A Comparative Analysis of Algorithms for Controlling the Attitude of an Unmanned Aerial Vehicle

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***Abstract*—Controlling behavior of Unnamed Aerial Vehicles is one of the stimulating problems in robotics. In this regard, we are conducting a survey which investigates an adaptive approach for controlling the attitude of flying robot. This paper aims to present the comparison between different controllers to be used in a dynamic model of a UAV's platform. A survey has been conducted on different types of PID algorithms, which has been considered in three structures with respect of optimal control signal applied to the actuators. A comparative analysis is performed on different stabilizing algorithms. For better performance of Quad-rotor during the hover mode, the cascade control system has been proposed. Another approach Linear Quadratic Regulator is discussed in this paper. All the results are based on the simulation. According to our survey, both PID and LQR should be used for most balanced Quad-copter.**

***Index Terms*— UAV's, PID, LQR, control, algorithm**

I. Introduction

UAV's like Quad-rotor also called Quad-rotor, is widely used in many applications and is being performed different tasks due to its hovering, landing strategies and flying characteristics. There are many Quad-rotors that are controlled by RC remote controllers but these Quad-rotors are not enough much stable. This instability is quite dangerous for human being because RC controlled Quad-rotors can unbalance at any time and may fall on any place during flying so it can be harmful, there must be a need to stabilize. First of all we focused on basics of quad-rotor and this is a very simple task to stable the quad-rotor because stability is achieved by just changing the speed of four rotors. By changing the speed of right and left rotors simply Quad-rotor can move left or right and by adjusting the speed of front and rear rotors, forward and backward movement is possible. . But there is a very critical algorithm exists behind this and different approaches can be used for the sake of stability. Pitch torque is the function of difference f_1-f_3 and roll is the function of difference f_2-f_4 and yaw is the function of

$$\tau_{M1}+\tau_{M2}+\tau_{M3}+\tau_{M4} \tag{1}$$

As shown in Fig. 1
where τMi is the motor torque that can vary by the motor and due to acceleration and drag force. According to newton's second law by neglecting shaft friction states that

$$I_{M\omega i}=-b\omega 2i+\tau_{Mi} \tag{2}$$

where I_M is the angular momentum that can change by i_{th} motor. Ever y external aspects must be included in software to manipulate the external effects (Hardware implementation into software).

Quad-rotor control is fundamental and interesting problem with six degree of freedom (three rotational and three translational) [1]. In order to achieve six degree of freedom, rotational and translational motion are coupled. The resultant dynamics are highly complex especially after considering the different aerodynamics effects. Unlike ground vehicles, Quad-rotor provide very less resistance during its motion to stable as discussed in [2].

A well-known approach for decoupling the problem solution is non-linear inverse dynamics (NID), but the problem with NID is, this is efficient when all the external parameters are known otherwise not. So to achieve this goal adaptive control is used in [3] and [4]. The solution of this problem under consideration of incomplete information about the plant and unknown external disturbances is the application of the Dynamic Contraction Method (DCM) [5] applied in [6]. Mostly problems occur for above mentioned approaches in real applications are proved high order of the controller equations and can influence for measurement of noise control quality. Approximations of higher derivatives amplify the measuring noise and cause abrupt changes for control signal. Therefore in this paper the different structures of PID controllers are described, which can reduce the adverse effects.

The most basic approach for stability is PID (proportional, integration and derivative) control that is much easy to understand. Basically this is the closed loop algorithm which can change the speed of rotors. There are different types of PID controllers, type A, B and C [2]. We compared all three types individually and their performances, PID controller is efficient for linear model

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not for dynamic model and Quad-rotor itself a dynamic model including different parameters.

The classic strategies which are assumed to obtained for controlling the helicopter or Quad-rotor for a particular operating point, but by using modern non-linear method we can increase the performance of UAV's in autonomous control. Another approach can be used for dynamic mode that is development of the LQR which can reduce the low vibrations based on a time variant model. The time-optimal control problem of a hovering Quad-rotor helicopter is addressed in [7]. Instead of utilizing the Pontryagin's Minimum Principle (PMP), in which higher order of non-linear are involved, nonlinear programming (NLP) method is proposed. Nonlinear control problems for hovering Quad-rotor helicopters such as feedback linearization control and back-stepping control laws were studied in [8].

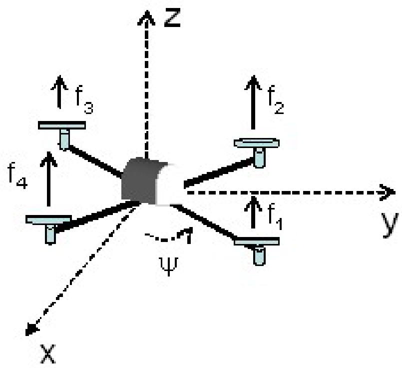

Figure 1. Parameters of quad-copter

II. KINEMETICS OF QUAD-COPTER

First we define the linear position, linear velocity, angular position and angular velocity. Linear position (x,y,z) Linear velocity (x',y',z') Angular position(ϕ,Θ,ψ) Angular Velocity (ϕ',Θ',ψ'**)**. However note that angular velocity and linear velocity is not the same thing so relationship between linear velocity and angular velocity is mentioned here

$$\omega=\begin{pmatrix}1 & 0 & -\mathrm{Sin}\,\theta\\ 0 & \mathrm{Cos}\,\phi & \mathrm{Cos}\,\theta\,\mathrm{Sin}\,\phi\\ 0 & -\mathrm{Sin}\,\phi & \mathrm{Cos}\,\theta\,\mathrm{Cos}\,\phi\end{pmatrix}\theta' \tag{3}$$

ω is angular velocity of the Quad-rotor. We can relate body and inertial frame by using rotational matrix R going from body frame to inertial frame. The matrix is derived from ZYZ Euler angle and successfully undergoing yaw, pitch and roll.

$$R=\begin{pmatrix}\mathrm{C}\phi\mathrm{C}\varphi-\mathrm{C}\theta\mathrm{S}\phi\mathrm{S}\varphi & -\mathrm{C}\varphi\mathrm{S}\phi-\mathrm{C}\phi\mathrm{C}\theta\mathrm{S}\varphi & \mathrm{S}\theta\mathrm{S}\varphi\\ \mathrm{C}\theta\mathrm{C}\varphi\mathrm{S}\phi+\mathrm{C}\phi\mathrm{S}\varphi & \mathrm{C}\phi\mathrm{C}\theta\mathrm{C}\varphi-\mathrm{S}\phi\mathrm{S}\varphi & -\mathrm{C}\varphi\mathrm{S}\theta\\ \mathrm{S}\phi\mathrm{S}\theta & \mathrm{C}\phi\mathrm{S}\theta & \mathrm{C}\theta\end{pmatrix} \tag{4}$$

where C=Cos and S=Sin.

In [9] complete dynamic modeling is performed which will be responsible for stability and controlled by some algorithm. These dynamic modeling techniques are necessary for a stable flight. Then we can control our flight by PID (Proportional, integration and derivative), LQR (Linear Quadratic Regulator).

There are three algorithms exist in PID controller proportional (p), integration (I) and derivative (d). These three algorithms perform a specific effect on Quad-copter's flight as discussed in Table I. P depends on current error, I is the accumulation of past errors. Here are the different effects which are individually mentioned in below table.

TABLE I. RESPONSES OF PID CONTROLLER

Parameter	RiseTime	Overshoot	Settling time	Steady-state Error	Stability
K_p	Decrease	Increase	Small Change	Decrease	Degrade
K_i	Decrease	Increase	Increase	Eliminate	Degrade
K_d	Minor Change	Decrease	Decrease	No effect in theory	Improve K_d small

$$u(t)=K_p+K_i\int_0^t e(\tau)d\tau+K\frac{dy(t)}{dt} \tag{5}$$

Ideally these parameters are:
where Kp is proportional, Ki is integration gain and Kd is derivative gain.

Different control schemes are discussed in [10] that are very much useful. In control applications, the refusal of external parameters and occurrence of disturbance chance can't be neglected it is necessary. In order to deal with such problems cascaded system is preferable, because this system has one input and multi outputs.

TABLE II. SPECIFICATIONS OF TERMINLOGIES

1- Rise time	Minimum
2- Overshoot	No
3- Settling time	Minimum
4- Steady state error	Negligible

Primary controllers and primary dynamics are the components of outer loop. There are two loops primary loop and secondary loop. Inner loop is the part of outer loop. Primary controller's calculate the set point for secondary controller and at the end the quad-rotor plant acquire at accurate values of yaw, roll and pitch. This cascaded system is shown in Fig. 4 Step response of cascaded system is shown in Fig. 5 Good response with low settling and steady state error can be experienced. But response has some overshoot which is undesirable. The requirement of a good PID controller is mentioned in Table II.

Aerobatic Flight

- Slightly higher value of P
- Slightly lower value of I
- Increasing value of D

Smooth Flight

- Lower value of P
- Higher value of I
- Decreasing value of D

A. *Types of PID Controllers*

There are different types of PID controller's i.e. type A, type B and type C also referred to as Fig. 2, 3 and 4 In control theory ideal PID controller parallel connection is shown in time domain whose equation is given below, but there is a problem with conventional PID controller. When step input is provided then it produces impulse response which is efficient only in ideal case. There are two sources of violent controller reaction one is proportional and the other is derivative.

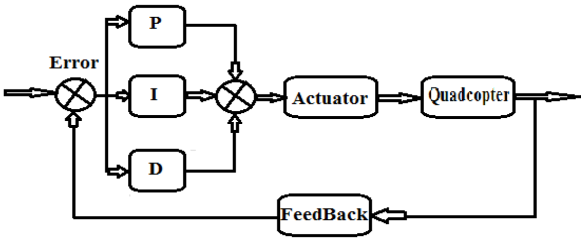

Figure 2. Type A

Problem can be resolve by derivative, or derivative and proportional both through the feedback and set point as integration. So know we have two combinations of circuits one is PI-D and I-PD. As suggested in [11], [12] the types are type B and type C respectively.

Type B: It is more suitable and efficient in practical exercise to take the derivative of output y(t). It is accomplished by putting derivative part of PID controller into feedback that takes the derivative of output and reduce the overshoot of output which may cause sharpness of control signal.

$$u(t) = K_p + K_i \int_0^t e(\tau)d\tau - K\frac{dy(t)}{dt} \tag{6}$$

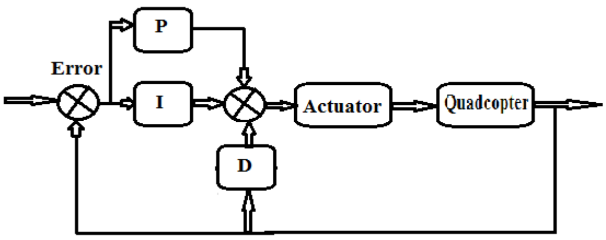

Figure 3. Type B

Type C: This structure is not so often as PID structure, but it has certain benefits. With this structure transfer of reference value to control signal is totally avoided and control signal has less sharp changes than other. Control law for this structure in Fig. 4 is given as:

$$u(t) = -K_pY(t) + K_i \int_0^t e(\tau)d\tau - K\frac{dy(t)}{dt} \tag{7}$$

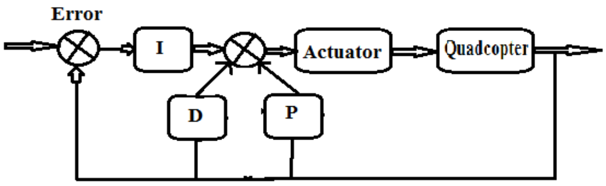

Figure 4. Type C

III. Linear Quadratic Regulator (LQR)

As explained and implemented in [13] a LQR. Now we are going to explain LQR system and then we shall see simulation results of that. The relationship of LQR relating with present states, next state and inputs are given below. As said in [13] LQR can handle multi inputs and multi outputs so the vector x and u is n*m and c*d matrices

$$x' = Ax + Bu \tag{8}$$

It is convenient approach to know the vector u that minimizes the cost of function in equation (2) must have to be minimized to achieve the desire position. More the low value of J, more the accurate position you can achieve.

$$J = \int_0^\infty (x.Qx + u.ru)dt \tag{9}$$

$$u = -Kx \tag{10}$$

We have to find the value of K that penalized the inputs and outputs to achieve the desire position. Its value need to be set that can minimize the cost of J in other words. Q is the square matrix adjusted to provide the most appropriate values of inputs which may be useful for achieving our goal. With the reference of [13] in step response of vertical position and vertical speed there is no over shoot founded in stimulation results and a very good rise time of 2 second which is very good for a stable flight. It was performed in [13] LQR control for all other movement for Quad- rotor where from satisfactory results were obtained.

Graph of speed response goes to zero after some time, this is just because of LQR controller. In actual or practical model there must be some disturbance in the environment due to air or drag force etc.

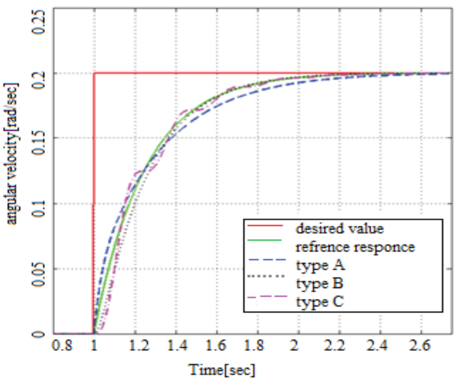

Figure 5. Comparison of Type (A,B,C)

IV. PID Controller with LQR Loop

In [13] define the complete methodology for tuning of PID controller. For better timing results, it is necessary to build another algorithm to insure measurement of desire value. In order to reach, an algorithm is performed in [13], which calculates the gain automatically and gain specific response. Development time increase radically and controllers could be tune for reasonable parameters for system. This scheme has most appropriate result that system is called critical damped function which is most appropriate in all other responses. There is no overshoot, less settling time and performance is very rapid. Response of that kind of controllers should very rapid because system should be respond quickly against the disturbance.

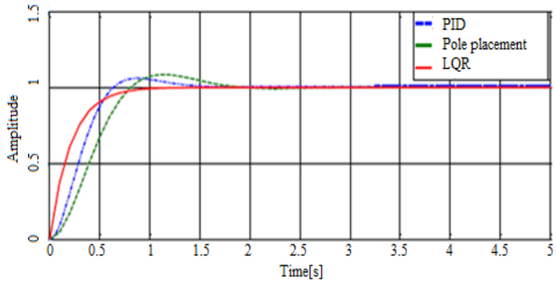

Figure 6. Comparison of PID and LQR

V. Summary

It is clear from above mentioned Fig. 5 and Fig. 6, that each controlling system has different response for the same attitude. So it is possible to choose the most applicable and appropriate system for the desired results according to their features.

PID controllers are used widely because they are easy to implement. But they can deal with linear systems single input and single output (SISO) but we require a system which can interact with multi inputs and multi outputs (MIMO) that is Linear Quadratic Regulator.

From figures it is easy to understand that PID controller tuned with LQR is fastest and the controller presented with no overshooting value and minimum rise time. Step response of PID controller tuned with LQR and classic LQR have the same settling time but with no overshoot.

Considering examination and results found in this paper, it is easy to observe that the plant was controlled with different controllers. Each of them shown the similar results, but with a little dissimilarity. It is known that classic LQR controller has minimum overshoot but there is a big transition delay which makes it an inappropriate choice when system needs for fast parameter updates. On the other hand, PID controller gives a faster response but with a robust gains as the other controllers. It can only handle one input but other controllers can handle multi inputs. Looking at the results above mentioned in this paper, we can conclude that PID tuned by the LQR controller shown a better performance when compared with others and hence, a classic PID tuned by LQR robust controller is versatile and easy implementable in terms of transient responses and complexity.

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