Wall Modeling LES Approach on Smooth Wall From Low to High Reynolds Numbers

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***Abstract*—A wall modeling LES approach is achieved for the smooth wall boundary layer simulation. By this method, the motions in the inner boundary layer will be modeled by a uniform wall modeling mesh using an equilibrium equation. After getting the feedback of wall shear stress from the wall-model, LES mesh will recalculate the flow field so that a low resolution mesh can be used for LES on wall bounded flow from low Reynolds number to high. Two cases, Re=3300 and 30000, are set up to verify the wall-model approach. Both cases show good agreement with the experiment or DNS result comparing to the pure LES mesh without wall modeling. The wall modeling LES approach could be used as a novel boundary layer simulation approach to avoid high computational cost.**[1]

***Index Terms*—wall-model, LES, smooth wall, boundary layer, Reynolds stress**

I. Introduction

When applied to turbulent boundary layers at high Reynolds numbers, the computational cost of large eddy simulation (LES) becomes highly prohibitive. Boundary layers are multi-scale phenomena where the energetic and dynamically important motions in the inner layer, 10% of the boundary layer, become progressively smaller in size as the Reynolds number increases, while the size of energetic motions in the outer layer is nearly independent of Reynolds number.

If these inner layer motions are resolved, the required grid resolution necessarily scales with the viscous length scale. Therefore, in order to make LES applicable to high Reynolds number wall bounded flows, the inner layer must be modeled while directly resolving only the outer layer.

There have been many proposed methods, Piomelli and Balaras [1] and Spalart [2], for modeling of the inner layer in LES. These approaches generally fall into one of two categories, methods that model the wall shear stress τ_w directly and methods that switch to includes hybrid LES/RANS and detached eddy simulation (DES).

In the wall-stress modeling approach [3], the LES is formally defined as extending all the way down to the wall but is solved on a grid that only resolves the outer layer motions. A wall-model takes as input the instantaneous LES solution at a height $y=h_{wm}$ above the wall and estimates the instantaneous shear stress τ_w at the wall y=0. This is then given back to the LES as a boundary condition.

II. Wall-model LES Approach

The inner layer wall model is a filtered equation [3]. Assuming equilibrium, the unresolved inner layer is modeled by solving

$$\frac{d}{d\eta}\left(\left(\nu+\nu_{t,wm}\right)\frac{du_{||}}{d\eta}\right)=0 \tag{1}$$

where η is the wall-normal direction, which should usually be aligned with y direction for regular geometries; $u_{||}$ is wall parallel velocity magnitude; ν is the kinematic viscosity.

The kinematic eddy viscosity $\nu_{t,wm}$ is obtained from the mixing-length model [4]

$$\nu_{t,wm}=\kappa\eta\sqrt{\tau_w/\rho}\,[1-\exp(-\frac{\eta^+}{A^+})]^2 \tag{2}$$

where A^+=17; κ=0.41 is the von Karman constant. The boundary condition at $\eta=0$ for Eq. 1 is the adiabatic no-slip condition. The wall parallel velocities from the instantaneous LES solution at $\eta=h_{wm}$ are interpolated to the upper boundary of the wall-model mesh.

Finally, the wall-shear stress τ_w is determined from the wall gradient of the inner solution [3]

$$\tau_w=\rho\nu\frac{du_{||}}{d\eta}\bigg|_{\eta=0}. \tag{3}$$

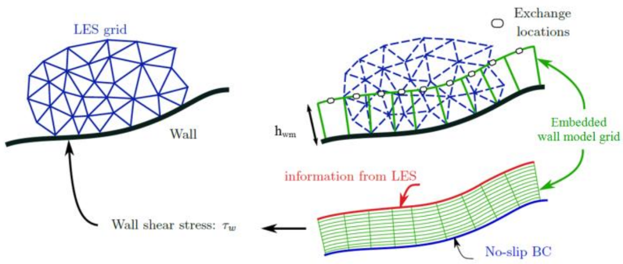

Figure 1. Smooth wall model for LES [5].

The wall-model mesh required by solving the wall-model equation is created using a simple extrusion of the wall surface by a thickness of $h_{wm}=0.1\delta$. The wall-model mesh so obtained should be logically structured

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and locally orthogonal. The procedure to couple the LES with the wall-model is showed in Fig. 1.

Since the instantaneous parallel velocities of the wall-model upper boundary are taken from the local LES cells, the wall-model upper surface is arranged at the exact center of a LES cell. According to the recommendation of Kawai and Larsson [6], five and a half LES cells are placed within the wall-model layer.

The wall shear stress which is given back to LES solver is computed from the following equation by assuming the viscous flux at the wall to be aligned with the velocity vector at the wall-model layer edge,

$$(\tau_{ij}n_j)_{w,LES} = \tau_{w,wm}e_{||,i} \tag{4}$$

where $e_{||}$ is a unit vector parallel to the wall and aligned with the velocity at the wall-model layer edge. Subscript wm indicate wall model; w indicate "at the wall".

III. Numerical Experiments

In order to verify the wall modeling LES approach, numerical experiments on smooth wall from low Reynolds number to high are hold. The Smagorinsky model is used to calculate the turbulent eddy viscosity. The solver in this project is based on OpenFOAM, in which the spatially filtered incompressible Navier-Stokes equations are solved.

This code uses a finite volume approach, with second order schemes in space and low numerical dissipation and a third-order Runge-Kutta scheme for explicit time advancement. The Smagorinsky subgrid scale model is used to account for the unresolved motions. The cell-centered formulation allow for a straightforward implementation of the flux-type boundary conditions that naturally arise from the wall-model.

A. Low Reynolds Number Channel Flow

The low Reynolds number channel flow case comes from Kim, Moin and Moser [7]. The computation domain for the wall-modeled LES is $4\pi\delta$, δ and $2\pi\delta$ in the stream-wise (x), wall-normal (y), and span-wise (z) directions, where δ is the half width of the channel.

The computation is carried out with 614400 grid points ($192\times20\times160$, in x, y, z) for a $Re_\delta = \frac{U_c\delta}{\nu} \approx 3300$, which is based on the mean centerline velocity U_c and the channel half width δ (a $Re_\tau = \frac{U_\tau\delta}{\nu} \approx 180$ based on the wall shear velocity U_τ). With this computational domain, the grid spacing in the stream-wise and span-wise directions is respectively $\Delta x^+ \approx 12$ and $\Delta z^+ \approx 7$ in wall units. Non-uniform meshes are used in the normal direction with uniform spacing Δy_w in the first 6 points near the wall, then a smoothly stretched grid up to the top boundary. The first mesh point away from the wall is at $\Delta y_1^+ \approx 4$. The LES mesh is more or less uniform with aspect radio less than 3 in any directions, and is approximately isotropic.

Slip-wall boundary condition is used at the top boundary and the bottom boundary y=0 is set to non-slip wall which is also receive data (τ_w) feedback from the wall model. The span-wise and stream-wise boundaries are all set as cyclic conditions. The height of wall-model mesh is 0.1δ with 100 non-equal spaced points in the normal direction. The LES and wall-model meshes are shown in Fig. 2.

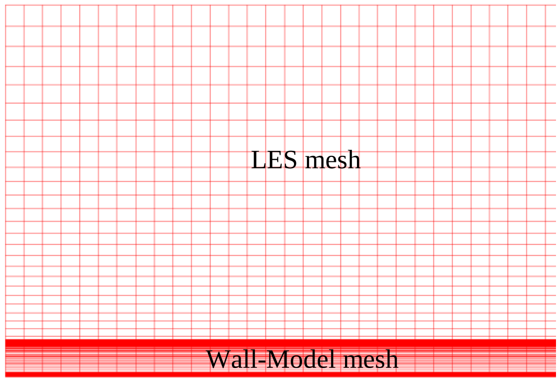

Figure 2. LES and wall-model mesh.

The time step is 5e-3s which makes the Courant number close to 0.5. A preliminary computation is performed during 20 time units on the pure LES grid to get a reasonable mean flow around the wall. Wall-modeling LES computations are then integrated during 100 time units to wash out the initial transients, and the averages are collected over the last 20 time units. The computation costs about 1000 core-hours on the Blueridge cluster in Virginia Tech.

The normalized mean velocity profile is shown in Fig. 3. The wall-model LES performs better than the LES without wall-model, and especially in the near wall region, the wall modeling LES result is closer to the DNS result.

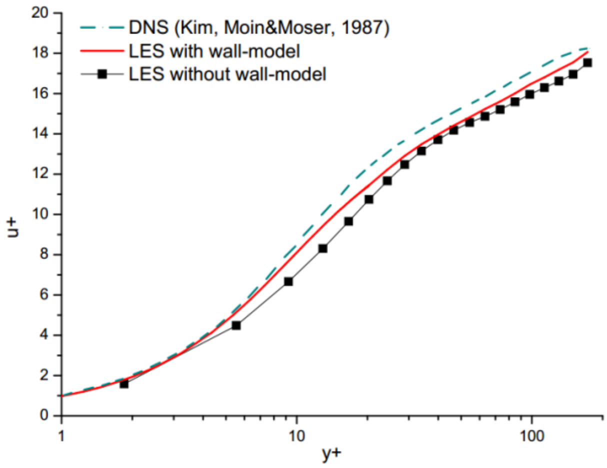

Figure 3. Normalized velocity profile of low Re.

B. High Reynolds Number Boundary Layer

In order to avoid developing a boundary condition for the inlet of turbulent boundary layer, channel flow is chosen to verify this wall modeling LES approach on smooth wall. The reference experiment comes from Degraaff and Eaton [8]. The computation domain for the wall-modeled LES is 12δ, δ and 5δ in the stream-wise (x), wall-normal (y), and span-wise (z) directions, where δ is the half height of the channel which is also equal to the boundary layer thickness (δ=35.58mm from the experiment).

The computation is carried out with 744000 grid points (240×31×100, in x, y, z) for a $Re_\delta = \frac{U_c \delta}{\nu} \approx 3.01 \times 10^5$, which is based on the mean centezrline velocity Uc and the boundary layer thickness δ (a $Re_\tau = \frac{U_\tau \delta}{\nu} \approx 10040$ based on the wall shear velocity U_τ).

With this computational domain, the grid spacing in the stream-wise and span-wise directions are respectively $\Delta x^+ = \Delta z^+ \approx 500$ and in wall units. Non-uniform meshes are used in the normal direction with uniform spacing Δy_w in the first 6 points near the wall, then a smoothly stretched grid up to the top boundary. The first mesh point away from the wall is at $\Delta y_1^+ \approx 200$. The LES mesh is more or less uniform with aspect radio less than 3.5 in any directions, and is approximately isotropic.

Slip-wall boundary condition is used at the top boundary and the bottom boundary y=0 is set to non-slip wall which is also receive data (τ_w) feedback from the wall model. The span-wise and stream-wise boundaries are all set as cyclic conditions.

The time step is 5e-5s which makes the Courant number close to 0.5. The time unit is 0.025s with the free-stream velocity 17.15m/s. A preliminary computation is performed during 3s (120 time units) on the pure LES grid to get a reasonable mean flow around the wall. Wall-modeling LES computations are then integrated during 200 time units to wash out the initial transients, and the averages are collected over the last 20 time units. The computation costs about 3200 core-hours on the Blueridge cluster in Virginia Tech.

The normalized height of wall-model is $h_{wm}^+ = 0.1Re_\tau \approx 1000$, and as is explained previous, the near wall region is modeled by wall-model and it only solve the parallel velocity and feedback the shear stress τ_w to LES mesh. As a consequent, the velocity profile is consisted by wall-model part and LES part as shown in Fig. 4. The Reynolds stressed are normalized by inner scales, U_τ and $\frac{\nu}{U_\tau}$.

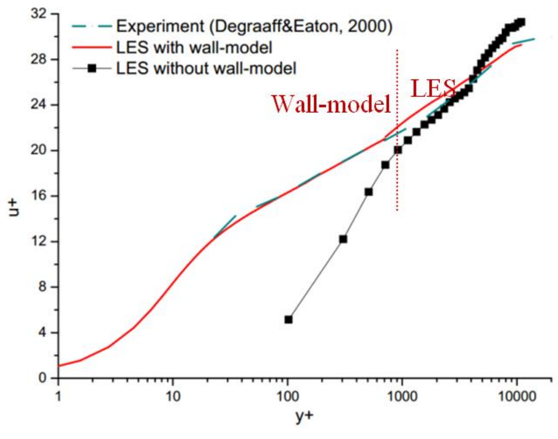

Figure 4. Normalized velocity profile on high Re.

Considering that the LES mesh does not solve the turbulence near the wall, the Reynolds stress only valid behind y+=1000 where the wall-model making data transfer to LES region. It can be seen from the Fig. 4 that the normalized velocity profile fits the experiment perfectly and the same LES mesh but without wall-model can't simulate the velocity trend with the first half region lower than the experiment and the other region opposite.

Basically, both the LES mesh and LES with wall-model mesh can resolve the stream-wise normal stress in the effective region in Fig. 5 but higher than the experiment in the beginning.

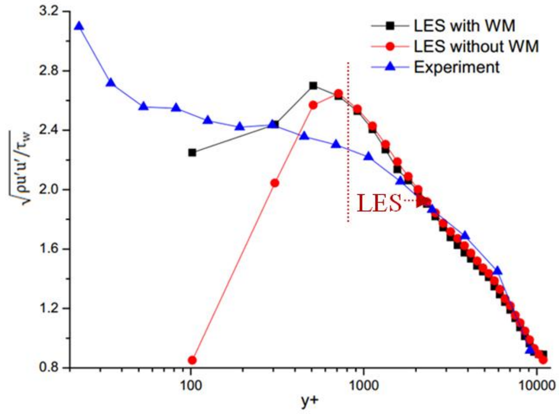

Figure 5. Stream-wise normal stress.

Comparing to pure LES, the LES with wall-model resolves the Reynolds shear stress better in Fig. 6, and the LES without wall-model overestimates the -v'u'.

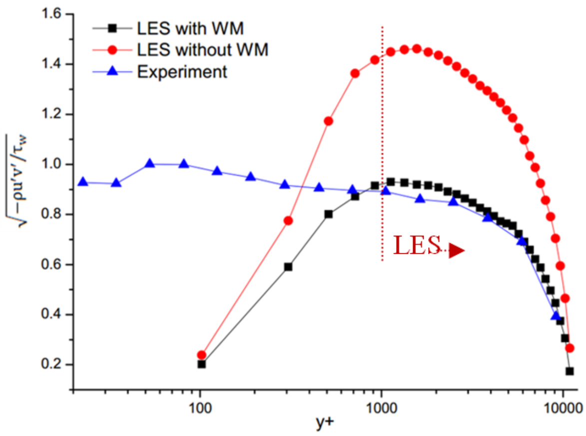

Figure 6. Reynolds shear stress.

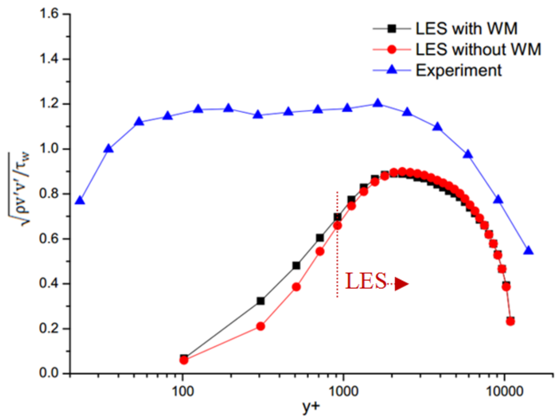

Figure 7. Wall-normal normal stress.

The resolved wall-normal normal stress for this test is shown in Fig. 7. The underestimation of v’v’ may come from the deviation of experiment measurement considering the fact that measurements of near wall variation of v’v’ with Reynolds-number are very rare.

IV. SUMMARY

The LES with wall-model can resolve the velocity profile perfectly from low Reynolds number to high. This method solves the near wall region by an equilibrium equation which can solve the parallel velocity and estimate the instantaneous wall shear stress. The wall model gives back the τ_w to LES mesh at a certain place as a boundary condition. As a result, the Reynolds stresses are well predicted in the LES region. The wall-model LES method can be used as an alternative solution in view that the computational cost of wall-resolved LES is enormous.

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