Influence of Cross-Saturation on the Various Models of Induction Machine

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Abstract—The analysis of saturated machines is generally performed with the stator and rotor currents, or mixed combinations of currents and flux linkages as state variables were also taken into account. A synthesis of possible models of induction machines is presented. Possible models contain explicit terms that describe cross-saturation except for the winding flux model. This paper treats the impact of cross-saturation on the various models of an induction machines. All found models are classified into four families: models with no, weak, mean and high sensitivity to the phenomenon.

Index Terms—asynchronous machines, modelling, cross saturation

I. INTRODUCTION

Magnetic saturation in electrical machine has been discussed in very many papers in the literature. Its introduction in the basic model of electrical machines was always a current problem. By incorporating the effects of magnetic saturation, the analytical models and simulation would gain in precision. In addition, in cases where the stability of the machine is an important criterion, the phenomenon of saturation can play a vital role. The first complete article concerning for modeling and simulation of transients regimes of saturated electrical machines, by considering main flux saturation, date of 1981 by P. Vas, [1]. A new general saturated model, where the currents are state variables, was developed in this article. Before this date, the proposed models were often incomplete because they did not include the phenomenon of cross-saturation, or concerned only of operation in steady regimes. The ideas developed in this article are the basis of many publications that have followed and discussed the effects of variable main flux saturation in the transient operation of cylindrical rotor Ac machines, [2]-[8]. A detailed analysis of nonlinear dynamic models for induction machine is proposed in [5, 6]. Analyses of saturated machines raised the presence of a magnetic coupling between the quadrature axes, called cross-saturation. To have a more realistic model it is necessary to consider the influence of this phenomenon, because it can have a relatively significant incidence on the characteristics of the machines. Indeed, the recent research in the domain electro-technical shows the importance of the cross-saturation who intervenes between the direct and quadrature axis of a saturated machine, [9]-[11].

As the magnetic saturation is a necessity for the modelling of an electric machine, then the cross-saturation presents its main disadvantage. Otherwise, the complete saturated models are very difficult to simulate, because of their complexity. On the other hand the principle of the vector controlled consists to realize an effective decoupling between the main variables of the machine, [12]-[14].

The objective of this paper is to examine the impact of the dynamic cross-saturation on the various models of IM and show that we can be neglected in some models. As shown in this study, one class of possible d-q models gives identical results as the full saturated model, although the cross-saturation is neglected. It then becomes possible to use a model that has the same structure as the linear model and give the possibility to assure the vector controlled even in presence the magnetic saturation.

II. SPACE VECTOR EQUATIONS INCORPORATING SATURATION EFFECT AND MODELS SYNTHESIS

A. Models Synthesis

In the analysis of the transient performance of electrical machines, either of two closely related methods may be used. One of these is the well known matrix method of generalized induction machine theory. In the followings, the voltage differential equations for a three-phase machine in terms of the space-vector will be established when saturation of the main flux-path is also considered.

Basic model of an induction machine may be given in terms of space vectors, in an arbitrary frame of reference \( \omega_s \), with the following set of equations:

\[
\bar{v}_r = R_f \bar{i}_r + \frac{d\bar{\lambda}_q}{dt} + j \omega_s \bar{\lambda}_d \tag{1} 
\]

\[
\bar{v}_s = R_s \bar{i}_s + \frac{d\bar{\lambda}_d}{dt} + j(\omega_s - \omega)\bar{\lambda}_q \tag{2}
\]

All rotor quantities are referred to the stator. Rotor winding are usually short-circuited, then \( \bar{v}_r = 0 \).
In this paper, this advantage is taken from beginning so that \( I_q = I_q = 1 \) is considered and \( L_y = L_y = L \) is resulted.

Stator and rotor flux linkages are:

\[
\bar{\lambda}_s = I_t \bar{t} + \bar{\lambda}_m \\
\bar{\lambda}_r = I_t \bar{t} + \bar{\lambda}_m
\]

(3) (4)

And

\[
\bar{\lambda}_m = L_m \bar{t} \\
\bar{\lambda}_m = \bar{t}_s + \bar{t}_r
\]

(5) (6)

The magnetizing characteristic of the machine is assumed to be a known non-linear function which gives dependence of magnetizing flux linkage space vector modulus on magnetizing current space vector modulus.

In any case, two vectors are sufficient to perform any transients. Their possible models, by changing the space state variables, can be obtained by: Choosing two vectors among \((\bar{t}_s', \bar{t}_r', \bar{t}_m', \bar{t}_s, \bar{t}_r, \bar{t}_m)\), except the pair \((\bar{t}_m', \bar{t}_m)\). According to the nature of their state variables, the resulting fourteen models are:

- **Current models:** \((\bar{t}_s', \bar{t}_s)\), \((\bar{t}_r', \bar{t}_r)\) and \((\bar{t}_m', \bar{t}_m)\).
- **Flux models:** \((\bar{\lambda}_s', \bar{\lambda}_s)\), \((\bar{\lambda}_r', \bar{\lambda}_r)\) and \((\bar{\lambda}_m', \bar{\lambda}_m)\).
- **Mixed models:** \((\bar{t}_s', \bar{\lambda}_s)\), \((\bar{t}_r', \bar{\lambda}_r)\), \((\bar{t}_m', \bar{\lambda}_m)\), \((\bar{t}_s', \bar{\lambda}_r)\), \((\bar{t}_s', \bar{\lambda}_m)\), \((\bar{t}_r', \bar{\lambda}_m)\) and \((\bar{t}_m', \bar{\lambda}_r)\).

**B. Classification of Possible Saturated Models**

Generalized flux linkage space vector is required to be aligned with magnetizing current and magnetizing flux space vectors, so that it can be represented in (5). Where \( L_m \) denotes an inductance which is dependent on the saturation level and remains for the time being unspecified. As there are six possible state-space variables, there are all together fourteen state-space models that can be formed (the pair of state-space variables that comprises magnetizing flux space vector and magnetizing current space vector cannot be selected). Living the selected pair of state-space variables, there are four possible types of models that may occur. These model types are mutually distinguishable with respect to the need to perform differentiation of inductance \( L_m \) as a time dependent variable in further derivations. The first types include models that will ask for \( L_m \). Secondary model type comprises all the models where \( \frac{dL_m}{dt} \) will be needed. The third group includes models that will ask for \( \frac{dL_m}{dt} \) and finally the four group encompasses models where \( \frac{dL_m}{dt} \) will be required. Possible saturated d-q models of IM are classified into four families:

- **Models with** \( L_m : (\bar{t}_s', \bar{\lambda}_s)\)

- **Models with** \( \frac{dL_m}{dt} : (\bar{t}_s', \bar{t}_s)\), \((\bar{t}_s', \bar{\lambda}_s)\), \((\bar{t}_s', \bar{t}_m)\), \((\bar{t}_m', \bar{\lambda}_s)\)

The theoretical treatments developed in [1 to 3], shows that the introduction of magnetic saturation raised the presence of magnetic coupling between the direct and quadrature axis. The cross-saturation coefficients in the four families are respectively \( L_{dq/0}, L_{dq/1}, L_{dq/2} \) and \( L_{dq/3} \).

where,

\[
L_{dq/0} = 0 \\
L_{dq/1} = (L_{sw0} - L_{sw}) \cos \alpha \sin \alpha \\
L_{dq/2} = L_{sw} \cdot L_{sw} - L_{sw} - \alpha \sin \alpha \\
L_{dq/3} = L_{sw} \cdot L_{sw} - L_{sw} - \alpha \sin \alpha
\]

**III. OF SATURATED MODELS WITHOUT CROSS-MAGNETIZING**

As the steady-state cross-saturation is retained in all the cases, equations (1)–(2) remain to be used in all the models. If the cross-saturation is to be neglected, it is necessary to equate terms \( L_{dq/1}, L_{dq/2} \) and \( L_{dq/3} \), respectively, to zero.

**A. Derivation of \( \frac{dL_m}{dt} \) Models: \((\bar{t}_s', \bar{t}_s)\)**

Deriving the linkage fluxes expressed by (3) and (4) leads to the time derivative of the magnetizing inductance \( L_m \). The leakage inductances are supposed to be invariable. Now let’s write:

\[
\frac{dL_m}{dt} = \frac{dL_m}{dt} \frac{dL_m}{dt} \frac{dL_m}{dt}
\]

(8)

The main system formed by equations (1) and (2) becomes:

\[
\begin{align*}
v_w &= R_l i_w + (L + L_m) \frac{dL_m}{dt} + L_m \frac{dL_m}{dt} \cdot \omega \left(L_{sw} + L_{sw} \right) \\
v_w &= R_l i_w + (L + L_m) \frac{dL_m}{dt} + L_m \frac{dL_m}{dt} \cdot \omega \left(L_{sw} + L_{sw} \right) \\
0 &= R_l i_m + L_m \frac{dL_m}{dt} + (L + L_m) \frac{dL_m}{dt} \cdot \omega \left(L_{sw} + L_{sw} \right) \\
0 &= R_l i_m + L_m \frac{dL_m}{dt} + (L + L_m) \frac{dL_m}{dt} \cdot \omega \left(L_{sw} + L_{sw} \right)
\end{align*}
\]

(9)

where

\[
L_{sw} = L_m + \cos^2 \alpha (L_{sw} - L_m) \\
L_{sw} = L_m + \sin^2 \alpha (L_{sw} - L_m)
\]

(10)

Argument \( \alpha \) is the angular position of space vector \( \bar{t}_m \) (or \( \bar{\lambda}_m \)) with respects to d-axis so that:
\[ \alpha = \tan^{-1} \frac{i_{qm}}{i_{dm}} \]  

\section{B. Derivation of \( \frac{dL_{1q}}{dt} \) Models: \((\vec{r}, \vec{\lambda})\)}

Derivation of the model \((\vec{r}, \vec{\lambda})\) requires the description of the currents \(i_{dr}, i_{dq}\) and fluxes \(\lambda_{dr}, \lambda_{dq}\) as function of the chosen state variables so that:

\[ \vec{r} = \frac{1}{L_n} (\vec{\tau} - L_{1r} \vec{\lambda}) \]  

\[ \vec{\lambda}_r = \left(1 + \frac{1}{L_n}\right) \vec{\lambda} - \left(1 - \frac{2}{L_n}\right) \vec{r} \]  

Deriving rotor linkage fluxes and rotor current, the finally equations of the model \((\vec{r}, \vec{\lambda})\) are:

\[ v_{sr} = R_i i_{sr} + \frac{d\lambda_{sr}}{dt} - \alpha_i \lambda_{sr} \]  

\[ v_{sr} = R_i i_{sr} + \frac{d\lambda_{sr}}{dt} + \alpha_i \lambda_{sr} \]  

\[ 0 = R_i (\lambda_{sr} - L_{1sr}) - (1 + L_{2r}) \frac{di_{sr}}{dt} - \omega \left(\frac{L_{1m}}{L_n} \lambda_{qr} - (1 + \frac{L_{2r}}{L_n}) i_{sr}\right) \]  

\[ 0 = R_i (\lambda_{sr} - L_{1sr}) - (1 + L_{2r}) \frac{di_{sr}}{dt} + \omega \left(\frac{L_{1m}}{L_n} \lambda_{qr} - (1 + \frac{L_{2r}}{L_n}) i_{sr}\right) \]  

where

\[ L_{2r} = \frac{L_{1m}}{L_n} + 1 \left(L_{1m}^{-1} - L_{1m}^{-1}\right) \cos^2 \alpha \]  

\[ L_{2q} = \frac{L_{1m}}{L_n} + 1 \left(L_{1m}^{-1} - L_{1m}^{-1}\right) \sin^2 \alpha \]

\section{C. Derivation of \( \frac{dL_{1r}}{dt} \) Models: \((\vec{r}, \vec{\lambda})\)}

Following, is shown how to proceed for deriving the \((\vec{r}, \vec{\lambda})\) model. The description of \(\vec{r}\) and \(\vec{\lambda}_r\) as function of the chooser state variables so that:

\[ \vec{r} = \frac{1}{L_{1w}} \left(1 - \frac{L_{1w}}{L_n}\right) \vec{r} \]  

\[ \vec{\lambda}_r = \left[1 - \frac{1}{L_{1w}}\right] \vec{\lambda}_r + \frac{L_{1w}}{L_n} \vec{\lambda} \]  

Finally the voltage equations of the model \((\vec{r}, \vec{\lambda})\) are:

\[ v_{sr} = R_i i_{sr} + (1 + L_{1r}) \frac{di_{sr}}{dt} + L_{1r} \frac{d\lambda_{sr}}{dt} - \omega \left(\frac{L_{1m}}{L_n} i_{sr} + (1 + \frac{L_{1m}}{L_n}) \lambda_{qr}\right) \]  

\[ v_{sr} = R_i i_{sr} + (1 + L_{1r}) \frac{di_{sr}}{dt} + L_{1r} \frac{d\lambda_{sr}}{dt} + \omega \left(\frac{L_{1m}}{L_n} i_{sr} + (1 + \frac{L_{1m}}{L_n}) \lambda_{qr}\right) \]  

\[ 0 = \frac{R_i}{L_n} (\lambda_{sr} - L_{1sr}) + \frac{d\lambda_{sr}}{dt} - \omega \lambda_{sr} \]  

\[ 0 = \frac{R_i}{L_n} (\lambda_{sr} - L_{1sr}) + \frac{d\lambda_{sr}}{dt} + \omega \lambda_{sr} \]  

where,

\[ L_{q1} = l \frac{L_m}{L} + l^2 (L_{1q}^{-1} - L_{1q}^{-1}) \cos^2 \alpha \]  

\[ L_{q1} = l \frac{L_m}{L} + l^2 (L_{1q}^{-1} - L_{1q}^{-1}) \sin^2 \alpha \]

\section{IV. CROSS-SATURATION SENSITIVITY OF POSSIBLE MODELS}

In order to emphasize the saturation in accordance with the carried out theory, the dynamic reversal process of an asynchronous motor with star connection and rated voltage 82V is considered. The motor’s parameters are:

\[ R_s = 3.45 \Omega; R_r = 2.95 \Omega; l = 0.01 H \]  

\[ L_{n\text{unsat}} = 0.1342 H \]

\section{Figure 1. No-load saturation curve for induction machine}

The magnetic characteristic \(\lambda_m(i_m)\) (Fig. 1) is approximated by:

If \(i_m \leq 0.484\) then \(\lambda_m = 0.1342^*i_m\)

If \(i_m \geq 0.742\) then \(\lambda_m = \frac{0.3051^*i_m}{1 + 0.5161^*i_m}\)

Else \(\lambda_m = 0.2046^*i_m + 0.2455^*i_m\)

In this section, we present comparative results of this paper were obtained in 4 cases:

a- Unsaturated model \((L_{nm} = \text{cst})\)

b- Full saturated model \((L_{gq} \neq L_{ds} ; L_{dq} \neq 0)\)

c- Saturated model without cross-saturation \((L_{gd} \neq 0)\)

d- Saturated model without cross-saturation \((L_{gd} = L_{gq} = L_{ds} = 0)\)

It has to be emphasized that the accuracy of saturation representation in all 14 models is the same. Otherwise, all found models have exactly the same ability to describe any transient regime. Fig. 1, shows the stator current to phase (a) with and without magnetic saturation. Moreover, this figure can be obtained using any of the 14 found methods.
models, especially with those fully developed in this paper.

Figure 2. Stator phase current predicted by full saturated model and by unsaturated model

The curves (3) to (5), show the simulation results related to the startup for three models $(\vec{i}_r, \vec{i}_s)$, $(\vec{i}, \vec{\lambda})$ and $(\vec{i}, \vec{\lambda}_s)$ of induction machine for nominal voltage $V = 82V$. The model without cross-saturation when $(L_d = L_q = L_m, L_{dq} = 0)$ gives practically identical results to those obtained with the full saturated model, Fig. 3 to Fig. 5.

This step is used to compare behavior of various models when cross-saturation is neglected. Thus it appears that cross-saturation can be fully ignored in a number of models, without any sacrifice in accuracy of the results. The mixed model which neglects the terms of cross-saturation gives identical results with the full saturated model.

As an example, it is impossible to distinguish between the two curves of Fig. 5 as the differences in values are of the order of 0.6 %.

This latter idea is confirmed by a comparative table (Table II) of stator current for various models with and without cross-saturation, in order to identify the models not sensitive to this phenomenon.

All the models that require time derivative of the magnetizing inductance are practically a high sensitivity to the omission of the cross-saturation representation. To the contrary, the family of models with $\frac{dl_m}{dt}$ yield very poor results when cross-saturation is neglected.

As a result, these models can be considerably simplified, enabling faster simulations. Additionally, the models can be used for construction of sufficiently simple flux observers for saturated machines in vector controlled drives.

<table>
<thead>
<tr>
<th>$\Delta I_m$ in %</th>
<th>V(V)</th>
<th>82</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\vec{i}_r, \vec{i}_s)$, $(\vec{i}_m, \vec{i}_r)$</td>
<td>117.014</td>
<td>125.033</td>
<td>119.23</td>
<td></td>
</tr>
<tr>
<td>$(\vec{i}, \vec{\lambda})$, $(\vec{i}, \vec{\lambda}_m)$</td>
<td>17.279</td>
<td>22.3036</td>
<td>28.6458</td>
<td></td>
</tr>
<tr>
<td>$(\vec{i}, \vec{\lambda}_s)$</td>
<td>5.5476</td>
<td>6.7229</td>
<td>8.2682</td>
<td></td>
</tr>
<tr>
<td>$(\vec{i}, \vec{\lambda}_r)$</td>
<td>0.5966</td>
<td>0.5955</td>
<td>0.6640</td>
<td></td>
</tr>
<tr>
<td>$(\vec{i}, \vec{\lambda}_s)$</td>
<td>0.5567</td>
<td>0.6381</td>
<td>0.6859</td>
<td></td>
</tr>
</tbody>
</table>

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The impact of cross-saturation on the various models of the asynchronous machine, allows us to establish the following classification:

- The models to no sensitivity: \((\lambda_s, \lambda_r)\)
- The models to weak sensitivity \((\lambda_s, \lambda_h, \lambda_r)\)
- The models to mean sensitivity \((\lambda_s, \lambda_h, \lambda_r, \lambda_r)\)
- The models to high sensitivity: \((\lambda_s, \lambda_h, \lambda_r, \lambda_r, \lambda_r, \lambda_r, \lambda_r)\)

V. CONCLUSION

In this paper, a detailed models synthesis and a method to take into account magnetic saturation for asynchronous machine has been presented. The synthesis proves the existence of fourteen possible models. The main feature of the presented procedure, for introducing saturation, is to avoid magnetic couplings between quadrature axes or cross-saturation. A detailed study on its influence on the various models when one eliminates the terms which characterize this phenomenon.

This study allowed us to establish a new classification of the possible models in four families: models with no, weak, mean and high sensitivity to cross-saturation. The use of models not sensitive to cross-saturation saves on the simplicity of model development and give the possibility to be able to assure the vector controlled even in presence the magnetic saturation.

NOMENCLATURE

- \(s, r\) Indices for stator and rotor respectively
- \(d, q\) Indices for direct and quadrature axis
- \(R_s, R_r\) Resistances per phases
- \(L_s, L_r\) Self inductances per phase
- \(l_s, l_r\) Leakage inductances per phase
- \(L_m\) Static magnetizing inductance
- \(L_{sd}, L_{sq}\) Dynamic magnetizing inductance
- \(\tilde{v}_d, \tilde{v}_q\) Space vector voltages
- \(\tilde{i}_d, \tilde{i}_q\) Space vector winding currents
- \(\tilde{\lambda}_d, \tilde{\lambda}_q\) Space vector winding fluxes
- \(\lambda_m\) Space vector magnetizing flux
- \(\tilde{\omega}\) Space vector magnetizing current
- \(\omega_o\) Frame speed
- \(\omega\) Actual rotor electrical speed
- \(\omega' = (\omega_o - \omega)\) Difference between from stator and rotor speeds

REFERENCES


Mohamed Arbi KHLIFI was born in Tunisia, in 1982. He received the M.S., and Ph.D. in Electrical Engineering from the Tunisian National University, Tunis, Tunisia. He joined the National college of Engineering of Tunis, as Professor in the Department of Electrical Engineering. He is currently a Professor in the Electrical Engineering Department, College of Engineering, Hail University. Prof. KHLIFI has published More than 50 research papers in many journals and international conferences. He has organized many national and international conferences and has been member of the international Board committee of many International Conferences, MELECON2012, ICESA2013, IREC2014 and 2015, ICSS2015. His is Associate Editor of IJECEE journal. His main is member IEEE FRANCE section and ASET (association for Tunisian electrical specialists). He is research laboratory in Signal Image and Energy Mastery in the University of Tunis. His main interest research areas are renewable energy, Smart grid wind, solar..., and control of high efficiency integrated electrical energy conversion in various industrial fields. Power electronics and control, which include electrical machine drives.