Amplification Control of a Chain of Integrators with a Delay in the Input under Uncertain AC and DC Sensor Noise

Sang-Young Oh
DONGHEE Industrial Co., LTD., Gwangmyeong R&D Center, Gyeonggi-do, Korea
Email: sangyoung.oh@donghee.co.kr

Min-Gil Jeong and Ho-Lim Choi
Department of Electrical Engineering, Dong-A University, Busan, Korea
Email: mgjeong@donga.ac.kr, hlchoi@dau.ac.kr

***Abstract*—We consider a control problem of a chain of integrators where there is a finite constant delay in the input and uncertain AC and DC sensor noise in the feedback channel. If a finite constant delay and uncertain sensor noise are included in a controller via the feedback channel, the signal is distorted and the entire system cannot work normally. Therefore, some appropriate action for a finite constant delay and uncertain AC and DC sensor noise effect is essential in the controller design. Our control scheme is equipped with a gain-scaling factor, an amplifier, and a compensator to keep the system states bounded and reduce the ultimate bound of the output arbitrarily small. Our result shows that the proposed method has a distinct advantage over the existing results.**

***Index Terms*—chain of integrators, amplification, compensator, input-delay, AC/DC sensor noise**

I. Introduction

Typically, the control system operates through the measured sensor data. There can be a case that noise enters into the feedback channel in an additive form so that the control signal is distorted and control performance is degraded. Also, there can be a delay in control input due to various reasons in practice. These input delay and sensor noise problems can occur and there have been many related results by far.

The measurement feedback control problems have been studied in [1]−[7]. In [1], a sensor noise canceling approach is proposed when the noise is assumed to be generated by a known exogeneous system. In [4], the authors propose a switching controller under only AC sensor noise. In [5], the authors deal with both AC and DC noise, but they require the known initial condition and there is no delay in the input. In [7], they deal with input delay and noise together, but the noise is limited to AC noise only.

In this letter, we newly consider a control problem for a chain of integrators under input delay and AC/DC sensor noise. Unlike [5], the initial condition information is not known in advance. In order to solve our problem, motivated by [8], [9], we add a signal amplification scheme to our considered system. Then, a newly designed controller with compensator and gain-scaling factor is introduced. Via system analysis, we show that our proposed controller derives the system output into arbitrarily small bound. An example is given to verify the advantage of our controller over the one in [7].

II. Amplification Scheme and Sensor

We consider a chain of integrators with delayed input

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \qquad i = 1, \cdots, n-1 \\ \dot{x}_n &= u(t-\tau) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, \cdots, x_n]^T \in R^n$ is the state, $u \in R$ is the input, $\tau \in (0, \infty)$ is a known finite constant. The initial condition is given as $u(\theta) = v(\theta)$, $-\tau \le \theta \le 0$ where $u_t(\theta) = u(t+\theta) = u_t$. If sensor noise is included in a controller via the feedback channel, the signal is distorted and the entire system cannot work normally. Therefore, controller is applied such as $u = \psi(\chi)$ where $\chi = x + S(t)$ is the state with measurement noise, $S(t) = [S_1(t), \cdots, S_n(t)]^T$ is the sensor noise. The following condition is assumed on the sensor noise.

Assumption 1: There exist uncertain constants d_i , α_i and ω_i such that

$$S_i(t) = d_i + \alpha_i \sin \omega_i t \qquad (i = 1, \cdots, n) \tag{2}$$

where $0 \le d_i \le \bar{d}_i$, $0 \le \alpha_i \le \bar{\alpha}_i$ and $\bar{\omega}_i \le \omega_i \le \infty$.

Remark 1: Our control goal is to keep the system states remain bounded and derive the system output into an arbitrarily small bound. Our control problem is generalized over [4] [5] [7] because input delay, unknown

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initial condition, and uncertain AC/DC sensor noise are combined altogether.

We set a reduction-type transformation [7].

$$\begin{aligned} z_i &= x_i, \qquad i = 1, \cdots, n-1 \\ z_n &= x_n + \int_{t-\tau}^{t} u(s)ds \end{aligned} \tag{3}$$

Then, from (1) and (3), the transformed system is

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{n-1} &= z_n - \int_{t-\tau}^{t} u(s)ds \\ \dot{z}_n &= u \\ y &= z_1 \end{aligned} \tag{4}$$

Due to DC noise with unknown initial condition, the control methods of [5], [7] may not achieve our control goal. To solve the problem, we modify our considered system with amplifiers as illustrated in Fig. 1. We add amplifiers and then we attach feedback sensors to measure the feedback signals such that sensors measure the amplified signals as similarly done in [8], [9].

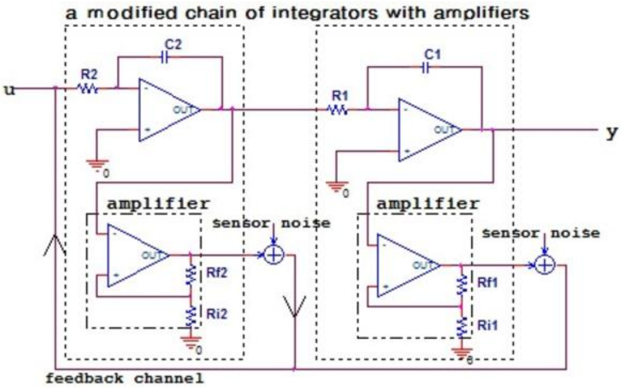


Figure 1. Modified system: A second-order case.

Let us set that the system signals are amplified by a factor of c via amplifiers. Then, after the amplifiers and sensor measurement, the signals in feedback channel become $cx_i + S_i(t)$. Now, with the transformation (3), the actual feedback signal to be used in controller is expressed as $cx_i + S_i(t)$.

III. Main Result

For notational convenience, let $\zeta = [\zeta_1, \cdots, \zeta_n]^T$, $\zeta_1 = cz_i + S_i(t)$. Now, we propose a measurement feedback controller as

$$\begin{aligned} u = {} & \frac{1}{\beta}\left[\frac{k_1}{\gamma^{n+1}}\zeta_1 + \cdots + \frac{k_n}{\gamma^2}\zeta_n\right] \\ & + \frac{1}{\beta^2} e^{\frac{ck_{n+1}}{\beta^2\gamma}t}\left[\frac{k_1}{\gamma^{n+1}}\int_0^t \zeta_1(\phi)(e^{-\frac{ck_{n+1}}{\beta^2\gamma}\phi})d\phi \right. \\ & \left. + \cdots + \frac{k_n}{\gamma^2}\int_0^t \zeta_n(\phi)(e^{-\frac{ck_{n+1}}{\beta^2\gamma}\phi})d\phi\right] \end{aligned} \tag{5}$$

where $\gamma, \beta, c \geq 1$ and $k_{n+1} < 0$. Our proposed controller is coupled with a gain-scaling factor γ and a compensator shown in integral forms.

Theorem 1: Assume that Assumption 1 holds. Select $k_1, \cdots, k_{n+1}$ such that $\lambda^{n+1} - k_{n+1}\lambda^n - \cdots - k_2\lambda - k_1 = 0$ is a Hurwitz polynomial. Then, all states of the closed-loop system (1) with (5) remain bounded. Moreover, the ultimate bound of y can be made arbitrarily small by adjusting γ, β, c.

Proof: From (4) and (5), the closed-loop system is

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{n-1} &= z_n - \int_{t-\tau}^{t} u(s)ds \\ \dot{z}_n &= \frac{1}{\beta}\left[\frac{k_1}{\gamma^{n+1}}\zeta_1 + \cdots + \frac{k_n}{\gamma^2}\zeta_n\right] \\ & \quad + \frac{1}{\beta^2} e^{\frac{ck_{n+1}}{\beta^2\gamma}t}\left[\frac{k_1}{\gamma^{n+1}}\int_0^t \zeta_1(\phi)(e^{-\frac{ck_{n+1}}{\beta^2\gamma}\phi})d\phi \right. \\ & \quad \left. + \cdots + \frac{k_n}{\gamma^2}\int_0^t \zeta_n(\phi)(e^{-\frac{ck_{n+1}}{\beta^2\gamma}\phi})d\phi\right] \end{aligned} \tag{6}$$

We let $\xi_1 = cz_i$, $i = 1, \cdots, n$ and set a virtual state as

$$\begin{aligned} \xi_{n+1} = {} & \frac{c}{\beta^2} e^{\frac{ck_{n+1}}{\beta^2\gamma}t}\left[\frac{k_1}{\gamma^{n+1}}\int_0^t \xi_1(\phi)(e^{-\frac{ck_{n+1}}{\beta^2\gamma}\phi})d\phi \right. \\ & \left. + \cdots + \frac{k_n}{\gamma^2}\int_0^t \xi_n(\phi)(e^{-\frac{ck_{n+1}}{\beta^2\gamma}\phi})d\phi\right] \end{aligned} \tag{7}$$

We can also express u as $u = \frac{1}{c}\delta + \frac{1}{c}\xi_{n+1} + \eta_1(t)$, $\delta = \frac{c}{\beta}B\tilde{K}(\gamma)\xi$, where $\xi = [\xi_1, \cdots, \xi_{n+1}]^T$, $B = [0, \cdots, 1]^T$, $\tilde{K}(\gamma) = \left[\frac{k_1}{\gamma^{n+1}}, \cdots, \frac{k_n}{\gamma^2}, 0\right]$, and

$$\begin{aligned} \eta_1(t) = {} & \frac{1}{\beta}\left[\frac{k_1}{\gamma^{n+1}}S_1 + \cdots + \frac{k_n}{\gamma^2}S_n\right] \\ & + \frac{1}{\beta^2} e^{\frac{ck_{n+1}}{\beta^2\gamma}t}\left[\frac{k_1}{\gamma^{n+1}}\int_0^t S_1(\phi)(e^{-\frac{ck_{n+1}}{\beta^2\gamma}\phi})d\phi \right. \\ & \left. + \cdots + \frac{k_n}{\gamma^2}\int_0^t S_n(\phi)(e^{-\frac{ck_{n+1}}{\beta^2\gamma}\phi})d\phi\right] \end{aligned} \tag{8}$$

Then, from (6)-(8), the closed-loop system is reorganized as

$$\dot{\xi} = A_{\bar{K}(\gamma)}\xi + \Delta_1(\cdot) + \Delta_2(\cdot) + \Delta_3(\cdot) + \delta + \eta(t) \tag{9}$$

where $A_{\bar{K}(\gamma)} = A + B\bar{K}(\gamma)$ with (A, B) the Brunovsky canonical pair, $\bar{K}(\gamma) = \left[\frac{k_1}{\gamma^{n+1}}, \cdots, \frac{k_{n+1}}{\gamma}\right]$, $\Delta_1(\cdot) = \left[0, \cdots, 0, -\int_{t-\tau}^{t}\delta(s)ds, 0, 0\right]^T$, $\Delta_2(\cdot) = \left[0, \cdots, 0, -\int_{t-\tau}^{t}\xi_{n+1}(s)ds, 0, 0\right]^T$,

$\Delta_3(\cdot)=\left[0,\cdots,0,-c\int_{t-\tau}^{t}\eta_1(s)ds,0,0\right]^T$, and $\eta(t)=[0,\cdots,0,\eta_1(t),0]^T$. From [10], since $A_{\bar{K}}=A+B\bar{K}$ is Hurwitz where $\bar{K}=\bar{K}(1)$, we can obtain a Lyapunov equation of $A_{\bar{K}(\gamma)}^T P_K(\gamma)+P_K(\gamma)A_{\bar{K}(\gamma)}=-\gamma^{-1}E_\gamma^2$ where $P_K(\gamma)=E_\gamma P_K E_\gamma$, $E_\gamma=\mathrm{diag}[1,\gamma,\cdots,\gamma^n]$, and $A_{\bar{K}}^T P_K+P_K A_{\bar{K}}=-I$. We set a Lyapunov function $V(\xi)=\xi^T P_K(\gamma)\xi$. Then, along the trajectory of (9)

$$\begin{aligned}\dot{V}(\xi)&=\dot{\xi}^T P_{K(\gamma)}\xi+\xi^T P_{K(\gamma)}\dot{\xi}\\&=-\frac{1}{2}\gamma^{-1}(E_\gamma\xi)^T(E_\gamma\xi)+2\xi^T P_{K(\gamma)}\delta\\&\quad-\frac{1}{4}\gamma^{-1}\|E_\gamma\xi\|^2-\frac{1}{4}\gamma^{-1}\xi^T E_\gamma E_\gamma\xi\\&\quad+2\xi^T E_\gamma P_K E_\gamma\{\Delta_1(\cdot)+\Delta_2(\cdot)+\Delta_3(\cdot)+\eta(t)\}\end{aligned}\tag{10}$$

The first two terms of (10) lead to the following

$$\begin{aligned}&-\frac{1}{2}\gamma^{-1}(E_\gamma\xi)^T(E_\gamma\xi)+2\xi^T P_{K(\gamma)}\delta\\&=-\frac{1}{2}\gamma^{-1}(E_\gamma\xi)^T(E_\gamma\xi)+2(E_\gamma\xi)^T P_K E_\gamma\delta\\&=-\frac{1}{2}\gamma^{-1}(E_\gamma\xi)^T(E_\gamma\xi)\\&\quad+2(E_\gamma\xi)^T\begin{bmatrix}p_{1,1}&\cdots&p_{1,n+1}\\ \vdots&\ddots&\vdots\\ p_{n+1,1}&\cdots&p_{n+1,n+1}\end{bmatrix}\begin{bmatrix}\frac{c}{\beta}\frac{k_1}{\gamma^{n+1}}\\ \frac{c}{\beta}\frac{k_1}{\gamma^{n-1}}\\ \vdots\\ \frac{c}{\beta}\frac{k_1}{\gamma^{3-n}}\\ 0\end{bmatrix}(E_\gamma\xi)\\&=-(E_\gamma\xi)^T\begin{bmatrix}\frac{1}{2\gamma}-*_1&-*_2&\cdots&-*_{n+1}\\ -*_2&\frac{1}{2\gamma}&&\\ \vdots&&\ddots&\vdots\\ -*_{n+1}&&\cdots&\frac{1}{2\gamma}\end{bmatrix}(E_\gamma\xi)\\&=:-(E_\gamma\xi)^T\prod(E\ \xi)\end{aligned}\tag{11}$$

where $*_1=\frac{c}{\beta}\left(\frac{k_1}{\gamma^{n+1}}2p_{1,1}+\frac{k_2}{\gamma^{n-1}}2p_{1,2}+\cdots+\frac{k_n}{\gamma^{3-n}}2p_{1,n}\right)$,

$*_i=\frac{c}{\beta}\left(\frac{k_1}{\gamma^{n+2-i}}p_{i,1}+\frac{k_2}{\gamma^{n-i}}p_{i,2}+\cdots+\frac{k_n}{\gamma^{4-n-i}}p_{i,n}\right)$, $(i=2,\cdots,n+1)$.

To investigate the positive definiteness of the matrix $\prod$, we see that

$$\begin{aligned}&\text{i)}\ \left(\frac{1}{2\gamma}-*_1\right)>0\\&\quad\rightarrow\frac{\beta}{2\gamma}>c\left(\frac{k_1}{\gamma^{n+1}}2p_{1,1}+\cdots+\frac{k_n}{\gamma^{3-n}}2p_{1,n}\right)\\&\text{ii)}\ \frac{1}{2\gamma}\left(\frac{1}{2\gamma}-*_1\right)-*_2^2>0\\&\quad\rightarrow\frac{\beta^2}{4\gamma^2}>\frac{c\beta}{2\gamma}\left(\frac{k_1}{\gamma^{n+1}}2p_{1,1}+\cdots+\frac{k_n}{\gamma^{3-n}}2p_{1,n}\right)\\&\qquad\qquad+c^2\left(\frac{k_1}{\gamma^n}p_{2,1}+\cdots+\frac{k_n}{\gamma^{2-n}}p_{2,n}\right)^2\\&\text{iii)}\ \frac{1}{(2\gamma)^2}\left(\frac{1}{2\gamma}-*_1\right)-\frac{1}{2\gamma}*_3^2-\frac{1}{2\gamma}*_2^2>0\\&\quad\rightarrow\frac{\beta^2}{8\gamma^3}>\frac{c\beta}{4\gamma^2}\left(\frac{k_1}{\gamma^{n+1}}2p_{1,1}+\cdots+\frac{k_n}{\gamma^{3-n}}2p_{1,n}\right)\\&\qquad\qquad+\frac{c^2}{2\gamma}\left(\frac{k_1}{\gamma^n}p_{2,1}+\cdots+\frac{k_n}{\gamma^{2-n}}p_{2,n}\right)^2\\&\qquad\qquad+\frac{c^2}{2\gamma}\left(\frac{k_1}{\gamma^{n-1}}p_{3,1}+\cdots+\frac{k_n}{\gamma^{1-n}}p_{3,n}\right)^2\\&\text{iv)}\ \frac{1}{(2\gamma)^3}\left(\frac{1}{2\gamma}-*_1\right)-\frac{1}{(2\gamma)^2}*_3^2>0\\&\quad\rightarrow\frac{\beta^2}{16\gamma^4}>\frac{c\beta}{8\gamma^3}\left(\frac{k_1}{\gamma^{n+1}}2p_{1,1}+\cdots+\frac{k_n}{\gamma^{3-n}}2p_{1,n}\right)\\&\qquad\qquad+\frac{c^2}{4\gamma^2}\left(\frac{k_1}{\gamma^{n-1}}p_{3,1}+\cdots+\frac{k_n}{\gamma^{1-n}}p_{3,n}\right)^2\\&\text{v)}\ \frac{1}{(2\gamma)^j}\left(\frac{1}{2\gamma}-*_1\right)>0,\qquad(j=4,\cdots,n)\\&\quad\rightarrow\frac{\beta}{(2\gamma)^{j+1}}>\frac{c}{(2\gamma)^j}\left(\frac{k_1}{\gamma^{j+1}}2p_{1,1}+\cdots+\frac{k_n}{\gamma^{3-j}}2p_{1,n}\right)\end{aligned}\tag{12}$$

Thus, from (12), we see that if $\beta>>\gamma$, $\beta>c$ and $k_i<0$, we have $\prod=\prod^T>0$.

Similarly to [10], we apply the Razumikhin theorem [11] with $\|E_\gamma\xi_t\|\le q\|E_\gamma\xi\|$, $q>1$. Here, we let $\xi_t=\xi(t+\theta)$, $-\tau\le\theta\le 0$ [11]. With this, we first have

$$\begin{aligned}\|E_\gamma\Delta_1(\cdot)\|&\le\tau\gamma^{n-2}\frac{c}{\beta}\|B\tilde{K}(\gamma)\|\|\xi(t+\theta)\|\\&\le\tau q\gamma^{-2}\frac{c}{\beta}\|B\tilde{K}(\gamma)\|\|E_\gamma\xi\|\\\|E_\gamma\Delta_2(\cdot)\|&\le\tau\gamma^{n-2}|\xi_{n+1}(t+\theta)|\le\tau q\gamma^{-2}\|E_\gamma\xi\|\end{aligned}\tag{13}$$

From (10) and (13), we have

$$\begin{aligned}\dot{V}(\xi)&\le-(E_\gamma\xi)^T\prod(E_\gamma\xi)\\&\quad-\frac{1}{4}\left(\gamma^{-1}-8\tau q\sigma\gamma^{-2}-8\tau q\sigma\gamma^{-2}\frac{c}{\beta}\|B\tilde{K}(\gamma)\|\right)\|E_\gamma\xi\|^2\\&\quad-\frac{1}{4}\gamma^{-1}\xi^T E_\gamma E_\gamma\xi+2\xi^T E_\gamma P_K E_\gamma\{\Delta_3(\cdot)+\eta(t)\}\end{aligned}\tag{14}$$

where $\sigma = \lambda_{\max}(P_K)$.

here, we define a notation. For any given $m \times n$ matrix (or vector) $M = [m_{i,j}]$, we let $|M| := [|m_{i,j}|]$, $1 \le i \le m$ and $1 \le j \le n$. With this notation, the last two terms of (14) lead to the following inequality

$$\begin{aligned} &-\frac{1}{4}\gamma^{-1}\xi^T E_\gamma E_\gamma \xi + 2\xi^T E_\gamma P_K E_\gamma \{\Delta_3(\cdot) + \eta(t)\} \\ &\le -\frac{1}{4}\gamma^{-1}|\xi|^T E_\gamma^2 |\xi| + \frac{1}{4}\gamma^{-1}|\xi|^T E_\gamma^2 \xi_b(t) \end{aligned} \tag{15}$$

where $\xi_b(t) = 8\gamma E_\gamma^{-1}|P_K|E_\gamma\{|\Delta_3(\cdot)| + |\eta(t)|\} =: [\xi_{b_1}(t), \cdots, \xi_{b_{n+1}}(t)]^T$.

Combining (14) and (15), we have

$$\begin{aligned} \dot{V}(\xi) &\le -(E_\gamma \xi)^T \Pi (E_\gamma \xi) \\ &-\frac{1}{4}\left(\gamma^{-1} - 8\tau\eta\sigma\gamma^{-2} - 8\tau\eta\sigma\gamma^{-2}\frac{c}{\beta}\|B\tilde{K}(\gamma)\|\right)\|E_\gamma \xi\|^2 \\ &-\frac{1}{4}\gamma^{-1}|\xi|^T E_\gamma^2 \{|\xi| - \xi_b(t)\} \end{aligned} \tag{16}$$

Thus, when $\gamma > 8\tau\eta\sigma + 8\tau\eta\sigma\frac{c}{\beta}\|B\tilde{K}(\gamma)\|$, all states of the closed-loop system remain bounded. Now, we investigate the ultimate bound of ξ_i i.e., $UB(\xi_i) = \lim_{t\to\infty}\xi_{b_i}(t)$. After a direct calculation, we can obtain

$$\xi_b(t) = 8\left\{\begin{bmatrix} \gamma^n |p_{1,n}| \\ \gamma^{n-1}|p_{2,n}| \\ \vdots \\ \gamma^0 |p_{n+1,n}| \end{bmatrix} |\eta_1(t)| + \begin{bmatrix} \gamma^{n-1}|p_{1,n-1}| \\ \gamma^{n-2}|p_{2,n-1}| \\ \vdots \\ \gamma^{-1}|p_{n+1,n-1}| \end{bmatrix} \left|c\int_{t-\tau}^{t}\eta_1(s)ds\right|\right\} \tag{17}$$

where $p_{i,j}$ is the (i, j) th element of P_K.

Under Assumption 1, we obtain the followings

$$\begin{aligned} \eta_1(t) &= \frac{1}{\beta}\frac{k_i}{\gamma^{n+2-i}}(d_i + \alpha_i \sin\omega_i t) \\ &+ \frac{1}{\beta^2}e^{Mt}\frac{k_i}{\gamma^{n+2-i}}\int_0^t (d_i + \alpha_i \sin\omega_i\phi)(e^{-M\phi})d\phi \\ &= \frac{1}{\beta^2}\frac{k_i}{\gamma^{n+2-i}}\left[d_i\left(\beta - \frac{\beta^2\gamma(1+e^{Mt})}{ck_{n+1}}\right) - \frac{\alpha_i \sin\omega_i t + \frac{1}{M}\alpha_i\omega_i e^{Mt}}{M + \omega_i}\right] \end{aligned} \tag{18}$$

And

$$\begin{aligned} c\int_{t-\tau}^{t}\eta_1(s)ds &= c\int_{t-\tau}^{t}\left\{\frac{1}{\beta}\frac{k_i}{\gamma^{n+2-i}}(d_i + \alpha_i \sin\omega_i s) + \frac{1}{\beta^2}e^{Ms}\frac{k_i}{\gamma^{n+2-i}}\int_0^s (d_i + \alpha_i\sin\omega_i\phi)(e^{-M\phi})d\phi\right\}ds \\ &= \frac{c}{\beta^2}\frac{k_i}{\gamma^{n+2-i}}\left[d_i\left[\beta - \frac{\beta^2\gamma\{1 + Me^{Mt}(1-e^{-Mt})\}}{ck_{n+1}}\right] + \frac{\frac{\alpha_i}{\omega_i}\{\cos\omega_i t - \cos\omega(t-\tau)\} - \alpha_i\omega_i e^{Mt}(1-e^{-Mt})}{M + \omega_i}\right] \end{aligned} \tag{19}$$

where $M = \dfrac{ck_{n+1}}{\beta^2\gamma}$, $1 \le i \le n$.

Since $\gamma, \beta, c \ge 1$ and $k_{n+1} < 0$, M is the negative. Therefore, $e^{Mt} \to 0$ as $t \to \infty$. To find the ultimate bound,

$$\lim_{t\to\infty}|\eta_1(t)| = \sum_{i=1}^{n}\frac{1}{\beta^2}\frac{k_i}{\gamma^{n+2-i}}\left[d_i\left(\beta - \frac{\beta^2\gamma}{ck_{n+1}}\right) - \frac{\alpha_i \sin\omega_i t}{\frac{ck_{n+1}}{\beta^2\gamma} + \omega_i}\right] \tag{20}$$

And

$$\lim_{t\to\infty}\left|c\int_{t-\tau}^{t}\eta_1(s)ds\right| = \sum_{i=1}^{n}\frac{c}{\beta^2}\frac{k_i}{\gamma^{n+2-i}}\left[d_i\left(\beta - \frac{\beta^2\gamma}{ck_{n+1}}\right) + \frac{\frac{\alpha_i}{\omega_i}\{\cos\omega_i t - \cos\omega(t-\tau)\}}{\frac{ck_{n+1}}{\beta^2\gamma} + \omega_i}\right] \tag{21}$$

If the value of β increases, AC and DC measurement noise is reduced. By increasing β as follows, we have

$$\lim_{t\to\infty}|\eta_1(t)| = \sum_{i=1}^{n}\frac{k_i}{\gamma^{n+2-i}}\left[-\frac{d_i\gamma}{ck_{n+1}}\right] \tag{22}$$

And

$$\lim_{t\to\infty}\left|c\int_{t-\tau}^{t}\eta_1(s)ds\right| = \sum_{i=1}^{n}\frac{k_i}{\gamma^{n+2-i}}\left[-\frac{d_i\gamma}{k_{n+1}}\right] \tag{23}$$

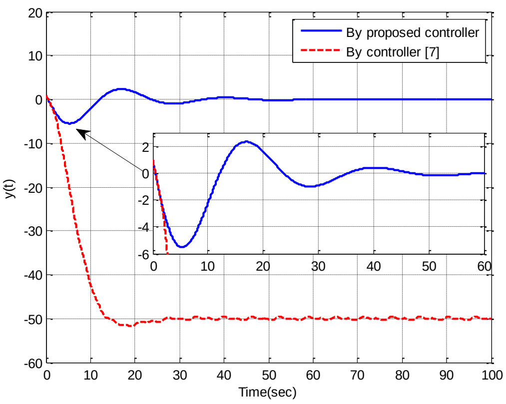
Then, as shown from (22), the magnitude of the DC measurement noise d_i can be reduced by adjusting c. If the value of c increases, DC measurement noise is reduced such as

$$\lim_{t\to\infty}|\eta_1(t)| \to 0 \tag{24}$$

However, as shown from (23), $\lim_{t\to\infty}\left|c\int_{t-\tau}^{t}\eta_1(s)ds\right|$ dose not approach to zero. Meanwhile, recall the relation between ξ and z as

$$\frac{\xi_i}{c} = z_i \tag{25}$$

Therefore, from (25), if the value of c increases, we can also reduce the ultimate bound of z. Therefore, the magnitude of $\xi_b(t)$ is reduced. As a result, the ultimate bound of $y(t) = x_1 = z_1$ can be made arbitrarily small.



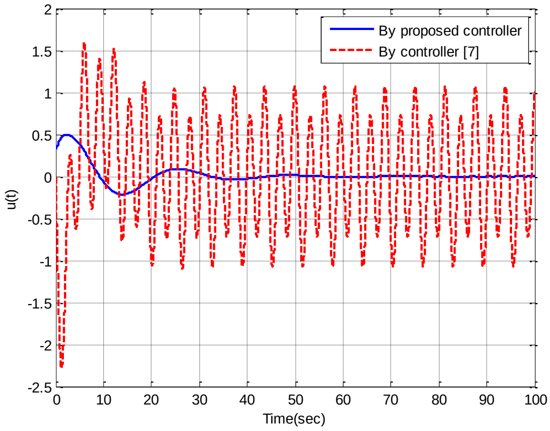


Figure 2. Simulation result of the proposed controller (5).

IV. Illustrative Example

We consider a second-order case. The control parameter K is set as $K = [-4, -4, -6]$. In our simulation, we set $S_1(t) = 5 + 5\sin t$, $S_2(t) = 6 + 4\sin 2t$ and $\tau = 1$. The initial conditions are set as $x_1(0) = 1$ and $x_2(0) = -2$.

In order to deal with AC/DC noise, other control parameters are set as $\gamma = 3$, $\beta = 1600(\beta >> \gamma)$ and $c = 700$. As shown in Fig. 2, while the controller [7] is unable to compensate to the measurement noise(in particular, DC noise), our proposed controller is effectively reduce the noise of AC and DC because of an amplifier and a newly designed compensator.

V. Conclusions

In this letter, we have proposed a controller for a chain of integrators where there exist known input delay and AC/DC sensor noise. Our controller equipped with a gain-scaling factor, an amplifier and a compensator to reduce the ultimate bound of the system output. We show the improved results over the existing results under AC and DC noise of sensor [7]. The benefit of our result is shown via comparison through simulation.

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Sang-Young Oh received the B.S.E. degree in 2013 and M.S. degree in 2015 from the department of electrical engineering, Dong-A university, Busan, Korea. Currently, he is an researcher at the Gwangmyeong R&D Center, DONGHEE Industrial Co., LTD., Gyeonggi-do, Korea. His research interests are control theory and nonlinear systems control. He is a member of ICROS, KIEE.

Min-Gil Jeong received the B.S.E. degree in 2014 and M.S. course from 2014 at the department of electrical engineering, Dong-A university, Busan, Korea. His research interests are input-output feedback linearization, switching control, and robust control. He is a member of ICROS, KIEE.

Ho-Lim Choi received the B.S.E. degree from the department of electrical engineering, The Univ. of Iowa, USA in 1996, and M.S. degree in 1999 and PhD degree in 2004, from KAIST(Korea Advanced Institute of Science and Technology), Deajeon, Korea. Currently, he is an associate professor at the department of electrical engineering, Dong-A university, Busan, Korea. His research interests are in the nonlinear control problems with emphasis on feedback linearization, gain scheduling, singular perturbation, output feedback, time-delay systems. He is a member of IEEE, IEICE, ICROS, KIEE