Development of a New Algorithm to Control Excitation of Particular Mode of a Building

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***Abstract*—In this study, an active control algorithm is developed in order to control a particular mode of a shear building. The location of the actuator is considered at the first floor level for an easy application of the control force. In order to achieve the desired control, the sliding surface is designed in such a way that the effect of a particular mode of the structure at an ideal sliding is nullified. The control force is designed using a signum function in order to achieve the reachability to the sliding surface. In order to demonstrate the effectiveness of the control algorithm, a four-story shear building with uniform mass distribution is considered under an earthquake ground excitation. A secondary structure is also attached to the shear building having its frequency tuned to the second mode of the primary structure. The algorithm found to work very well in suppressing the second mode of the shear building and provides a tremendous reduction in the responses of the secondary structure.**

***Index Terms*—sliding mode control, secondary structures, structural vibration**

I. Introduction

Active control strategy is achieving a wide acceptance all over the world for control of response in civil structures subjected to wind and earthquake or any other loads. In active control strategy, the behavior of a structure can be adapted and hence, this strategy is preferred over the passive one under a constantly changing environment. Performance of a structure [1] can be enhanced easily by the combination of passive along with active and/or semi-active controls. For example, during a strong shaking, a base isolated structure (passive control) is subjected to a huge base as well as super structure displacements. In a recent study, [2] has demonstrated that a combination of passive and active control strategies can reduce the superstructure motions without increasing the inter-storey drifts. However, in spite of huge potential, the main challenge in implementing such (active) control technology remains in power requirement, cost effectiveness, adaptability of gain at different frequency regimes, robustness of algorithm etc. For control of structural response, many algorithms have been utilized for optimal gain design such as linear quadratic Gaussian (LQG), sliding mode control, pole placement, and fuzzy control. A good review of such algorithms can be found in [3]. Apart from the existing conventional algorithms, researchers have developed new algorithms as well for specific problems by modifying the conventional one. To name a few, Feng, Shinozuka and Fujii [4], [5], Fujii and Feng [6], [7] used instantaneous optimal control and bang-bang control algorithm; Yang, Wu, Reinhorn, and Riley [8] used a well-known sliding mode control; Amini and Vahdani [9] combined three control algorithms such as probabilistic optimal control, fuzzy logic-based control and optimal control theory; Pnevmatikos and Gantes [10] proposed a modified pole placement algorithm; Cetin, Zergeroglu, Sivrioglu, and Yuksek [11] developed a nonlinear adaptive controller for a magneto-rheological MR damper through Lyapunov-based techniques; Kim [2] used skyhook control and Fuzzy logic-based control; Ozbulut Bitaraf and Hurlebaus [12] used an Adaptive Fuzzy Neural Controller (AFNC) and Simple Adaptive Control (SAC) to compare with LQG control algorithm; Park and Park [13] proposed a minmax algorithm. It may be noted that most of these studies deal with the response reduction of base isolated structures. From literature, the idea of a sliding mode control is first evolved in Russia in the early 1960s.However, it become popular in mid 1970s from the work done by Utkin [14].The concept of sliding mode control have widely applied in the area of flight control, space system and robots, control of electric motors or many other adaptive schemes. However, in case of civil structure like building, bridges the application is limited. The sliding mode controlled system is often termed as the variable structure control system. In this, the system becomes a class of systems for which the control law is changed intentionally by some defined rules, which are framed based on the states of the system. The rules for change in the control law or switching can be obtained from a condition, known as the sliding surface, which provides the desired behavior of the system. The control law is designed in such a way to bring the system to the sliding surface. An ideal sliding is established whenever the system reaches the sliding surface. However, depending upon the switching of control force, the system oscillates about the sliding surface. If an infinite switching is possible, the ideal sliding can be achieved. In this study a control algorithm is developed using the sliding mode control that intends to control a particular mode of a primary structural system.

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II. Formulation

An n-degree-of-freedom building system is considered with columns having stiffness k_1, k_2,k_3...k_n and masses m_1, m_2, m_3,...m_n as shown in Fig. 1. For a damped vibration, the structure can be idealized as a spring-mass-damper system. The actuator location is assumed to be at the first floor of the building satisfying the controllability criteria. The justification of selecting such an actuator location will be discussed later in this section. Let X_i denotes the displacements of the i^{th} floor of the primary system where, each displacement is considered with respect to the ground at time instant t.

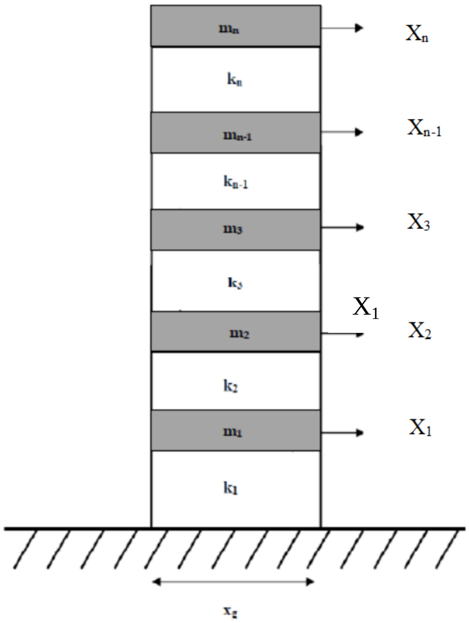

Figure 1. Shear building model.

A. System Equations

The equation of motion of the system may be described as follows:

$$[m]\{\ddot{X}\}+[k]\{X\}+[c]\{\dot{X}\}=\{b\}u+\{F\} \tag{1}$$

where *[m]*, *[k]* and *[c]* denote the mass, initial stiffness and the damping matrices of the system respectively. Here damping is considered as the Raleigh damping. Here, *{b}* is an $n\times 1$ location vector with the first row as unity and rest of the rows are zero; *u* is the control force to be applied by the actuator; *{F}*is an external excitation. The state space formulation of (1) can be written as follows:

$$\begin{bmatrix}[O] & [m]\\ [m] & [O]\end{bmatrix}\begin{Bmatrix}\ddot{X}\\ X\end{Bmatrix}+\begin{bmatrix}-[m] & [O]\\ [c] & [k]\end{bmatrix}\begin{Bmatrix}\dot{X}\\ X\end{Bmatrix}=\begin{Bmatrix}\{o\}\\ \{b\}\end{Bmatrix}u+\begin{Bmatrix}\{o\}\\ \{F\}\end{Bmatrix} \tag{2}$$

In (2), *[O]* is an $n\times n$ null matrix and *{o}* is an $n\times 1$ null vector. Eq. (2) can be modified as follows:

$$[M_a]\{\dot{X}_a\}+[K_a]\{X_a\}=\{B\}u+\{F\}_a \tag{3}$$

where *[M]* is a $2n\times 2n$ matrix defined as $[M_a]=\begin{bmatrix}[O] & [m]\\ [m] & [O]\end{bmatrix}$ $[K_a]$ is a $2n\times 2n$ matrix defined as $[K_a]=\begin{bmatrix}-[m] & [O]\\ [c] & [k]\end{bmatrix}$; *[m]*, *[c]* and *[k]*are defined earlier. here, $\{B\}=\begin{Bmatrix}\{o\}\\ \{b\}\end{Bmatrix}$ and $\{F_a\}=\begin{Bmatrix}\{o\}\\ \{F\}\end{Bmatrix}$ is the external source of excitation in states pace form. $\{X_a\}$ is state vector combining the velocity and the displacement of the floors $\{X_a\}=\begin{Bmatrix}\{\dot{X}\}\\ \{X\}\end{Bmatrix}$.

B. Sliding Mode Control

The main purpose of a sliding mode control algorithm is to bring the system to an ideal sliding surface where the desirable behavior of the system can be achieved. Hence, the equation of sliding surface can be considered in the following way to control the p^{th} mode of the structure.

$$s=\{S\}\{X_a\}=\left[\{\phi_P\}^T[m]\quad \{o\}^T\right] \tag{4}$$

where $\{o\}^T$ is an$1\times n$ row matrix with all the elements as zero; $\{\phi_P\}$is the mass normalized modal vector for p^{th} mode of the system (1) and $s = \{S\}\{X_a\} = 0$ is the equation of the sliding surface where $\{S\}=[\{\phi_p\}^T[m]\ \{o\}^T]$. This provides $\{S\}\{X_a\} =\{\phi_p\}^T[m]\{\dot{X}\}$. Eq.(4) takes the following form:

$$s=\{\phi_P\}^T[m]\sum_{1}^{n}\{\phi_r\}\dot{\eta}_r=\dot{\eta}_P=0 \tag{5}$$

where $\dot{\eta}_P$ is the modal velocity for p^{th} mode for the system. It can be observed from (5) that the switching surface nullify the effect of the p^{th} modal velocity once an ideal sliding is established. In other words, the p^{th} modal component is diminished at an ideal sliding. One may note that in this study, the velocity feedback is considered for designing the sliding surface which is same for all coordinates systems defined in Section A. Thus, for verifying an ideal sliding condition, only the knowledge of the instantaneous velocity of the system is required.

C. Control Design

Number footnotes here, the control force to be applied for bringing the system to the ideal sliding condition is derived. It is assumed that the location vector *{B}* (2) and (3) satisfies the controllability condition for a normal shear building. The state space formulation (2) can be written as,

$$\{\dot{X}_a\}=[\Lambda_a]\{X_a\}+\{\bar{B}\}u \tag{6}$$

where, $[\Lambda_a] = [M_a]^{-1}[K_a]$ and $\{\bar{B}\} = [M_a]^{-1}\{B\}$. Control force *u* has to be considered in such a way that it bring the system to the sliding surface $s = 0$. The condition needed to be satisfied for reaching the sliding surface written as follows:

$$s\dot{s}<0 \tag{7}$$

It can be easily understood from (7) if s and $\dot{s}$ are of opposite sign, the system always moves towards the sliding surface $s = 0$. This criteria is known as reachability condition. Thus in sliding mode control, the choice of the sliding surface governs the performance of the system whereas the control law is designed to guarantee reachability condition. The control force is selected as the following:

$$u = -(\{S\}\{\bar{B}\})^{-1} n\, sig(s) \tag{8}$$

From (6) and (8), $\dot{s}$ can be written as follows:

$$s = \{S\}[\Lambda_a]\{X_a\} - \{S\}\{\bar{B}\}(\{S\}\{\bar{B}\})^{-1} n\, sig(s) \tag{9}$$

where n is a positive integer. *sig(s)* is the signum function of sliding surface s. The control force u has a constant magnitude changing its sign depending on the sign of the sliding surface. The expression for reachability thus can be obtained as below

$$s\dot{s} = -s\{\phi_P\}^T[k]\{X\} - s\{\phi_P\}^T[c]\{\dot{X}\} - s\, n\, sig(s) + s\{\phi_P\}^T\{F\} \tag{10}$$

Simplifying (10), we may obtain the following form.

$$s\dot{s} = -s(\eta_P\omega_P{}^2 + 2\xi_P\omega_P\dot{\eta}_P) - n\,|\,s\,| + sF^P \tag{11}$$

where η_p is the p^{th} modal coordinate; F_p is the p^{th} modal component of the external excitation. In case of ground excitation, F_p will be $\alpha_P\ddot{x}_g$ where α_P is the modal participation factor for the p^{th} mode and $\ddot{x}_g$ is the ground acceleration. From the equation of sliding surface, (5) the following condition is obtained in order to guarantee reachability for a ground excitation.

$$|\,\eta_P\omega_P{}^2 - \alpha_P\ddot{x}_g\,| < n + 2\xi_P\omega_P\,|\,\dot{\eta}_P\,| \tag{12}$$

For simplicity the damping related term may be neglected and the condition for reachability is obtained as below.

$$|\,(\eta_P)_{\max}\,\omega_P{}^2\,| + |\,\alpha_P\ddot{x}_g\,| < n \tag{13}$$

In (12), $(\eta_p)_{max}$ can be obtained by analyzing the structure with ground excitation considering a constant value n and hence the control force. Thus, by iterations the value of n has to be fixed for which the reachability is satisfied. As the sliding surface is reached, a high frequency switching between two control actions $u = -(\{S\}\{\bar{B}\})^{-1} n\, sig(s)$ takes place as the system trajectory repeatedly cross the sliding surface. If an infinite frequency switching is possible, the system is bound to lie on the sliding surface and an ideal sliding takes place. During such an ideal sliding the system behaves as a reduced order system and the system dynamics can be obtained by an equivalent control action [14]. At sliding, an equivalent control action is obtained as follows:

$$u_{eqv} = -(\{S\}\{\bar{B}\})^{-1}\{S\}[\Lambda_a]\{X_a\} \tag{14}$$

The equivalent control law (14) provides the modified system dynamics as follows:

$$\{\dot{X}_a\} = [[I_n] - \{\bar{B}\}(\{S\}\{\bar{B}\})^{-1}\{S\}][\Lambda_a]\{X_a\} \tag{15}$$

Since $S \epsilon R^{1\times n}$ has full rank, the order of the modified system $([[I_n] - \{\bar{B}\}(\{S\}\{\bar{B}\})^{-1}\{S\}][\Lambda_a])$ is reduced by *1*. The modified system (15) can be analyzed to verify the stability.

III. Numerical Illustration

The algorithm developed in this study aims to control a particular mode of a shear building. An excitation of higher modes primarily increases the building floor accelerations that directly affect the vibration of a secondary system attached to the building. In case of a secondary structure, such as a piece of equipment or a machine (having its frequency same to any of the mode of the primary structure) is subjected to huge vibration when the structure is subjected to ground excitation. This algorithm can be used to control the responses of such important secondary structures. The proposed algorithm is applied to a four story shear building with a secondary system attached to it. The mass of the shear building is considered to be *32000* kg for all the floors and the stiffness of the building is *41293.8* kN/m for all the floors except ground floor where the stiffness is *21801.5*kN/m. The secondary mass is assumed to be attached to the first floor of the system as the lower floor levels are sensitive to particularly to the second mode. The mass of the secondary system is considered to be *0.5%* of the floor mass (*32000* kg) or *160*kg. The combined system can be considered as an *n + 1* degree of freedom system. The additional equation of motion for the secondary mass can be written as follows:

$$m_s x_s + k_s(x_s - x_1) + c_s(\dot{x}_s - \dot{x}_1) = -m_s\ddot{x}_g \tag{16}$$

where m_s, k_s and c_s are the mass, stiffness and damping of the secondary structure, respectively; x_s and x_1 are the displacements with respect to the ground for the secondary mass and the first floor, respectively. The equation of motion for the floor masses will remain the same as described earlier except the first floor, where the effect of the secondary mass is considered. The effect of the secondary structure is insignificant on the primary structure because of the small value of the secondary mass (*0.5%* of the floor mass). However, the reverse is not true. The study is conducted for the value of k_s(i.e. 169 kN/m) for which the natural frequency of the secondary structure is tuned to the second mode of the primary structure. The primary system is assumed to have *2%* Raleigh damping for the first two modes of vibration and the secondary system is assumed to have low viscous damping of *1%*. The time history analysis of the structure is carried out for the 1980 Cape-Mendocino (UNAM/UCSD station 6604) ground motion selected from the PEER strong motion database. A detail description of the responses for the primary and secondary structures is considered for the selected ground motion.

A. Control Force

The control force obtained from the proposed algorithm is applied at the first floor of the primary structure. In generating the control force, the primary structure is only considered as per the algorithm, although the velocity feedback of the primary structure is considered from the combined system analysis. This assumption is justified as the mass of the secondary system is too small to affect the modal properties and the responses of the primary system. Further, the direct application of control force as obtained from the algorithm may induce chattering in the structure for which the high frequencies are excited. This may increase the floor acceleration at the initial phase of vibration when the excitation is very less. Thus, to reduce this adverse effect, the control force is applied after a certain amplification of the first floor acceleration. Also the force application increases linearly from zero to the designed value. The control force to be applied is selected through iterations by i) satisfying the reachability condition and ii) observing the chattering in the responses. One may note that these conditions are contradictory to each other and hence, the control force thus obtained is an optimal value. The maximum control force applied is *240.2*kN, which is *0.19* times the total weight of the primary structure. It should be noted that the applied force is very small as compared to the total weight of the structure.

B. Structural Response

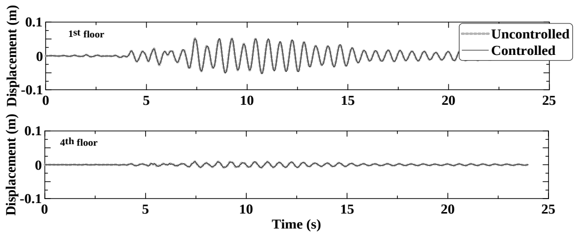

Figure 2. Relative velocity time history at 1st and 4th floor

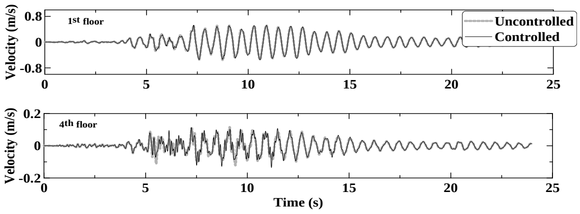

Figure 3. Relative velocity time history at 1st and 4th floor

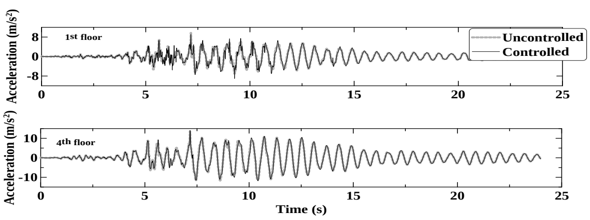

Figure 4. Absolute acceleration time history at 1st and 4th floor levels.

The displacement, velocity and the acceleration of the primary structure is demonstrated in Fig. 2-Fig. 4. The change in the response of a primary structure is insignificant for the controlled case as compared to the uncontrolled case as can be seen from these figures. A slight chattering can be observed at the initial part of the floor acceleration time history (Fig. 4) although it does not increase the floor acceleration. Fig. 5 demonstrates the results for the time history analysis of the secondary structure for displacement, velocity and acceleration. A huge amplification of the responses of the secondary structure is observed as compare to the case when no control is applied. The Fourier amplitude spectrum for the acceleration of the secondary mass of the system is also shown in Fig. 6. Two peaks can be observed for the secondary mass, one at the fundamental and another near the second modal frequency of the primary structure for the uncontrolled case. The Fourier amplitude near the second mode of the primary structure is much larger than the peak corresponding to the fundamental mode. This amplitude reduces to a significant amount (even less than the Fourier amplitude at the fundamental mode of the primary structure) by the application of control force as can be observed from Fig. 6 for the controlled case. This reduction at the second mode of the primary structure can also be observed from the Fourier amplitude of the first floor. Alight excitation of the high frequency can also be observed from the figure because of the chattering in the sliding process. The peak responses of secondary structure are for the control and uncontrolled cases along with the percentage reduction in response for the controlled case are given in Table I. Thus, the algorithm is found to be effective in controlling the response of a particular mode of the structure by applying a control force which is nominal in comparison to the weight of the structure.

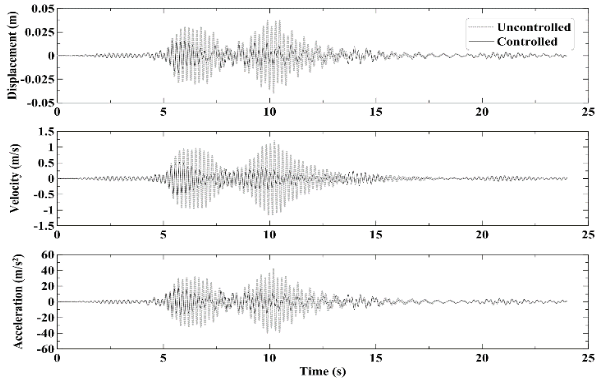

Figure 5. Relative displacement, relative velocity and acceleration time histories for secondary mass.

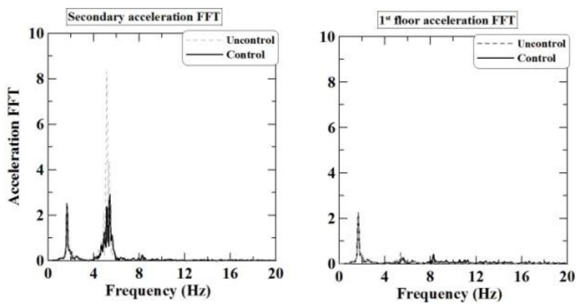

Figure 6. Fourier amplitude spectra for first floor acceleration and the secondary acceleration responses

TABLE I. PEAK RESPONSES OF THE SECONDARY STRUCTURE

Response	Uncontrolled	Controlled	% Reduction
Displacement (m)	0.04	0.0125	68.75
Velocity (m/s)	1.4	0.4	71.43
Acceleration (m/s^2)	45	15	67

IV. CONCLUSION

An active control algorithm is developed by considering sliding mode control in order to control a particular mode of a shear building. The location of the actuator is considered at the first floor level for an easy application of the control force. In order to achieve the desired control, the sliding surface is designed using the velocity response of the structure in such a way that the effect of a particular mode of the structure at an ideal sliding is nullified. A signum function is used for designing the control force in order to achieve the reachability to the sliding surface. The effectiveness of the control algorithm is demonstrated using a four-story shear building with uniform mass distribution subjected to earthquake ground excitation. A secondary structure is also attached to the shear building having its frequency tuned to the second mode of the primary structure. The algorithm found to work very well in suppressing the second mode of the shear building and provides a tremendous reduction in the responses of the secondary structure. Further as it uses only the velocity response of the structure, the reduction can be achieved through a much lesser number of sensors.

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