Design of Interval Type-2 Fuzzy Sliding Mode Controller for Hypersonic Aircraft

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Abstract—**The interval type-2 fuzzy, combined with sliding mode control, is proposed in this paper to design a novel nonlinear robust controller for a hypersonic aircraft. In this method, sliding mode reaching law is designed to keep the system stable. In order to overcome the uncertain parameters and external disturbances which exist in aerospace, we utilize an interval type-2 fuzzy approach represented by 9-point representation. Simulation results indicate that this method of interval type-2 fuzzy sliding mode controller can provide robust flight control to ensure good tracking performance of hypersonic aircraft.**

Index Terms—**sliding mode control, interval type-2 fuzzy, flight control, hypersonic aircraft**

I. INTRODUCTION

A hypersonic aircraft, with either rockets or scramjet as the main power plant, is an aircraft which can achieve Mach-5 flight speed, either winged or wingless. It is, however, more difficult to control the flight of hypersonic aircraft, with their features of having rapidly time-varying, strongly nonlinearity, strong-coupling and uncertainty.

The following are some methods that have been proposed to handle these problems. The nonlinear dynamic inversion was used to deal with the uncertain parameters of hypersonic aircraft to achieve the robust nonlinear control [1]. In [2], the robust linear output feedback control was used to provide robust tracking in the presence of model uncertainties for an air-breathing hypersonic vehicle. A nonlinear robust adaptive controller was designed for a flexible air-breathing hypersonic aircraft by combination of nonlinear sequential loop closure and adaptive dynamic inversion [3]. Terminal sliding mode control has been combined with a second-order sliding mode control approach to solve the parameter uncertainties of a hypersonic vehicle [4].

Interval type-2 fuzzy [5], as a novel kind of fuzzy, can deal with more uncertain problems. Recently, the type-2 fuzzy has also begun to be used to control aircraft. A type-2 fuzzy and a cerebellar model articulation controller were combined and used in the automatic landing system [6]. It can also be combined with neural networks to solve the parameter uncertainty problem for hypersonic flight vehicles [7].

In this paper, the combination of sliding mode control and the interval type-2 fuzzy control based on the full-state feedback is proposed to longitudinal control of hypersonic aircraft.

II. THE MODEL OF HYPERSONIC AIRCRAFT

Using the longitudinal force and moment equilibrium of hypersonic aircraft, the longitudinal model of hypersonic aircraft [8] can be obtained as follows:

$$\dot{V} = \frac{T\cos\alpha - D}{m} - \frac{\mu\sin\gamma}{r^2} \tag{1}$$

$$\dot{\gamma} = \frac{L + T\sin\alpha}{mV} - \frac{(\mu - V^2 r)\cos\gamma}{Vr^2} \tag{2}$$

$$\dot{q} = \frac{M_y}{I_y} \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$

$$\dot{h} = V\sin\gamma \tag{5}$$

$$\ddot{\beta} = -2\xi\omega\dot{\beta} - \omega^2\beta + \omega^2\beta_c \tag{6}$$

In this model, V, γ, q, α, and h are the aircraft's velocity, flight path angle, pitch rate, angle of attack, and altitude, respectively; β, ω, and ξ are the throttle setting, natural frequency, and damping coefficient; and m, μ, r, M_y, and I_y are the mass, gravitational constant, radial distance from the earth's center, pitching moment, and moment of *inertia*. For the lift L, drag D, thrust T, we have: $L = 0.5\rho V^2 sC_L$, $D = 0.5\rho V^2 sC_D$, $T = 0.5\rho V^2 sC_T$. Uncertain parameters as an additive variance are modeled to the nominal values used for controller design as follows: $m = m_0(1+\Delta m)$, $I_y = I_{y0}(1+\Delta I_y)$, $s = s_0(1+\Delta s)$, $\bar{c} = \bar{c}_0(1+\Delta c)$, $c_e = c_{e0}(1+\Delta c_e)$, $\rho = \rho_0(1+\Delta\rho)$.

$$\begin{bmatrix} \dddot{V} \\ h^{(4)} \end{bmatrix} = \begin{bmatrix} f_V \\ f_h \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \beta_c \\ \delta_e \end{bmatrix} \tag{7}$$

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where, f_V, f_h, b_{11}, b_{12}, b_{21}, b_{22} can refer to [11].

III. DESIGN OF INTERVAL TYPE-2 SLIDING MODE CONTROLLER OF HYPERSONIC AIRCRAFT

In this paper, considering the nonlinear model of hypersonic aircraft with uncertain parameters and extra disturbances in aerospace, the interval type-2 fuzzy combined with the sliding mode control to provide longitudinal control of hypersonic aircraft. The system structure is shown in Fig. 1.

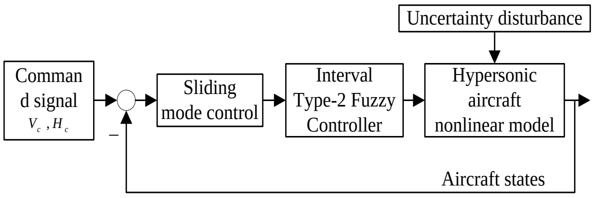

Figure 1. The system structure of type-2 fuzzy sliding mode control of hypersonic aircraft

A. Sliding Mode Reaching Law

According to the Lyapunov stability theory, define the Lyapunov function $V = \frac{1}{2}S^2$, where S is the sliding surface [9-11]. If $\dot{V} = S\dot{S} < 0$, the sliding surface satisfies the sliding condition and the system is stable. Therefore, define the sliding mode surfaces as:

$$\begin{cases} S_V = (d/dt + \lambda_V)^2 e_V(t) = \ddot{V} - \ddot{V}_d + 2\lambda_V \dot{e}_V + {\lambda_V}^2 e_V \\ S_h = (d/dt + \lambda_h)^3 e_h(t) = \dddot{h} - \dddot{h}_d + 3\lambda_h \ddot{e}_h + 3{\lambda_h}^2 \dot{e}_h + {\lambda_h}^3 e_h \end{cases} \quad (8)$$

Choose the reaching law:

$$\begin{cases} \dot{S}_V = -k_V sat(S_V) \\ \dot{S}_h = -k_h sat(S_h) \end{cases} \quad (9)$$

where k_V and k_h are positive integers, and sat is the saturation function.

Finally, the control law can be obtained as follows:

$$\begin{bmatrix} \beta_c \\ \delta_e \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \dddot{V}_d - f_V - 2\lambda_V \ddot{e}_V - \lambda_V^2 \dot{e}_V - k_V sat(S_V) \\ h_d^{(4)} - f_h - 3\lambda_h \dddot{e}_h - 3\lambda_h^2 \ddot{e}_h - \lambda_h^3 \dot{e}_h - k_h sat(S_h) \end{bmatrix} \quad (10)$$

B. Interval Type-2 Fuzzy Logic System

In the elevator channel, δ_e and $\dot{\delta}_e$ are taken as the two inputs of the interval type-2 fuzzy respectively. Finally, the output u_{δ_e} of interval type-2 fuzzy is taken as the real input to the hypersonic aircraft.

In the interval type-2 fuzzy controller, the fuzzy set of δ_e has an upper set $\bar{X}_{\delta_e}$ and a lower fuzzy set $\underline{X}_{\delta_e}$. Then we use $\tilde{X}_{\delta_e}$ to stand for the fuzzy set of δ_e. Similarly, we use $\tilde{X}_{\dot{\delta}_e}$ to stand for the fuzzy set of $\dot{\delta}_e$, which has an upper fuzzy set $\bar{X}_{\dot{\delta}_e}$ and a lower fuzzy set $\underline{X}_{\dot{\delta}_e}$.

In terms of Mamdani fuzzy rule system, the interval type-2 fuzzy rules can be described in the following form:

R^j: If x_1 is $\tilde{X}_1{}^j$ and x_2 is $\tilde{X}_2{}^j$ and $\cdots$ and x_n is $\tilde{X}_n{}^j$; Then y is Y^j, $j = 1, \cdots, N$.

R^j is the label of j-th rule, x_i, $i = 1, \cdots, n$ are the inputs of the fuzzy system, $\tilde{X}_i{}^j$, $i = 1, \cdots, n$; $j = 1, \cdots, N$ are fuzzy subsets of the inputs, Y^j, $j = 1, \cdots, N$ are fuzzy subsets of the output..

$$R^j = (\tilde{X}_1^j \times \tilde{X}_2^j \times \cdots \times \tilde{X}_n^j) \times Y^j \quad (11)$$

R^j is described by the membership function $\mu_{R^j}(x_1, x_2, \cdots, x_n, y)$.

$$\mu_{R^j}(x_1, x_2, \cdots, x_n, y) = \mu_{\tilde{X}_1^j} \wedge \mu_{\tilde{X}_2^j} \wedge \cdots \wedge \mu_{\tilde{X}_n^j} \wedge \mu_{B^j} \quad (12)$$

Therefore, for the elevator of hypersonic aircraft, there is:

$$\mu_R(\delta_e, \dot{\delta}_e, u_{\delta_e}) = \bigvee_{j=1}^{N} [\mu_{R^j}(\delta_e, \dot{\delta}_e, u_{\delta_e})] \quad (13)$$

The 9-point representation of an interval fuzzy set [12] is utilized in this paper to describe the interval type-2 fuzzy set, as shown in Fig. 2.

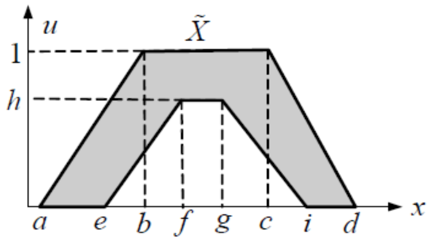

Figure 2. The 9-point representation of an IT2 FS

As shown in Fig. 3, the membership functions of $\tilde{X}$ can be represented as [a, b, c, d, e, f, g, i, h].

The type reducer [13], [14] and defuzzifier are designed by Karnik- Mendel (KM) algorithms [15]. Then, y_l and y_r can be computed.

$$F^n = \left[\mu\underline{X}_{\delta_e} \times \mu\underline{X}_{\dot{\delta}_e}, \mu\bar{X}_{\delta_e} \times \mu\bar{X}_{\dot{\delta}_e}\right] = \left[\underline{f}^n, \bar{f}^n\right] \quad (14)$$

$$f^n = \frac{\underline{f}^n + \bar{f}^n}{2}, \ n = 1, 2, \cdots, N \quad (15)$$

$$y_l = \frac{\sum_{n=1}^{N} \underline{y}^n f^n}{\sum_{n=1}^{N} f^n} \quad (16)$$

$$y_r = \frac{\sum_{n=1}^{N} \bar{y}^n f^n}{\sum_{n=1}^{N} f^n} \quad (17)$$

From the type-reducer, the interval set $[y_l, y_r]$ can be obtained, the average of which is used to count the defuzzified output.

$$u_{\delta_e} = y = \frac{y_l + y_r}{2} \quad (18)$$

IV. EXPERIMENTS

A. Set Up of Experiments

Assume a hypersonic aircraft with the following model parameters and initial states:

$m_0 = 136820kg$, $s_0 = 334.73m^2$, $\bar{c}_0 = 24.38m$, $I_{y0} = 9490740kg \cdot m^2$.

$$C_L = 0.6203\alpha \tag{19}$$

$$C_D = 0.645\alpha^2 + 0.0043378\alpha + 0.003772 \tag{20}$$

$$C_T = \begin{cases} 0.02576\beta & if\ \beta < 1 \\ 0.0224 + 0.00336\beta & if\ \beta > 1 \end{cases} \tag{21}$$

$$\begin{aligned} &|\Delta m| \le 0.02; |\Delta I_y| \le 0.01; |\Delta s| \le 0.01; \\ &|\Delta \bar{c}| \le 0.01; |\Delta c_e| \le 0.01; |\Delta \rho| \le 0.02. \end{aligned} \tag{22}$$

For the interval type-2 fuzzy system, the form of input and output membership functions can be seen in the following Fig. 3.

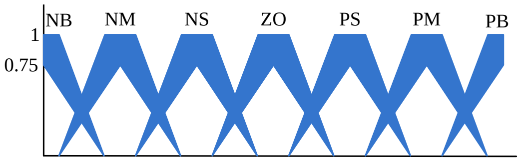

Figure 3. The membership functions of inputs and output

As for the input δ_e , it has been divided into seven fuzzy sets {NB, NM, NS, ZO, PS, PM, PB}. The interval type-2 fuzzy rules can be obtained in Table I.

TABLE I. INTERVAL TYPE-2 FUZZY RULES

u_{δ_e}		$\dot{\delta}_e$						
		PB	**PM**	**PS**	**ZO**	**NS**	**NM**	**NB**
δ_e	**PB**	PB	PM	PM	PM	PM	PM	PB
	PM	PM	PM	PM	PM	PM	PM	PM
	PS	PM	PS	PS	PS	PS	PS	PM
	ZO	PS	PS	PS	PS	PS	PS	PS
	NS	PS	PS	PS	ZO	PS	PS	PS
	NM	ZO	ZO	ZO	ZO	ZO	ZO	ZO
	NB	ZO	ZO	ZO	ZO	ZO	ZO	ZO

B. Simulation Results

We should design the controller without external disturbances at first. The control effects of velocity and altitude are shown in Fig. 4.

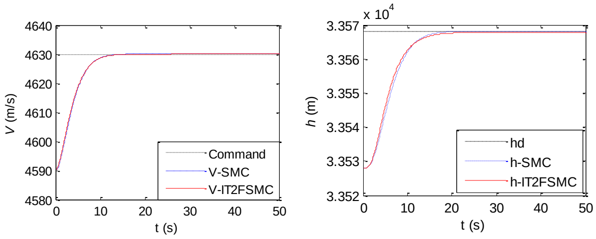

Figure 4. The comparison of control effects without uncertain parameters and external disturbances

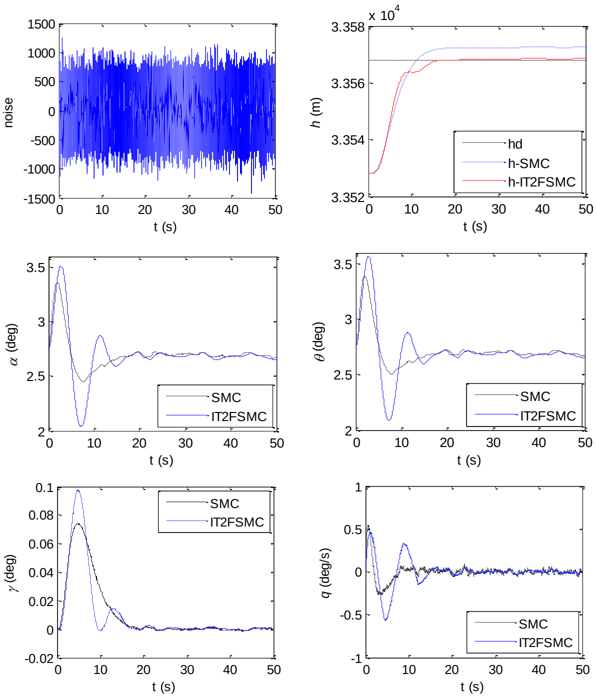

Figure 5. The comparison of control effects with uncertainty parameters and external disturbances

From Fig. 4, we can see that the control effects are excellent when the uncertain parameters and external disturbances are ignored. The velocity and altitude of hypersonic aircraft can track the ideal command precisely. However, the problems of uncertainties and external disturbances are inevitable. Consequently, we have to consider the influences of uncertainties and external disturbances.

White noise was added to the velocity component of the hypersonic aircraft respectively to imitate the environment with external disturbances. The control effects are shown in Fig. 5.

From the simulations results shown in Fig. 5, we can see that the hypersonic aircraft with the controller of interval type-2 fuzzy sliding mode control can precisely track the trajectory, while the controller with sliding mode control cannot track the ideal trajectory, which illustrates the proposed controller possesses robustness.

V. CONCLUSION

In this paper, considering uncertainties of parameters and inevitable external disturbance, the interval type-2 fuzzy represented by 9-point representation was combined with sliding mode control to design a novel flight controller of hypersonic aircraft. The effectiveness of the novel controller was illustrated by comparisons of control effects in presence of uncertainty parameters and strong external disturbances.

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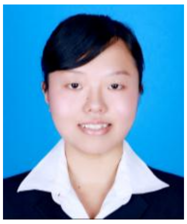

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