Approximation of the Piecewise Function Using Neural Fuzzy Networks with an Improved Artificial Bee Colony Algorithm

Cheng-Hung Chen, Yao-Cheng Tsai, and Rong-Zuo Jhang
Department of Electrical Engineering, National Formosa University, Taiwan
Email: chchen.ee@nfu.edu.tw

***Abstract*—The artificial bee colony (ABC) algorithm is inspired by the behavior of honey bees. It is a relatively new optimization algorithm that has been proved competitive with conventional biology-inspired algorithms. The IABC algorithm is used, with the differential evolution (DE) algorithm added to the new solution search equation of ABC, to improve convergence speed. The IABC adopts the reward-based roulette wheel selection mechanism initially to divide all solutions suitably into feasible and infeasible solutions; thereafter, it divides them based on feasible and infeasible solutions for the implementation of incentives and punishments. Finally, the proposed method is applied to nonlinear system control problems. The experimental results of this study demonstrate the performance of IABC against that of other algorithms in nonlinear problems.**

***Index Terms*—artificial bee colony algorithm, differential evolution, neural fuzzy networks, nonlinear system problems, reward-based roulette wheel selection**

I. Introduction

Neural fuzzy networks (NFNs) are powerful techniques and have been used to solve engineering problems [1]-[3] in recent decades. For the traditional TSK-type NFN [4], [5], the consequent part is a linear combination function of the input variables to complete, and its network output locally approximates the function of the target output. However, the consequent part of the traditional TSK-type NFN cannot be provided a complete mapping capability for highly nonlinear problems. In this paper, a specific NFN based on previous research is adopted [6], and the functional link neural network (FLNN) [7] is used to construct the consequent part. The consequent part of the specific NFN is used in the nonlinear combination of input variables mapped to the function expansion, which can increase the dimension of the input vector, and then simplify the creation of nonlinear decision boundaries and identification of complex nonlinear functions.

Furthermore, methods have been developed to model the intelligent behaviors of a honeybee colony for solving combinatorial type problems [8], [9]. For optimizing numerical functions, Karaboga introduced a bee swarm algorithm called the artificial bee colony (ABC) algorithm that simulates the foraging behavior of bees [10].

This study presents an improved artificial bee colony (IABC) for NFNs. The IABC method is a mixture of the original ABC algorithm and the DE algorithm, and allows the solution the opportunity to explore wider ranges. Moreover, the reward-based roulette wheel selection method was developed to replace the traditional roulette wheel selection method in IABC, which can strengthen the performance of the original ABC algorithm's choice solution.

II. Neural Fuzzy Networks

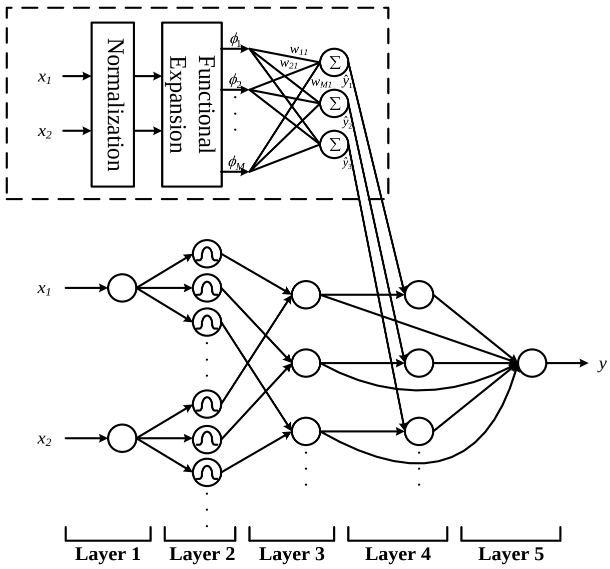

Figure 1. Structure of the specific NFN.

This section describes the specific NFN [6], wherein a complex nonlinear combination of the input variables is adopted as the consequent part of the fuzzy rules. The complex nonlinear combination of the input variables is generated by FLNN. Fig. 1 shows the structure of the specific NFN model which realizes a fuzzy if-then rule in the following form:

Manuscript received October 3 2014; revised December 21, 2014.
This study was sponsored by the Ministry of Science and Technology, Taiwan, R.O.C., under Grant MOST 103-2221-E-150-036.

doi: 10.12720/joace.3.6.503-506

Rule-j:
IF x_1 is A_{1j} and x_2 is A_{2j} …and x_i is A_{ij} …and x_N is A_{Nj}

$$\begin{aligned}\text{THEN } \hat{y}_j &= \sum_{k=1}^{M} w_{kj}\phi_k \\ &= w_{1j}\phi_1 + w_{2j}\phi_2 + \ldots + w_{Mj}\phi_M\end{aligned} \tag{1}$$

where x_i is the input variable; $\hat{y}_j$ is the output variable; Rule-j is the jth rule, A_{ij} is the linguistic term of the precondition part with Gaussian membership function, N is the number of input variables, w_{kj} is the link weight of the local output, ϕ_K is the basis trigonometric function of the input variables, and M is the number of basis functions.

III. Improved Artificial Bee Colony Algorithm for Neural Fuzzy Networks

The IABC algorithm is a hybrid of the original ABC algorithm and the DE algorithm with an additional new incentive-based roulette selection, improving the following two points: 1) the mutation operation of the DE algorithm becomes an updated formula for the employed bees' and onlooker bees' mating operations; and 2) the reward-based roulette wheel selection was developed to replace the employed bee probability value calculation formula. The whole learning process of the IABC algorithm is provided:

A. Initialization Food Source Positions

1) Step 1: Coding step

The coding step in IABC is the coding of the specific NFN into a solution. An example of NFN coded into a solution is shown in Fig. 2, where parameters m_{ij} and σ_{ij} represent a Gaussian membership function with mean and deviation, with the ith input variable and jth rule. Moreover, w_{ij} represents the weight corresponding parameters of the consequent part.

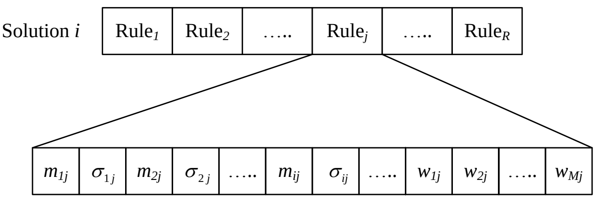

Figure 2. Coding the specific NFN into a solution.

2) Step 2: Initial population

In the initialization step, the initial population of solutions is produced randomly within the range of the boundaries of the parameters. The operation can be represented as follows:

$$x_{i,j} = x_{\min,j} + rand[0,1]\left(x_{\max,j} - x_{\min,j}\right) \tag{2}$$

where i=1, 2,…, SN, j=1, 2,…, D. SN is the population size, D is the number of optimization parameters, $x_{i,j}$ represents the jth parameter of the ith solution, and $x_{\min,j}$ and $x_{\max,j}$ are the lower and upper bounds of parameter j, respectively.

B. Calculate the Nectar Amounts

After the initialization phase, the fitness function is used to evaluate solution performance. In this study, good performance solutions have a lower fitness function. The fitness function is obtained using the following formula:

$$f(x) = \sqrt{\frac{1}{N_t}\sum_{k=1}^{N_t}\left(y_k - \bar{y}_k\right)^2} \tag{3}$$

where y_k represents the model output of the kth data, $\bar{y}_k$ represents the desired output of the kth data, and N_t represents the number of training data.

C. Determine the New Food Source Positions for Employed Bees

In IABC, each employed bee takes advantage of the DE mutation strategy to produce a new food source position (a solution), which allows the employed bee to have the opportunity to explore a wider range. The search equation used to calculate a new food source is as follows:

$$v_{i,j}^t = \begin{cases} x_{r1,j}^t + F \cdot \left(x_{r2,j}^t + x_{r3,j}^t\right) & \text{if } (rand_j[0,1) \le CR), \\ x_{i,j}^t & \text{otherwise.} \end{cases} \tag{4}$$

where F is a scaling factor; $r1$, $r2$, and $r3$ are randomly-selected solutions in which $r1 \neq r2 \neq r3 \neq i$; t is the generation number; and $rand_j[0,1)$ is the jth optimization parameters of a uniform random number generator. After each new food source position $v_{i,j}^t$ is produced, it is evaluated and greedy selection is used for comparison with the old food source $x_{i,j}^t$.

D. Calculate the Probability Value

In this phase, the solutions are divided into feasible and infeasible solutions. A solution within the range of the search space is referred to as a feasible solution; conversely, a solution that deviates from the range of the search space is referred to as an infeasible solution.

After all employed bees complete the feasible-infeasible solution separation process, reward-based roulette wheel selection is used to calculate the probability value. The reward-based roulette wheel selection incentivizes the behavior of bees that obtained feasible solutions, and punishes the behaviors of those that obtained infeasible solutions. The probability value is calculated using the following form:

$$p_i = \begin{cases} \dfrac{0.5 + \left(\dfrac{f_i}{\sum_{n=1}^{SN_1} f_n}\right) * 0.5}{SN_1 + 1}, & \text{if solution is feasible} \\[2ex] \dfrac{0.5 - \left(\dfrac{\frac{1}{v_i}}{\sum_{n=1}^{SN_2} \frac{1}{v_n}}\right) * 0.5}{SN_2 - 1}, & \text{if solution is infeasible} \end{cases} \tag{5}$$

where SN_1 represents the number of the feasible solution, SN_2 represents the number of the infeasible solution, f_i is the fitness value of the *i*th feasible solution x_i, and v_i is the fitness value of the *i*th infeasible solution x_i. A review of Eq.(5) shows that the feasible solution will be relatively large because of the way in which incentives affect the probability value, or because punishment diminishes the probability value of infeasible solutions.

By calculating probability values using roulette wheel selection, the food source position for the onlooker bees is chosen. In roulette wheel selection, a larger probability value has a higher opportunity for selection, and a smaller probability value has a lower opportunity for selection; thus, feasible solutions are chosen more often than infeasible solutions are.

E. Determine the New Food Source Positions for Onlooker Bees

In this phase, after all the onlooker bees disperse to the food source positions, they use the same DE mutation strategy (4) to produce new food source positions and evaluate them. This process will also provide onlooker bees with the opportunity to explore a wider range. After locating every new food source position, greedy selection is used for comparison with the older source.

F. Produce New Positions for the Exhausted Food Sources

In IABC, a predetermined number of cycles must initially be set, which is called a "*limit*". Moreover, if a food source position cannot be further improved and exceeds its predetermined "*limit*", the food source is considered an abandoned solution. Thereafter, the scout bee discovers a new food source position to replace. This operation is defined in (3).

G. Memorize the Position of the Best Food Source

In the final phase, if the fitness value of any bee food source is better than the fitness value of the best food source position thus far, the new food source position replaces the best food source position.

IV. Experimental Results-Approximation of the Piecewise Function

This example is the approximation of a signal variable piecewise function, which has been studied frequently in the literature. This function is defined as follows:

$$f(x)=\begin{cases} -2.186x-12.864, & -10\le x<-2, \\ 4.246x, & -2\le x<0, \\ 10e^{-0.05x-0.5}\sin[(0.03x+0.7)x], & 0\le x\le 10 \end{cases} \quad (6)$$

The piecewise function is continuous and analyzable. However, it is inefficient and often fails when using traditional analytical tools. This failure may result because wide-band information is not only hidden at the turning point, but is also the coexistence of linearity and nonlinearity.

In this example, the sample data is distributed uniformly over [-10, 10], and 200 points are selected as training data. Ten fuzzy rules are applied to the NFN in this example. The maximal number of fitness function evaluations (Max_NFFEs) is set as 5000 for each method and was repeated 30 times with different random seeds. The final mean root-mean-square error (RMSE) of the IABC method is approximately 0.1824. The learning curves of the average performance of all algorithms are shown in Fig. 3.

A comparison of the results of the best RMSE and mean RMSE with standard deviation (Std Dev) between IABC and the other methods is shown in Table I. Table I shows that the performance of IABC with respect to RMSE is evidently better than those achieved using other methods.

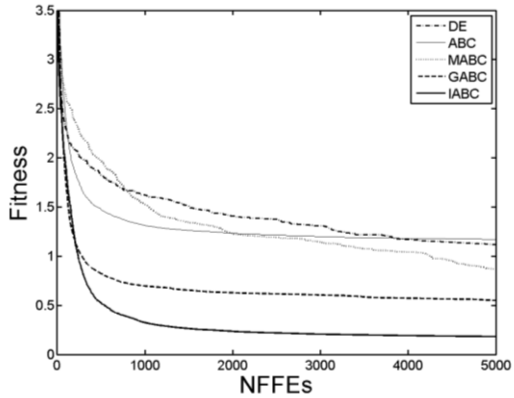

Figure 3. Learning curves of average performance of the DE, ABC, MABC, GABC, and IABC.

TABLE I. Comparison of Results Obtained by the DE, ABC, MABC, GABC and IABC Algorithms.

Method	DE	ABC	MABC	GABC	IABC
Best RMSE	0.6361	0.6189	0.3633	0.2379	0.0661
Mean RMSE	1.117	1.1708	0.8671	0.5496	0.1824
Std Dev	0.0585	0.0861	0.1072	0.1002	0.0304

V. Conclusion

This study proposed an IABC algorithm for the specific NFN to solve nonlinear control problems. The mutation strategy of DE was adopted in the proposed IABC algorithm to produce a new food source for the employed and onlooker bees, and the reward-based roulette wheel selection was used to make the better feasible solutions has a higher opportunity to be improved. The simulation results show that IABC can solve the problem in the experiment efficiently, and achieves a better performance than other existing algorithms.

References

[1] C. F. Juang and C. T. Lin, "An online self-constructing neural fuzzy inference network and its applications," *IEEE Trans. Fuzzy Systems*, vol. 6, no. 1, pp. 12-32, Feb. 1998.

[2] P. P. Angelov and D. P. Filev, "An approach to online identification of Takagi-Suigeno fuzzy models," *IEEE Trans. Syst., Man, Cybern. B*, vol. 34, no. 1, pp. 484-498, Feb. 2004.

[3] C. F. Hsu, "Self-orgranizing adaptive fuzzy neural control for a class of nonlinear systems," *IEEE Trans. Neural Networks*, vol. 18, no. 4, pp. 1232-1241, July 2007.
[4] H. Takagi and M. Sugeno, "Derivation of fuzzy control rules from human operator's control actions," in *Proc. IFAC Symp. Fuzzy Inform., Knowledge Representation and Decision Analysis*, 1983, pp. 55-60.
[5] J. S. R. Jang, "ANFIS: Adaptive-network-based fuzzy inference system," *IEEE Trans. Syst., Man, and Cybern.*, vol. 23, pp. 665-685, 1993.
[6] C. H. Chen, C. J. Lin, and C. T. Lin, "A functional-link-based neuro-fuzzy network for nonlinear system control," *IEEE Trans. Fuzzy Systems*, vol. 16, no. 5, pp. 1362-1378, Oct. 2008.
[7] J. C. Patra, R. N. Pal, B. N. Chatterji, and G. Panda, "Identification of nonlinear dynamic systems using functional link artificial neural networks," *IEEE Trans. Syst., Man, Cybern. B*, vol. 29, no. 2, pp. 254-262, Apr. 1999.
[8] V. Tereshko and A. Loengarov, "Collective decision-making in honeybee foraging dynamics," *Computing and Information Systems*, vol. 9, no. 3, pp. 1-7, Oct. 2005.
[9] H. Drias, S. Sadeg, and S. Yahi, "Cooperative bees swarm for solving the maximum weighted satisfiability problem, computational intelligence and bioinspired systems," in *Proc. 8th International Workshop on Artificial Neural Networks*, IWANN 2005, Vilanova, Barcelona, Spain, June 8-10 2005.
[10] D. Karaboga, "An idea based on honeybee swarm for numerical optimization," *Tech. Rep. TR06*, Erciyes University, Engineering Faculty, Computer Engineering Department, 2005.

Cheng-Hung Chen received the B.S. and M.S. degrees in computer science and information engineering from the Chaoyang University of Technology, Taiwan, R.O.C., in 2002 and 2004, respectively, and the Ph.D. degree in electrical and control engineering from the National Chiao-Tung University, Taiwan, R.O.C., in 2008. Currently, he is an Associate Professor of Electrical Engineering Department, National Formosa University, Yunlin County, Taiwan, R.O.C. His current research interests are fuzzy systems, neural networks, evolutionary algorithms, intelligent control, pattern recognition, and image processing. He has authored or coauthored more than 50 papers published in the referred journals and conference proceedings. He is also a member of the Chinese Fuzzy Systems Association (CFSA), the Taiwanese Association for Artificial Intelligence (TAAI), and the IEEE Computational Intelligence Society.

