications like overshoot, rise time, settling time, and steady state error. In this paper, The Bees Algorithm optimizes the weighting matrices of LQR controller be able to move the cart in reference input with the minimum deflection of the pendulum's angular position. The tuning aim is to minimize the objective function which consists of time domain responses of system in MATLAB/Simulink. The paper gives the simulation results obtained for the system demonstrating the efficiency and robustness of the proposed design method of LQR controller.**

***Index Terms*—the bees algorithm, LQR controller tuning, optimal control, inverted pendulum**

I. Introduction

The single Inverted Pendulum (IP) system with two degrees of freedom (the angle of the pendulum and the linear position of the cart) constitutes the theoretical background of SEGWAY since it is highly unstable. The stabilizing of the system is significant an issue in the field of control engineering to verify modern control theories.

There are presented variety methods for Inverted Pendulum; classical methods such as PID and LQR controllers and artificial intelligent control techniques such as Fuzzy Logic Control [1].

The performance of model based controllers depend on selection and tuning technique of one or more design parameters which are usually determined with using trial and error techniques. Inverted pendulum is one of the benchmark problems of control theory for illustration and comparison of optimization methodologies. There are different optimization algorithm to obtain optimally LQR controller for stable motion of Inverted Pendulum such as Memetic Algorithm (MA) [2], Multi-Objective Differential Evolution Algorithm (MODE) [3], Genetic Algorithm (GA) [4], Quantum Particle Swarm Optimization Algorithm (QPSO) [5] and Artificial Bee Colony Algorithm (ABC) [6].

Following a definition of the system and LQR controller in sections 1-2, The Bees Algorithm is briefly described in section 3. In section 4, the paper gives the simulation results of the Inverted Pendulum demonstrating the of the optimal LQR controller design.

II. Modelling of the Inverted Pendulum System

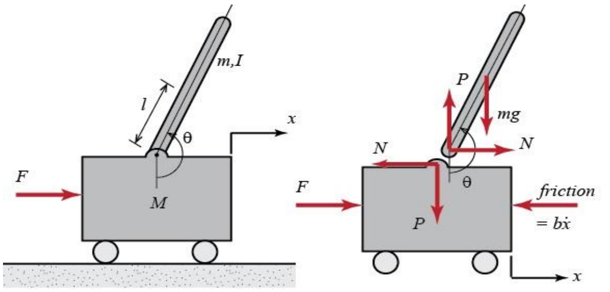

Figure 1. Model and free body diagram of IP system

Fig. 1 shows that the general model of the Inverted Pendulumsystem [7]. The model consists of a free moving pendulum with the mass *m* and length *l* located in the vertical direction to the cart with mass *M*, which is free to move in the *x* direction when a force *F* is applied to the system.

TABLE I. Parameters of the IP System

Symbol	Parameter	Value	Unit
M	Mass of the cart	0.5	[kg]
m	Mass of the pendulum	0.2	[kg]
b	Friction of the cart	0.1	[N/m/s]
l	Length of the pendulum	0.3	[m]
I	Inertia of the pendulum	0.006	[kgm2]
g	Gravity	9.81	[m/s2]

As show in Fig. 1, the free body diagram of the system. From the free body diagram, the following linearized (about $\theta=\phi+\pi$, ϕ represents a small angle from the vertical upward direction) dynamic equations in the

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horizontal direction and vertical direction are determined in Eq. (1) and Eq. (2). The dynamic equations can be represented in a state-space form and an output equation as stated in Eq. (3) and Eq. (4). The parameters of the Inverted Pendulum system are shown in Table I.

$$(I+ml^2)\ddot{\phi}-mgl\phi=ml\ddot{x} \tag{1}$$

$$(M+m)\ddot{x}+b\dot{x}-mg\ddot{\phi}=u \tag{2}$$

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -0.1818 & 2.6727 & 0\\ 0 & 0 & 0 & 1\\ 0 & -0.4545 & 31.1818 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ 1.8182\\ 0\\ 4.5455\end{bmatrix}u \tag{3}$$

$$y=\begin{bmatrix}1 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ 0\end{bmatrix}u \tag{4}$$

III. LQR Controller Design for the IP System

LQR control problem may be stated to find the optimal input $u(t) = -Kx(t)$ sequence that minimizes the quadratic performance index (J) which is defined in Equation (5). Where $Q = diag\ [q_1 \ldots q_n]$ and $R = diag\ [q_1 \ldots q_m]$ are the symmetric, positive state and control weighting matrices. The state feedback gain matrix is defined as $K = (R^{-1} B^T P)$. Where P is the unique symmetric, positive semi-definite solution to the algebraic Riccati Equation as depicted $PA + A^TP + Q - PBR^{-1} B^TP = 0$.

$$J=\int_0^{\infty}[x^TQx+u^TRu]dt \tag{5}$$

This paper focuses on using The Bees Algorithm to select the weighting matrices in LQR controller design. The controller design criteria: The maximum percent overshoot (*%OS*) is less than *22.5%*, the settling time (t_s) is less than *5* second, the steady-state error (e_{ss}) is less than *2%* for the cart's position (x) and the pendulum's angle (ϕ). LQR control block diagram of the system is present in Fig. 2.

IV. The Bees Algorithm for the LQR Controller Design

This section summarizes the main steps of The Bees Algorithm. Fig. 4 shows the pseudo-code for the algorithm in its simplest form. Reference [8]-[10] describes The Bess Algorithm in detail. The algorithm requires a number of parameters to be set, namely: number of scout bees (n), number of sites selected for exploitation out of n visited sites (m), number of top-rated (*elite*) sites among the m selected sites (e), number of bees recruited for the best e sites (*nep*), number of bees recruited for the other (m-e) selected sites (*nsp*), initial size of each patch (a patch is a region in search space that includes a visited site and its neighborhood and stopping criterion).

The algorithm starts with the n scout bees being placed randomly in the search space. The fitnesses of the sites visited by the scout bees are evaluated in step 2. In step 3, the m sites with the highest fitnesses are designated "*selected sites*" and chosen for neighborhood (*ngh*) search. In steps 4 and 5, the algorithm conducts searches in the neighborhood of the selected sites, assigning more bees to the best e sites. Selection of the best sites can be made directly according to the fitnesses associated with them. Alternatively, fitness values can be used to determine the probability of sites being selected. Searches in the neighborhood of the best e sites which represent the most promising solutions are made more detailed by recruiting more bees for them than for the other selected sites. Together with scouting, this differential recruitment is a key operation of The Bees Algorithm. In step 5, for each patch, only the one bee that has found the site with the highest fitness (the "*fittest*" bee) will be selected to form part of the next bee population. In steps 6-8, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions. These steps are repeated until a stopping criterion is met. At the end of each iteration (*itr*), the colony will have two parts to its new population - representatives from each selected patch and other scout bees assigned to conduct random searches.

The idea is to search for the optimal values of the weighting matrices of LQR controller with respect to a determined objective function. The objective function (J_e) is created according to existed studies [11-14]. It is defined in Equation (6).

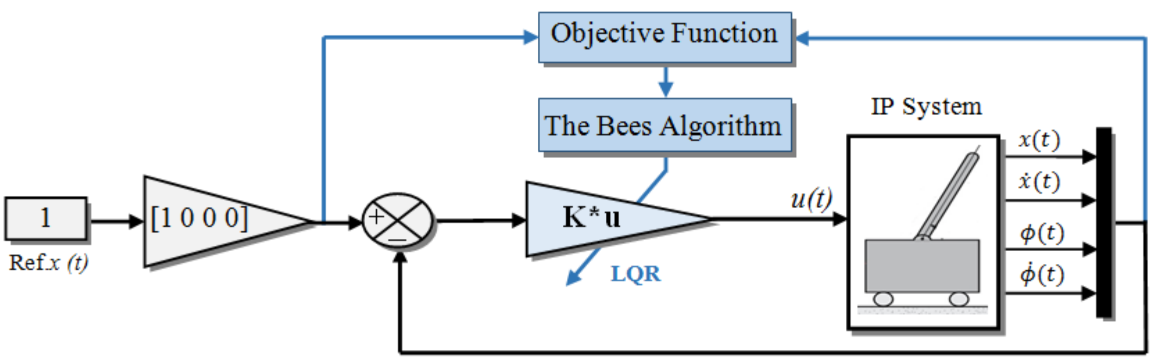

Figure 2. The block diagram of LQR controller tuning

$$J_e = \left(5Xt_r + 2{,}5Xt_s + Xt_p + 10X_{max} + 10^4 Xe_{ss}\right) + \left(20\phi_{norm} + 2\phi t_s + 20\phi t_p + 170\phi_{max} + 2.10^6 \phi e_{ss}\right) \quad (6)$$

where t_r is the rise time, t_s is the settling time, t_p is the peak time, *max* is the maximum overshoot, e_{ss} is the steady state error, *norm* is angle of matrix norm. The constant values are weight factors.

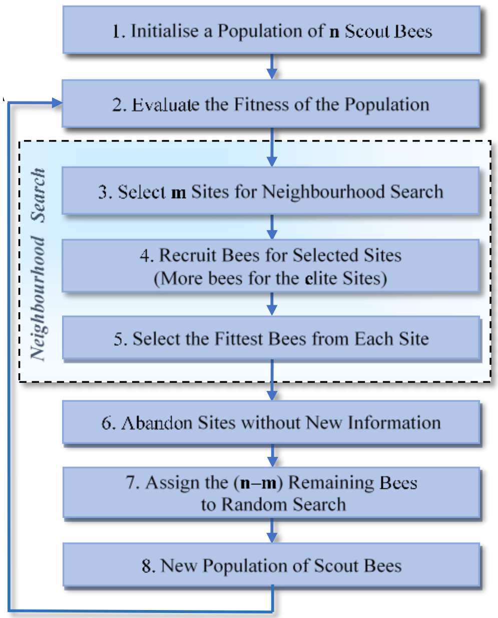

Figure 3. The block diagram of LQR controller tuning

TABLE II. THE BEES ALGORITHM PARAMETERS

n	m	e	nep	nsp	ngh	itr
50	20	10	20	15	0.01	100

TABLE III. OPTIMIZATION RANGE OF PARAMETERS

	Q_1	Q_2	Q_3	Q_4	R_1
Max.	100	10	1000	10	2
Min.	0	0	0	0	0

V. SIMULATION RESULTS

The parameters of The Bees Algorithmare demonstratedin Table II and the optimization range of LQR parameters are set as in Table III. In Fig. 2, the block diagram of LQR controller for the Inverted Pendulumis modelled in MATLAB/Simulink. After optimization, the tuned weighting matrices of the LQR controller are obtained as *Q = diag [22.733, 7.993, 46.269, 3.431]* and *R= diag [0.1671]* and the state feedback gain matrix is *K = [-4.078, -5.216, 31.275, 6.107]* and the control system is run for 7 seconds in Simulink.

Fig. 4 show that the graphical results obtained by the LQR controller tuned by The Bees Algorithm for the angular displacement of the pendulum and the linear displacement of the cart. The LQR controller responses of system's outputs in time domain specification are presented in Table IV.

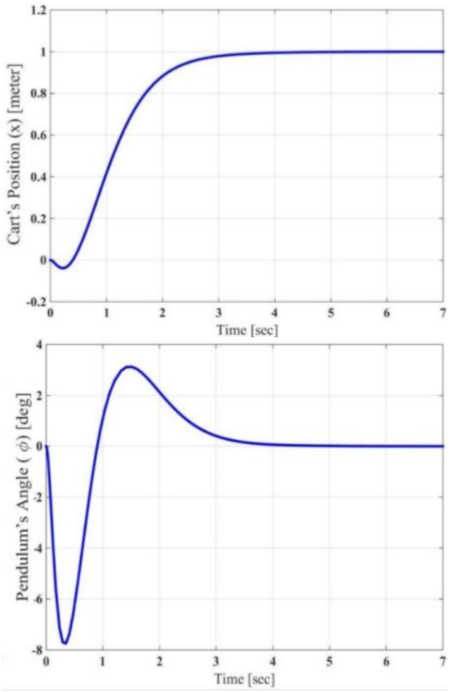

Figure 4. The cart's position and the Pendulum's angle response

TABLE IV. CONTROL PERFORMANCE OF THE CART'S POSITION AND THE PENDULUM'S ANGLE

<table>
<tr><th></th><th>Cart Position</th><th colspan="2">Pendulum Angle</th></tr>
<tr><td>Rise Time (tr)</td><td>1.51 [s]</td><td colspan="2">---</td></tr>
<tr><td>Settling Time (ts)</td><td>3.02 [s]</td><td colspan="2">3.5 [s]</td></tr>
<tr><td>Peak Time (tp)</td><td>6.9 [s]</td><td colspan="2">0.135 [s]</td></tr>
<tr><td rowspan="2">Overshoot (OS)</td><td rowspan="2">0.1 %</td><td>Max.</td><td>3.12 [deg]</td></tr>
<tr><td>Min.</td><td>-7.75 [deg]</td></tr>
<tr><td>Steady State Error (ess)</td><td>0 [m]</td><td colspan="2">0 [deg]</td></tr>
</table>

The Bees Algorithm was programmed in MATLAB and run on an *Intel(R) Core(TM) i7-4700HQ CPU 2.40 GHz PC* with *16.0 GB* memory. The algorithm run for *100* iterations to find a minimum value for the objective function (Equation (6)) with the settings given for the algorithm. Computing time of the algorithm is *1188.7 second* and the ratio of CPU usage is *12%*. The Bees Algorithm complexity analysis and comparing with other optimization algorithm are extensively investigated in benchmarking studies [10].

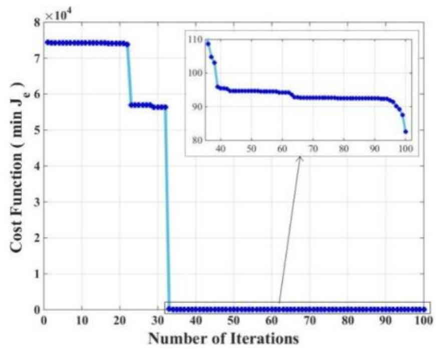

Figure 5. The performance analysis of the bees algorithm for the LQR controller design

Because of the algorithm is used for a LQR controller design as a diversity method in this paper, it was not compared to other algorithms. So the performance analysis of The Bees Algorithm to design of a LQR controller for the Inverted Pendulum system is presented in Fig. 5.

VI. CONCLUSION

The tuning of LQR controller for an unstable Inverted Pendulum with The Bees Algorithm is reported in this paper. The Bees Algorithm is a diversity method to select optimal weighting matrices of LQR controller. Q and R matrices are tuned using The Bees Algorithm to minimize the angle of the pendulum and the positioning errors of the cart. The control responses of system in time specification are given in Table IV. The results obtained show that the optimal LQR controller produced with the tuning method is very successfully to control of the system within controller design criteria. Moreover, the optimization technique proposed in this study ensures to tuning the expected specifications of time domain response of Inverted Pendulum system in optimization. It can improve performances in terms of control accuracy and speed of response by changing the objective function, the optimization parameters and ranges.

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REFERENCES

[1] P. Ponce, A. Molina, and E. Alvarez, "A review of intelligent control systems applied to the inverted-pendulum problem," *American Journal of Engineering and Applied Sciences*, vol. 7, no. 2, pp. 194-240, 2014.

[2] J. Zhang, L. Zhang, and J. Xie, "Application of memetic algorithm in control of linear inverted pendulum," in *Proc. 2011 IEEE International Conference on Cloud Computing and Intelligence Systems*, China, 2011, pp. 103-107.

[3] I. B. Tijani, R. Akmeliawati, and A. I. Abdullateef, "Control of an inverted pendulum using mode-based optimized LQR controller," in *Proc. 2013 IEEE 8th Conference on Industrial Electronics and Applications* , Australia, 2013, pp. 1759-1764.

[4] C. Wongsathan and C. Sirima, "Application of GA to design LQR controller for an inverted pendulum system," in *Proc. IEEE Int. Conf. Robotics and Biomimetics*, Thailand, 2008, pp. 951-954.

[5] K. Hassani and W.S. Lee, "Optimal tuning of linear quadratic regulators using quantum particle swarm optimization," in *Proc. International Conference of Control, Dynamic Systems and Robotics*, no. 59, Ottawa, Canada, 2014.

[6] B. Ata and R. Coban, "Artificial bee colony algorithm based linear quadratic optimal controller design for a nonlinear inverted pendulum," in *Proc. International Conference on Advanced Technology &Sciences*, Antalya, Turkey, 2014, pp. 47-52.

[7] Information. [Online]. Available: http://ctms.engin.umich.edu/CTMS

[8] D. T. Pham, et al, "The bees algorithm–A novel tool for complex optimization problems," in *Proc. the 2nd Virtual International Conference on Intelligent Production Machines and Systems*, 2006, pp. 454-459.

[9] D. T. Pham, A. Ghanbarzadeh, E. Koc, S. Otri, S. Rahim, and M. Zaidi, *The Bees Algorithm Technical Note*, Manufacturing Engineering Centre, Cardiff University, UK., 2005.

[10] D. T. Pham and M. Castellani, "Benchmarking and comparison of nature-inspired population-based continuous optimization algorithms," *Soft Computing-A Fusion of Foundations, Methodologies and Applications*, vol. 18, pp. 871-903, 2013.

[11] D. T. Pham, E. Koc, M. Kalyoncu, and M. Tınkır, "Hierarchical PID controller design for a flexible link robot manipulator using the bees algorithm," in *Proc. 6th International Symposium on Intelligent Manufacturing Systems*, Sakarya, Turkey, 2008, pp. 757-765.

[12] D. T. Pham and M. Kalyoncu, "Optimization of a fuzzy logic controller for a flexible single-link robot arm using the bees algorithm," Cardiff University, UK, 2009.

[13] A. A. Fahmy, M. Kalyoncu, and M. Castellani, "Automatic design of control systems for robot manipulators using the bees algorithm," in *Proc. the Institution of Mechanical Engineers, Part I, Journal of Systems and Control Engineering*, vol. 226, no. 4, 2012, pp. 497-508.

[14] M. A. Şen and M. Kalyoncu, "Optimization of a PID controller for an inverted pendulum using the bees algorithm," in *Proc. Int. Conference on Mechatronics and Automation Science, Applied Mechanics and Material Journal, Section: Manufacturing Science and Technology VI*, vol. 789-790, Paris, France, 2015, pp. 1039-1044.

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