Kalman Consensus Based Multi-Robot SLAM with a Rao-Blackwellized Particle Filter

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***Abstract*—This paper addresses a multi-robot SLAM approach based on the Kalman consensus filter (KCF). Under the unknown initial condition, a reference robot designates the initial poses of other robots when the first rendezvous between them occurs. Accordingly, past and current poses and maps of these robots are estimated by an acausal filter and a causal filter. If initialized robots meet again, their current poses are updated using the KCF. Accordingly, their past poses and maps until the most recent rendezvous are also compensated through the acausal filter. In two simulations, the FastSLAM algorithm, which is a special case of Rao-Blackwellized particle filters, was employed for SLAM. The performance of the proposed approach was verified by comparing conventional approaches.**

***Index Terms*—Kalman consensus filter, Rao-Blackwellized particle filter, Multi-robot SLAM, FastSLAM**

I. Introduction

Multiple robots should know their surroundings and their poses concurrently before performing some missions such as mineral resources exploration and rescuing people. For this, the cooperation of them can be considered for time efficiency and map accuracy, which is called multi-robot SLAM or Cooperative SLAM [1].

Teresa A *et al*. in [2], [3] concentrated the cooperation between aerial and ground robots. They consider some events between robots such as rendezvous, feature correspondences and absolute localization measurements for loop-closing. But they have an assumption that the robots know their pose relative to one another.

Rao-Blackwellized particle filters for a single robot are extended for multiple robots in [4], [5]. A. Howard designs a multi-robot SLAM framework under the unknown initial condition. A reference robot incrementally builds a map and localizes its poses. Other robots just accumulate their control input and observation obtained from equipped sensors. If they meet with the reference robot, they are initialized at that time, and their past and current poses and surrounding maps are estimated in the unified coordinate.

Chen *et al*. in [6] presented a multi-robot FastSLAM algorithm by combining Kalman-Consensus Filter (KCF) to improve the accuracy of localization and mapping. In the feature update part, the KCF is performed. However, they basically assumed known data association for features and the known initial condition.

To perform Multi-robot SLAM, the information filter and the information consensus filter are used together in [7]. They compare the results from the information consensus filter and covariance intersection (CI). But the known conditions are still assumed in the simulation.

In our previous work [8], [9], we proposed a multi-robot SLAM framework. Under the unknown initial condition, robots initialize their poses when the first rendezvous with the reference robot occurs. Subsequently, the poses and maps between the *N*-1th and the *N*th rendezvous are compensated whenever the *N*th rendezvous occurs again. For the compensation, current poses for two robots are fused by Covariance Intersection (CI).

Therefore, this paper presents a Rao-Blackwellized particle filter based multi-robot SLAM using the KCF in the event of rendezvous. Unlike the conventional approach, we consider several rendezvous between robots. The robots are initialized at the first meeting with a reference robot. In the case of the second rendezvous or more rendezvous, the current pose and covariance of two robots are fused via the procedure of the KCF. Based on these poses, their past poses and maps are compensated through backtracking until the most recent rendezvous point. In two simulations, we show the performance of the proposed approach in terms of the accuracy of the robot pose and map. First the conventional approach for the multi-robot SLAM framework and its problems are described in Chapter 2. In Chapter 3, the proposed approach is explained in detail. Chapter 4 shows the accuracy of the robot pose and map through the simulations. This paper is summarized in Chapter 5.

II. Problem Description

Under the unknown initial condition, the coordination of multiple robots should be unified in one frame. To tackle this problem simply, we put a reference robot R_f as described in [4]. R_f incrementally estimates its pose x_t^f and map m_t. Other robots just accumulate their control input u_t and sensor measurement z_t over time. In this paper, a Rao-Blackwellized particle filter are used for single robot SLAM. Although the state of the *i*th particle should be written by $x_t^{(i)}$, *i* is omitted to simplify

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following expressions. Suppose that R_f meets an arbitrary robot R_n in time $t = a$. R_f measures the relative transformation vector Δ_a^n between R_f and R_n. In addition, the pose of R_n is initialized on the frame of R_f as follows:

$$x_a^n = \Delta_a^n \oplus x_a^f \tag{1}$$

where the operator $\oplus$ indicates an appropriate 2D coordinate transform, and x_a^n is the initialized pose of R_n at $t=a$. We assume that the uncertainty of Δ_a^n is negligible.

After the initialization, the past poses $x_{1:a-1}^n$ and maps $m_{1:a-1}^n$ of R_n are estimated using the accumulated control input $u_{1:a}$ and sensor measurement $z_{1:a}$. The current pose x_t^n and map m_t^n of R_n are also estimated using the current control input u_t and sensor measurement z_t, simultaneously. In those estimations, two estimators are exploited, which are an acausal filter and a causal filter. The posterior for R_f and R_n is now represented as follows:

$$\begin{aligned}&p(x_{1:t}^n, x_{1:t}^f, m \mid z_{1:t}^f, u_{0:t-1}^f, x_0^f, z_{1:t}^n, u_{0:t-1}^n, \Delta_a^n) = \\ &p(m \mid x_{1:t}^f, z_{1:t}^f, x_{1:a-1}^n, z_{1:a-1}^f, x_{a+1:t}^n, z_{a+1:t}^f) \cdot p(x_{1:t}^f \mid z_{1:t}^f, u_{0:t-1}^f, x_0^f) \cdot \\ &p(x_{1:a-1}^n \mid z_{1:a-1}^n, u_{0:a-1}^n, x_a^f, \Delta_a^n) \cdot p(x_{a+1:t}^n \mid z_{a+1:t}^n, u_{a:t-1}^n, x_a^f, \Delta_a^n)\end{aligned} \tag{2}$$

where $z_{1:t}^f$, $u_{0:t-1}^f$, $z_{1:t}^n$ and $u_{0:t-1}^n$ are the measurements and the control inputs of R_f, and the measurements and the control inputs of R_n until t, respectively.

If R_n met another robot R_m at $t=b$, R_m is also initialized using the estimated state of R_n at $t=a+(a-b)$ as computed in (1).

In this framework, the second or third rendezvous is not considered because the unknown initial condition problem is already solved at the first meeting. However, at each rendezvous point, the uncertainties of two robots can be dramatically reduced. To do this, in our previous work, we adopted Covariance Intersection (CI) for the reduction of the uncertainties at the Nth rendezvous. In case of R_f, CI is applied as follows:

$$\left(P_{a'}^{newf}\right)^{-1} = w\left(P_{a'}^f\right)^{-1} + (1-w)\left(P_{a'}^n\right)^{-1} \tag{3}$$

$$\left(x_{a'}^{newf}\right) = \left(P_{a'}^{newf}\right)^{-1}\left(w\left(P_{a'}^f\right)^{-1} x_{a'}^f + (1-w)\left(P_{a'}^n\right)^{-1} x_{a'}^n\right) \tag{4}$$

where $P_{a'}^f$ and $P_{a'}^n$ denote the covariance matrices of R_f and R_n at $t = a'$, respectively. $P_{a'}^{newf}$ and $x_{a'}^{newf}$ are the updated covariance and state of R_f. The estimated covariance and state of R_n are also updated by following the same procedure.

But CI does not always guarantee the convergence of the filter. In addition, it has worse performance than the Kalman consensus filter (KCF), which is empirically verified in [6].

III. Proposed Approach

As we mentioned, the errors of the robot poses and maps are consistently accumulated over time. If rendezvous between robots occurs more than two times, the errors can be dramatically reduced. For the error reduction, we adopt the KCF to overcome the problem of CI. In this section, the proposed approach is described by focusing on the error reduction.

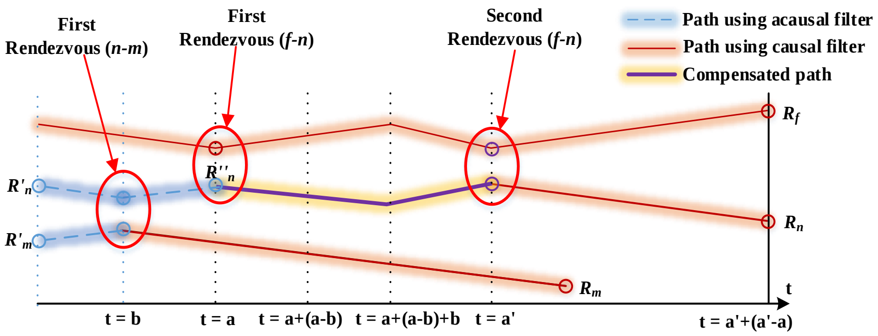

Figure 1. An example for the proposed multi-robot SLAM framework.

If a reference robot R_f meets an arbitrary robot R_n for the first time, the initialization of R_n is conducted using (1). As shown in Fig. 1, if the second rendezvous occurs between them at $t = a'$, two robots exchange their information vector $v_{a'}^i$, information matrix $V_{a'}^i$ and prior/predicted state estimate $\hat{x}_{a|a-1}^i$. In case of R_f, the information vectors and matrices are updated, respectively, as follows:

$$b_{a'}^f = v_{a'}^f + v_{a'}^n, \quad B_{a'}^f = V_{a'}^f + V_{a'}^n \tag{5}$$

Like (5), the fused information vector and matrix for R_n are calculated via the same procedure. Based on the fused data, the state of R_f can be computed as follows:

$$\hat{x}_{a'|a'}^f = \bar{x}_{a'|a'-1}^f + M_{a'}^f(b_{a'}^f - B_{a'}^f \bar{x}_{a'|a'-1}^f) + \gamma (J_{a'}^f)^{-1}\left(\bar{x}_{a'|a'-1}^n - \bar{x}_{a'|a'-1}^f\right) \tag{6}$$

where Kalman gain $M_{a'}^f$ is determined by $(J_{a'}^f + B_{a'}^f)^{-1}$, consensus gain is represented by $\gamma (J_{a'}^f)^{-1}$, and γ is defined by $\varepsilon / \left(1 + \| (J_{a'}^f)^{-1} \|\right)$. In the consensus term, $\bar{x}_{a'|a'-1}^n$ indicates the average state of particles to estimate the pose of R_n. Likewise, R_n is also updated as follows:

$$\hat{x}_{a'|a'}^n = \bar{x}_{a'|a'-1}^n + M_{a'}^n(b_{a'}^n - B_{a'}^n \bar{x}_{a'|a'-1}^n) + \gamma (J_{a'}^n)^{-1}\left(\bar{x}_{a'|a'-1}^f - \bar{x}_{a'|a'-1}^n\right) \tag{7}$$

where Kalman gain $M_{a'}^n$ is determined by $(J_{a'}^n + B_{a'}^n)^{-1}$ and consensus gain is represented by $\gamma (J_{a'}^n)^{-1}$. In the consensus term, $\bar{x}^f_{a'|a'-1}$ denotes the average state of particles to estimate the pose of R_f. In addition, their covariance matrices $P^f_{a'}$ and $P^n_{a'}$ are also updated by $M^f_{a'}$ and $M^n_{a'}$. Based on $\hat{x}^n_{a'|a'}$ and $\hat{x}^f_{a'|a'}$, two acausal filters are generated, which are carried out from the second rendezvous point to the first rendezvous point. It also has map $M(z^f_{1:b+\eta}, x^f_{1:b+\eta})$ and $M(z^n_{1:b+\eta}, x^n_{1:b+\eta})$, which implies that the quality of the map around the first rendezvous point is relatively reliable. A reliability parameter η is defined to set a reliable range.

Fig. 2 shows the proposed multi-robot SLAM framework. Each robot basically estimates its pose and map using a Rao-Blackwellized particle filter. This filter has better performance than the extended Kalman filter due to multi-hypothesis data association and time complexity. In the first rendezvous with the reference robot, the initializations of robots are conducted in the frame of the reference robot. A causal filter and an acausal filter are generated to estimate its past poses and maps as well as its current pose and map. If the second or more rendezvous occurs, the current poses of two robots are promptly updated via the process of the KCF. Subsequently, two acausal filters with early maps around the first rendezvous point are generated for the pose compensation. Finally, these acausal filters are terminated when the pose at the previous rendezvous point is updated. In addition, the acausal filter, which is generated at the first rendezvous, is terminated when the pose and the map estimations at the start point of the robot are finished.

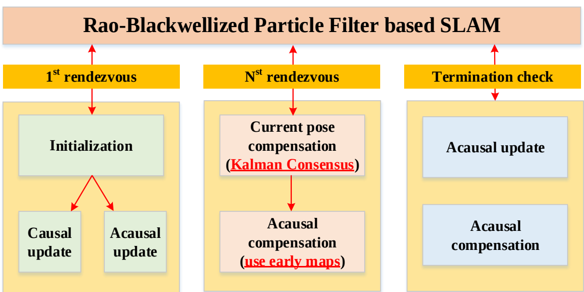

Figure 2. The proposed multi-robot SLAM framework

IV. SIMULATIONS

To evaluate the performance of the proposed approach, we extend and refine the simulator made by Time Bailey [10]. In two different simulations, robots localize their poses and build maps by assuming the unknown initial condition. These robots move at a maximum speed of 3m/s. The period of the update for the control input is 0.5s. The period of the update for observation is 1.6s. In addition, they have non-holonomic constraints (e.g. maximum steering angle: 30 degree and maximum rate of change in steer angle: 20 degree). Their control noise Q and observation noise R are defined as follows:

$$Q = \begin{bmatrix} \sigma_V & 0 \\ 0 & \sigma_G \end{bmatrix}, \quad R = \begin{bmatrix} \sigma_R & 0 \\ 0 & \sigma_B \end{bmatrix} \tag{8}$$

where σ_V, σ_G, σ_R and σ_B are 0.33, 3*rad*, 0.1, 1*rad*, respectively.

For single robot SLAM, the FastSLAM algorithm is used, which is a special case of Rao-Blackwellized particle filters [11]. Ten particles are used to operate the FastSLAM algorithm.

A. Simulation I

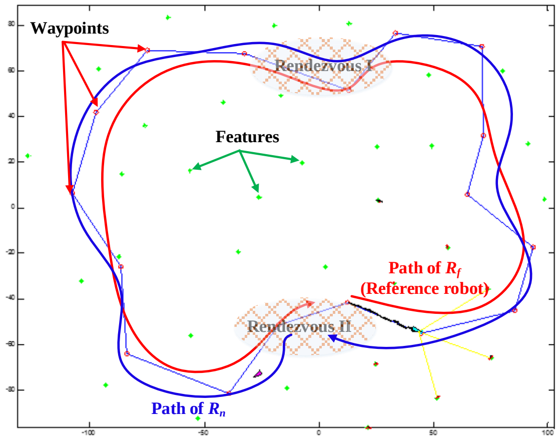

Figure 3. The environment of the simulation I. The path of R_n is represented as the blue line. The path of R_f is represented as the red line. They move in opposition directions. There are 16 way points and 35 features.

In this simulation, there are 16 way points and 35 features as shown in Fig. 3. A reference robot R_f and an arbitrary robot R_n move in opposition directions. During their journey, they meet two times. Until the first rendezvous point, the poses and maps of R_f are only estimated. The control input and the observations of R_n are just accumulated. After the first rendezvous, the past poses and maps of R_n are estimated through an acausal filter and its current pose and map is updated by a causal filter. The acausal filter is operated from the first rendezvous point to the start point of R_n. In this simulation, the time step t at the first rendezvous is about 200. A boundary constant of reliability η is defined as 50, which implies that the map updated from $t = 200$ to $t = 250$ is used for the compensation after the second rendezvous.

TABLE I. COMPARISON OF POSE ERRORS AT RENDEZVOUS

	No fusion	KCF fusion
For R_n	1.7634	0.7936
For R_f	0.4461	0.3871

When the second rendezvous occurs, the current poses of two robots are updated using the KCF. The accuracy for the poses is described in Table I. The errors of both poses are reduced using the KCF. In addition, the covariance of both robots is decreased, which implies that the robot poses can be more correctly estimated. Subsequently, the poses and the map of each robot are also updated based on the compensated current pose. It is conducted between the first rendezvous point and the second rendezvous point.

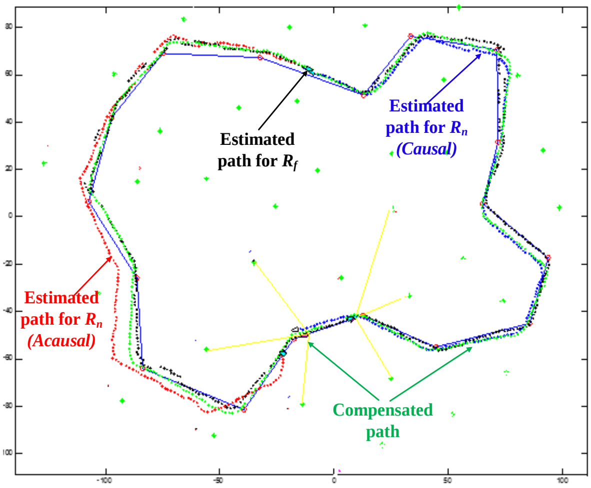

Figure 4. The result of the simulation I. The estimated acausal and causal paths for R_n are represented as red and blue. The estimated path for R_f is represented as black. The compensated paths of R_n and R_f are represented as the green lines.

Fig. 4 shows the final result of the simulation. The estimated path for R_f is represented as black. The estimated path for R_n is divided by a path obtained from the causal filter and a path obtained from the acausal filter. The compensated paths are described as green lines. As shown in the figure, the path of each robot estimated between the first rendezvous and the second rendezvous is sophisticatedly compensated. Total errors for the robot poses and features are computed by

$$\varepsilon_{Pose} = \frac{\sum_t \sqrt{\left(\left(\sum_i x^{t,[i]}\right)\Big/N - x_{true}(t)\right)^T \left(\left(\sum_i x^{t,[i]}\right)\Big/N - x_{true}(t)\right)}}{total\ runtime} \quad (9)$$

$$\varepsilon_{Features} = \frac{\sum_t \sum_i \left(\sum_j^{N_f} \sqrt{\left(m_{j,t}^{[i]} - m_{j,true}(t)\right)^T \left(m_{j,t}^{[i]} - m_{j,true}(t)\right)} \right)}{N_f \cdot N \cdot total\ runtime} \quad (10)$$

where N is the number of particles, $x^{t,[i]}$ is the ith particle at t, $x_{true}(t)$ is the true vehicle pose at t, The jth feature of the ith particle is defined as $m_{j,t}^{[i]}$, and N_f is the number of features in the map.

TABLE II. COMPARISON OF TOTAL ERRORS

	Conventional		Proposed	
	Pose	Feature	Pose	Feature
For R_n	3.8912	2.5185	1.9126	1.8414
For R_f	2.8543	2.7269	1.7323	2.5085

The robot pose and feature errors of the conventional approach and the proposed approach are described in Table II, respectively. The errors are more compensated in the proposed approach. If the constant parameter of the consensus gain ε is defined more properly, the errors can be more reduced.

B. Simulation II

As shown in Fig. 5, two robots have different trajectories along their waypoints shaped like 'M'. They meet two times that are described as Rendezvous I and Rendezvous II in the figure. In addition, they only communicate at two rendezvous points. Likewise with the previous simulation, the initialization of R_n is conducted at the first rendezvous with R_f. It occurs when the time is about 150. A boundary constant of reliability η is defined as 20, which implies that the map updated from $t = 150$ to $t = 170$ is used during the compensation after the second rendezvous. The compensation and the fusion of information occur at the second rendezvous. The errors of the fused pose for both R_f and R_n are described in Table III. In the case of the proposed approach, the errors are remarkably reduced due to the KCF, which affects subsequent compensation.

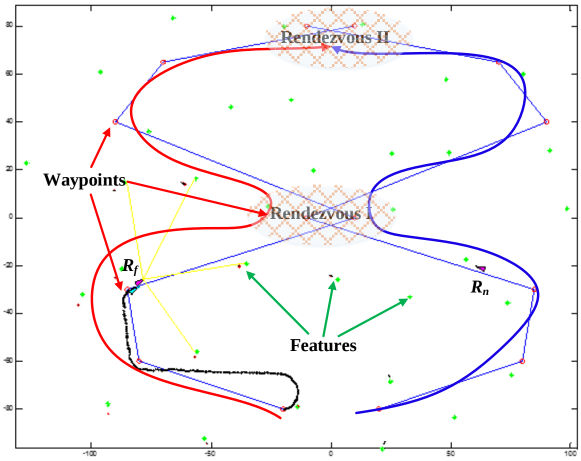

Figure 5. The environment of the simulation II. The path of R_n is represented as the blue line. The path of R_f is represented as the red line. There are 7 way points for each robot.

TABLE III. POSE ERRORS AT THE SECOND RENDEZVOUS

	No filter	KCF fusion
For R_n	5.6575	1.7399
For R_f	2.1133	1.7678

The robot pose and feature errors of the conventional approach and the proposed approach are described in Table IV. Based on the compensation of the current pose for both R_n and R_f, the errors for the robot poses and features are more correctly compensated in the proposed approach.

TABLE IV. COMPARISON OF TOTAL ERRORS

	Conventional		Proposed	
	Pose	Feature	Pose	Feature
For R_n	1.4752	1.1585	1.1926	1.0441
For R_f	1.3544	1.2296	1.3233	1.0885

The result of the simulation II is represented in Fig. 6. The path of R_f (black) is incrementally estimated since it starts. When the first rendezvous between R_f and R_n occurs, the paths of R_n are estimated by the causal filter (blue) and the acausal filter (red). In addition, its path and map are represented in the frame of R_f. After the second rendezvous, the paths and maps of R_f and R_n are compensated by the KCF and backtracking. As shown in the figure, their paths and maps are compensated more accurately.

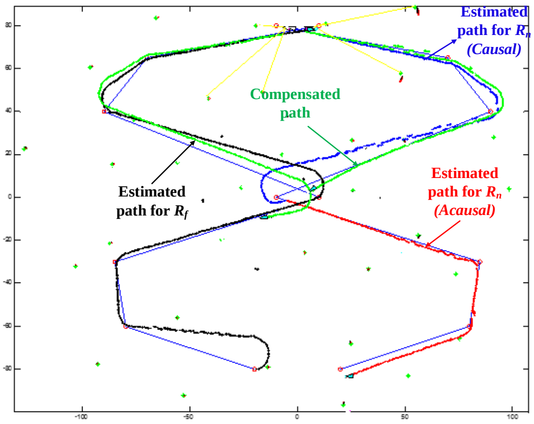

Figure 6. The result of the simulation II. The estimated acausal and causal paths for R_n are represented as red and blue. The estimated path for R_f is represented as black. The compensated paths of R_n and R_f are represented as the green line. The compensation is operated between the first rendezvous and the second rendezvous.

V. Conclusion

This paper addresses a multi-robot SLAM approach using the Kalman Consensus Filter (KCF) in the rendezvous situation. Under the unknown initial condition, a reference robot estimates its own pose and map using a Rao-Blackwellized particle filter before rendezvous with other robots. The reference robot designates the initial poses of other robots when the first rendezvous between them occurs. Accordingly, past and current poses and maps of these robots are estimated by a causal filter and an acausal filter. When initialized robots meet again, their current poses are promptly updated by the Kalman consensus filter. Their past poses and maps between the current rendezvous point and the most recent rendezvous point are also compensated through backtracking. In two simulations, the FastSLAM algorithm, which is a special case of Rao-Blackwellized particle filter, was employed for single robot SLAM. The performance of the proposed approach is verified in terms of pose accuracy and map accuracy.

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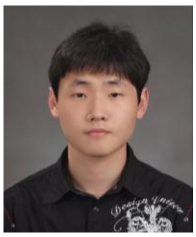

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