A Synthesis Method of Robust Cascade Control System

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***Abstract*—The article presents a general synthesis method of controllers in the cascade system. At first, it determines structure for each of equivalent control loops in the cascade system and imposes robust conditions for these loops. After that, the structures of controllers in the system are defined. Then the proportional coefficients of controllers are retuned based on the "soft characteristic". Finally, we show an example of using proposed method.**

***Index Terms*—cascade control system, soft characteristic, soft oscillation index**

I. Introduction

The cascade control is one of the most popular structures to improve performance of the single-loop control systems, by that arose the cascade control system. In the published works [1] [2] [3] [4] [5] [6], the authors deal with various concepts of a cascade control system. However, there is no identical view of cascade control system among them. The authors give some cascade structure and refer to system with two control loops at the end. In [7], the author outlined a clear definition and most complete structure of a cascade control system which is a system having many closed-loops nested inside.

The most industrial control systems have to work in conditions when dynamic behaviour of object (process) often changes. At the same time, the disturbances occur out of rules and are difficult to determine. In such circumstance the problem synthesizing control system has to meet two principle requirements, as follows:

The system must have maximum stability margin to anticipate the uncertainty of the object.

System output is close to the set point as precisely as possible, under the action of disturbances.

The researchers of [1]-[6], for tuning cascade controllers, have not completely solved the robust stability problem of the system and just stop at cascade structure with two loops. This article will propose a new synthesis method based directly on the robust conditions of the system [7].

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II. Background Theory

A. The Viewpoint of High-Robust Control System [7]

Let a control system has the transfer function: $W(s)=\frac{A(s)}{D(s)}$, where, s – complex variable; $D(s)$ – the characteristic polynomial with q real roots: $s_i = -\beta_i$ and $2p$ pairs of conjugate-complex roots: $s_k = -\beta_k \pm j\omega_k$, $\omega_k \geq 0$, $j=\sqrt{-1}$; $q+2p = n$; n – the order of $D(s)$. Thereafter, we call roots of the polynomial symply characteristic roots. Let acting step input with Laplace transform: $z(s) = 1/s$.

Our purpose is to construct robust control system therefore we consider only stable ones. So, the output system response can be written in the form:

$$y(t)=c_0+\sum_{i=1}^{q}c_i e^{-\beta_i t}+\sum_{k=q+1}^{q+p}c_k e^{-\beta_k t}\sin(\omega_k t+\varphi_k) \quad (1)$$

where, $c_{k(k=0,1,\ldots,q+p)}$ – constants; ω_k – frequency; φ_k – phase of the oscilation; t – time variable.

We consider the oscilating component of (1):

$$y_k(t)=c_k e^{-\beta_k t}\sin(\omega_k t+\varphi_k) \quad (2)$$

The oscilation decay of process $y_k(t)$ is defined as: $d_k=\frac{y_k(t+T)}{y_k(t)}$, where $T = 2\pi/\omega_k$ – period. Substituting $y_k(t)$ from (2), it's easy to come to the relationship:

$$d_k=e^{-2\pi m_k},\ m_k=\beta_k/\omega_k \quad (3)$$

Clearly that the factor m_k determines oscilation behaviour of the $y_k(t)$-component (therefore in [8] it's named "oscilation index"). Further, the greater m_k corresponds smaller decay d_k and less oscilation of the component. In the ultimate case, if $m_k = +\infty$ then $y_k(t)$ will not be oscilatory ($d_k = 0$).

So, the system is more robust (more stable) if all the $y_k(t)$ are less oscilatory. Consequently, the system will be most robust, if all $y_k(t)$ are not oscilatory, according to ultimate $m_k = +\infty$.

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In last case all characteristic roots trend into negative real and the system becomes a pure inertial (a lag system without oscilation). Among pure inertial systems, the simplest and most reliable is the first-order one.

Such, the most robust (high-robust) system has the transfer function:

$$W(s)=\frac{K}{1+\theta s},\ \theta>0 \tag{4}$$

where, K - transfer coefficient; θ - inertial constant.

B. High Performance of High-Robust System [7]

As shown in [7], a system under structure (4) will have best performance, when coefficient K = 1 and inertial constant θ is small as possible.

So, the high-robust system with high (best) performance has the transfer function:

$$W(s)=\frac{1}{1+\theta s},\ \ \theta>0 \tag{5}$$

where, θ - inertial constant (the lag of desired system).

We call the system as high-quality system. Suppose, it is constructed as a single loop shown in Fig. 1, where, z - reference; y - output; d - disturbance; $O(s)$, $R(s)$ - the transfer functions of object and controller.

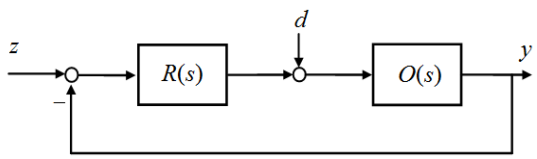

Figure 1. Generic block diagram of a single-loop control system

According to the diagram and formula (5), we have transfer function of the open-loop:

$$H(s)=\frac{W(s)}{1-W(s)}=\frac{1/(1+\theta s)}{1-1/(1+\theta s)}=\frac{1}{\theta s} \tag{6}$$

And the controller:

$$R(s)=H(s)/O(s)=\frac{1}{\theta s}O(s)^{-1} \tag{7}$$

It is there clear that, if the object is known, there is only parameter θ needed to determine for defining regulator.

III. Synthesis of Robust Cascade System

The problem synthesising cascade control system, is implies to ensure high-robust stability and high-performance of the system. It is stated as follows: Assuming that the control object changes uncertainly, the disturbance is arbitrary. Let's determine controllers so that the stability margin of system is the largest and the control error is smallest.

A. Transfer Function of the Open-Loop Cascade System

As defined in [7], the cascade control system has many nested loops, in which each loop contains a cascade subsystem and its own dynamic blocks (Fig. 2).

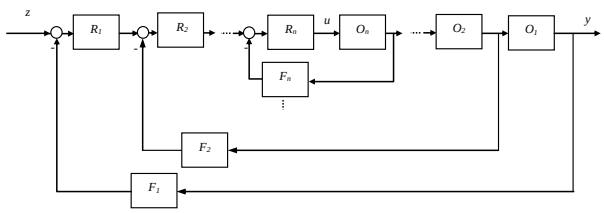

Figure 2. Cascade control scheme

The notes in the Fig. 2 are: z, y, u – respectively reference, output and control signal; F_i, O_i, R_i – own dynamic blocks of i-th loop.

We take equivalent transformation of the diagram in Fig. 2 to single loop as in Fig. 3, where:

W_k – transfer function of subsystem, counting from the output of R_k to input of O_k;

O_{Tk} – equivalent transfer function of partstructure, counting from the output of O_k to input of R_k.

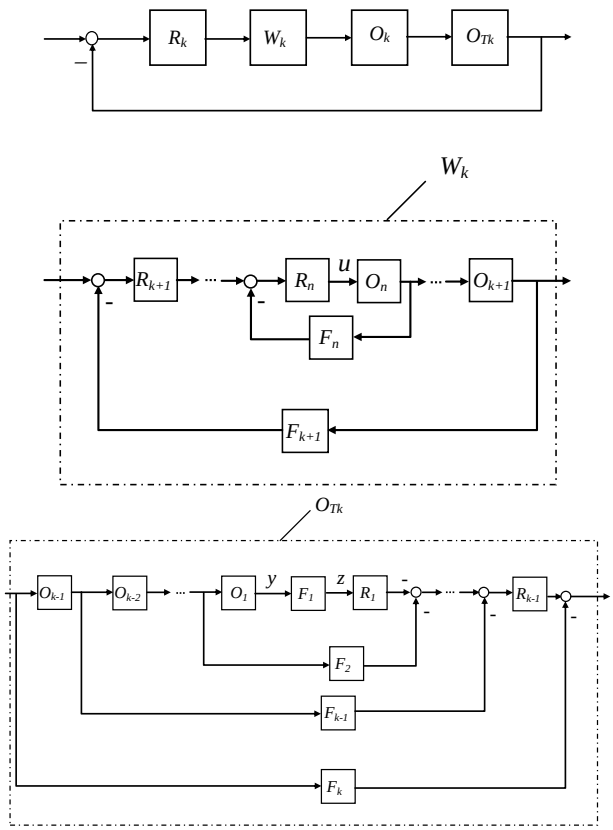

Figure 3. The equivalent single loop related to R_k controller

According to the Masson rule we have:

$$W_k=\frac{\prod_{i=k+1}^{n}R_iO_i}{1+\sum_{j=k+1}^{n}F_j\prod_{i=j}^{n}R_iO_i}\ ,\quad O_{Tk}=\frac{\sum_{j=1}^{k}F_j\prod_{i=j}^{k}R_iO_i}{O_kR_k}$$

From here, the transfer function of k-th open-loop (break out at the input of R_k) is defined as below:

$$H_k=R_kW_kO_kO_{Tk}=\frac{\sum_{j=1}^{k}F_j\prod_{i=j}^{n}R_iO_i}{1+\sum_{j=k+1}^{n}F_j\prod_{i=j}^{n}R_iO_i} \tag{8}$$

B. Robust Cascade Controllers Synthesis

A cascade control system is high-quality one if the high-quality condition is satisfied for each equivalent single loop. Thus, by (4) each equivalent single open-loop satisfies following condition:

$$H_k = \frac{1}{\theta_k s} \tag{9}$$

where, H_k - the open-loop transfer function of the k-th loop; θ_k - the inertial constant of the loop.

Subtituting (9) into (8) for all loops, we have following equations to find controllers (assume all F_k=1 without loss of generality):

$$\begin{cases} \dfrac{\sum_{j=1}^{k}\prod_{i=j}^{n} R_i O_i}{1+\sum_{j=k+1}^{n}\prod_{i=j}^{n} R_i O_i} = \dfrac{1}{\theta_k s}, k=\overline{1,n} \end{cases} \tag{10}$$

$$\Leftrightarrow \begin{cases} \dfrac{\sum_{j=1}^{k}\prod_{i=j}^{n} R_i O_i}{1+\sum_{j=k+1}^{n}\prod_{i=j}^{n} R_i O_i} = H_k, k=\overline{1,n} \end{cases} \tag{11}$$

Let's note: $x_j = \prod_{i=j}^{n} R_i O_i, j \le n$, x_{n+1}=1.

The system of equations (9) will become:

$$\begin{cases} \dfrac{\sum_{j=1}^{k} x_j}{1+\sum_{j=k+1}^{n} x_j} = H_k, k=\overline{1,n} \end{cases} \tag{12}$$

$$\Leftrightarrow$$

$$\begin{cases} H_1^{-1}x_1 & -x_2 & -x_3 & -\ldots & -x_{n-1} & -x_n & =1 \\ H_2^{-1}x_1 & +H_2^{-1}x_2 & -x_3 & -\ldots & -x_{n-1} & -x_n & =1 \\ \ldots \\ H_{n-1}^{-1}x_1 & +H_{n-1}^{-1}x_2 & +H_{n-1}^{-1}x_3 & +\ldots & +H_{n-1}^{-1}x_{n-1} & -x_n & =1 \\ H_n^{-1}x_1 & +H_n^{-1}x_2 & +H_n^{-1}x_3 & +\ldots & +H_n^{-1}x_{n-1} & +H_n^{-1}x_n & =1 \end{cases} \tag{13}$$

Solving the linear equations related x_i ($i=\overline{1,n}$), we recieve controllers:

$$R_k = \frac{x_k}{x_{k+1}} O_k^{-1}; k=\overline{1,n}. \tag{14}$$

From here the controllers may be determined by following steps.

<u>*Step 1*</u>: Determine x_n

Multiply H_n^{-1} and H_{n-1}^{-1} turn in the last two equations of (13), then subtract down and solve:

$$H_n^{-1}H_{n-1}^{-1}x_n + H_n^{-1}x_n = H_{n-1}^{-1} - H_n^{-1}$$

$$\Rightarrow x_n = \frac{H_{n-1}^{-1} - H_n^{-1}}{H_n^{-1}\left(H_{n-1}^{-1}+1\right)} \tag{15}$$

<u>*Step 2*</u>: Determine $\dfrac{x_k}{x_{k+1}}$

Subtract down two consecutive equations of (13) and solve, we have:

$$x_k = \frac{H_{k-1}^{-1} - H_k^{-1}}{H_k^{-1}+1}\sum_{i=1}^{k-1} x_i$$

With k=1:

$$\frac{x_1}{x_2} = \frac{H_2^{-1}+1}{H_1^{-1}-H_2^{-1}} \tag{16}$$

With $k=\overline{2,n}$:

$$\frac{x_k}{x_{k+1}} = \frac{\left(H_{k+1}^{-1}+1\right)\left(H_{k-1}^{-1}-H_k^{-1}\right)}{\left(H_k^{-1}-H_{k+1}^{-1}\right)\left(H_{k-1}^{-1}+1\right)} \tag{17}$$

Replace H_k in the equations (15), (16), (17) by expression in (9), we have the following results:

$$x_n = \frac{\theta_{n-1}-\theta_n}{\theta_n\left(\theta_{n-1}s+1\right)} \tag{18}$$

$$\frac{x_1}{x_2} = \frac{\theta_2 s+1}{\left(\theta_1-\theta_2\right)s} \tag{19}$$

$$\frac{x_k}{x_{k+1}} = \frac{\left(\theta_{k+1}s+1\right)\left(\theta_{k-1}-\theta_k\right)}{\left(\theta_k-\theta_{k+1}\right)\left(\theta_{k-1}s+1\right)} \tag{20}$$

Replace the results (17) (18) (19) into (14), we receive cascade controllers:

$$\begin{aligned} R_1 &= \frac{\theta_2 s+1}{\left(\theta_1-\theta_2\right)s} O_1^{-1} \\ &\ldots \\ R_{n-1} &= \frac{\left(\theta_n s+1\right)\left(\theta_{n-2}-\theta_{n-1}\right)}{\left(\theta_{n-1}-\theta_n\right)\left(\theta_{n-2}s+1\right)} O_{n-1}^{-1} \\ R_n &= \frac{\theta_{n-1}-\theta_n}{\theta_n\left(\theta_{n-1}s+1\right)} O_n^{-1} \end{aligned} \tag{21}$$

From (21), we see that a controller at each loop depends only on the coefficients θ_k of the previous loop, the current loop and the next loop, and just determine the value θ_k we can determine the controller R_i ($i=\overline{1,n}$). If the dead time of the object in each loop is eliminated, then the structure of controllers is more simple and easy to use.

<u>*Special case:*</u>

If delay of any loop is much smaller than that of the next outer loop, the formulas (21) can be simplified as followings:

$$\begin{cases} R_1 = \dfrac{\theta_2 s+1}{\theta_1 s} O_1^{-1} \\ \ldots \\ R_{n-1} = \dfrac{\theta_n s+1}{\theta_{n-1}s} O_{n-1}^{-1} \\ R_n = \dfrac{1}{\theta_n s} O_n^{-1} \end{cases} \tag{22}$$

We see that, to have the controller R_k ($k=\overline{1,n-1}$) just determine θ_k and θ_{k+1}, and to have the controller R_n just determine θ_n.

IV. Retuning the Control Coefficients Based on Soft Characteristic

Based on definition of the soft oscillation index in [8]:

$$m(\omega) = m_0 \frac{1-e^{-\tau\omega}}{\tau\omega}$$

where: m_0 = const; τ – dead time of object; $\omega \geq 0$ – the frequency. Hereafter, the symbol m is similar to $m(\omega)$.

Replace $s = -m\omega + j\omega$, $j = \sqrt{-1}$, into (8) we have the "soft characteristic" of k-th loop: $H_k(-m\omega+j\omega)$ presented in Fig. 4.

Let $H_k(-m\omega+j\omega) = P_k(\omega) + jQ_k(\omega)$. It will cut across the horizontal axis at frequency ω^*, when:

$$H_k(-m\omega^* + j\omega^*) = P_k(\omega^*),\ Q_k(\omega^*) = 0$$

Consider relation with critical point: $(-1,j0)$, we have:

$$KP_k(\omega^*) = -1 \Leftrightarrow K = \frac{-1}{P_k(\omega^*)} \tag{23}$$

It's easy to see that in order to the soft characteristic gos through the point $(-1, j0)$, it is need only to multiply amplitude of the soft characteristic by K.

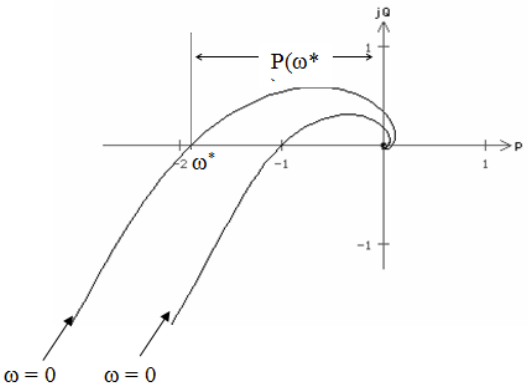

Figure 4. The soft characteristic of the open-loop system

Suppose that the controller of k-th loop is expressed in the form: $R_k = K_k \frac{A(s)}{B(s)}$, where K_k – proportional coefficient. Putting the form of R_k into (8), we receive the soft characteristic of k-th loop:

$$H_k(s) = K_k \frac{A(s)}{B(s)} W_k(s) O_k(s) O_{Tk}(s),\ s = -m\omega + j\omega$$

So, in order to the soft characteristic goes through critical point, it is need only multiply proportional coefficient of controller R_k by correcting coefficient K defined by (23).

V. Example

Let's consider a cascade system with two loops (see Fig. 5). Where, the objects O_1 and O_2 have the following transfer functions:

$$O_1(s) = \frac{e^{-0,32s}}{(1+0,9s)(1+0,38s)(1+0,22s)}$$

$$O_2(s) = \frac{e^{-0,035s}}{1+0,16s}$$

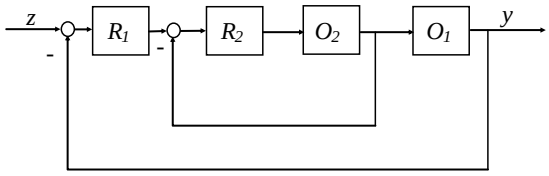

Figure 5. The cascade system with two loops

It is obvious in the example the delay of the inner loop is much smaller than that of the outer loop. Therefore we can use the formulas (22). Eliminating dead time of the objects, we have the robust controllers:

$$R_1 = \frac{\theta_2 s+1}{\theta_1 s}(1+0,9s)(1+0,38s)(1+0,22s)$$

$$R_2 = \frac{1}{\theta_2 s}(1+0,16s)$$

We have:

τ_1 = 0,320 minutes – the dead time of O_1,

τ_2 = 0,035 minutes – the dead time of O_2.

Following [8], we choose oscillation indexes of inner and outer loops as: m_{c1} = 0,461, m_{c2} = 0,922. Then we calculate the lag-time coefficients of the loops [8]:

$\theta_1 = 1,348\tau_1$ = 1,348*0,320 = 0, 43 minutes.

$\theta_2 = 1,943\tau_2$ = 1,943*0,035 = 0,068 minutes.

Consiquently, the controllers are follows:

$$R_1 = \frac{1+0,068s}{0,43s}(1+0,9s)(1+0,38s)(1+0,22s)\times\Phi(s)$$
$$= \frac{2,318(1+0,068s)(1+0,9s)(1+0,38s)(1+0,22s)}{s(1+0,022s)^3}$$

$$R_2 = \frac{1}{0,068s}(1+0,16s) = \frac{14,65(1+0,16s)}{s}$$

where: $\Phi(s) = \frac{1}{(1+0,1T_{\min}s)^3} = \frac{1}{(1+0,022s)^3}$ – a filter chosen so that the R_1 will be proper; $T_{\min}$ – the filter's time constant chosen less or equal the smallest time constant of R_1.

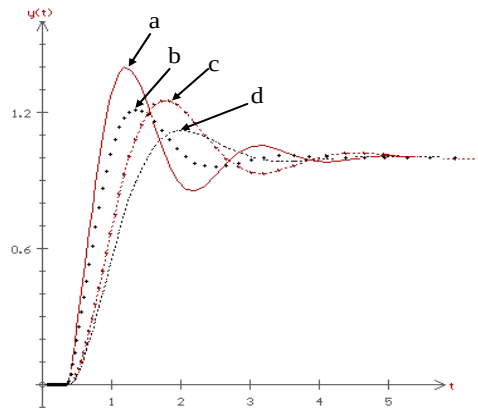

Figure 6. The system step response

With the controllers, the step response of the system is shown in Fig. 6 (process a), and the soft characteristic of open outer loop shown in Fig. 7 (curve a).

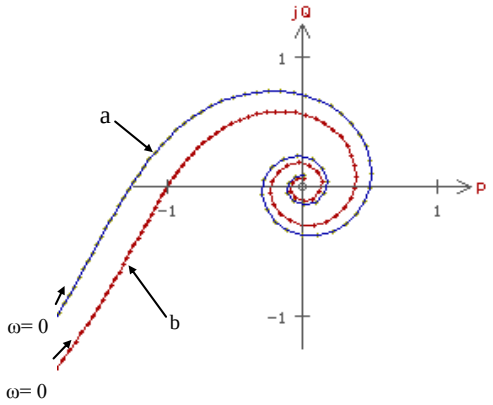

Figure 7. The soft characteristic of the open outer loop.

According to simulation results, the step response has overshoot: $\delta\% = 40\%$, settling time: T_s = 3,24 minutes. The overshoot is still large and settling time is long.

In this case, the soft characteristic of open outer loop encompasses critical point. So we have to retune the proportional coefficient of controller R_1 in order to the soft characteristic is shrinked and crosses the critical point and by this condition the oscillation index (m_c) of the system will be guaranteed. After retuning we have:

$$R_1 = \frac{1{,}818(1+0{,}068s)(1+0{,}9s)(1+0{,}38s)(1+0{,}22s)}{s(1+0{,}022s)^3}$$

The R_2 is not need to change. Simulation results after the retuning are shown in Fig. 6 (process b) and Fig. 7 (curve b).

In last case the overshoot become: δ = 20,5%, and the settling time: T_s = 1,92 minutes.

The transfer function R_1 is rather complicated, but no problem if realize it by programmable microprocessor.

Below, we show other recipe to receive controller R_1 in mode of PID-structure for industry application.

Let V_1 is the equivalent object (Fig. 8).

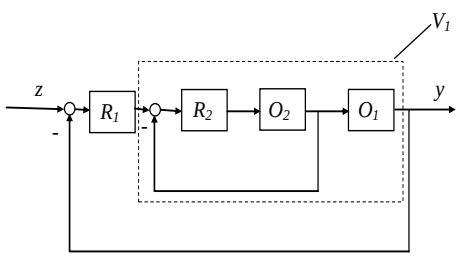

Figure 8. The equivalent single loop structure of the cascade system

We have $V_1 = \frac{R_2 O_2 O_1}{1 + R_2 O_2}$. Approximating V_1 by model of second-order lag transfer function plus dead time (SOPDT) with using cleft-over-step algorithm of optimization [9], we receive the model:

$$MV_1 = \frac{e^{-0{,}529s}}{(1+0{,}667s)(1+0{,}667s)}$$

With model *MV1* after removing its dead time, we have:

$$MV_{1PT} = \frac{1}{(1+0{,}667s)(1+0{,}667s)}.$$

$$R_1 = \frac{1}{\theta_1 s} MV_{1PT}^{-1} = \frac{1}{0{,}43s}(1+0{,}667s)(1+0{,}667s)$$

$$= 3{,}102 + \frac{2{,}326}{s} + 1{,}035s$$

Thus R_1 is the PID controller. Simulation result is shown in Fig. 6 (process c).

Retuning the proportional coefficient of the regulator by above methodology, we receive:

$$R_1 = 1{,}962 + \frac{1{,}437}{s} + 0{,}66s$$

Simulation result is shown in Fig. 6 (process d). The considered example also shows the capability of the cascade system to reject disturbance. Specifically, according to graphics in the Fig. 9, the disturbance response of cascade system almost decreases by 14 times comparing with single loop, from 0,421 to 0,029.

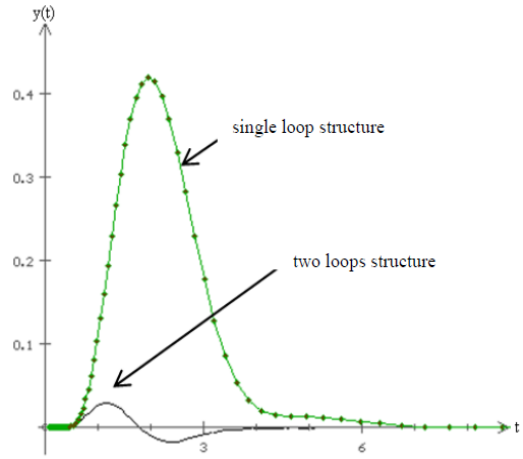

Figure 9. The disturbance response in cases of single loop and of two loops

VI. Conclusions

This article proposes a new method synthesizing controllers of cascade system based directly on the robust conditions applied to the system.

The proposed methodology of implementation is simple, easy to understand, easy to apply and is highly universal. It can be effectively applied to tune industrial control systems.

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