Iterated Modified Gain Extended Kalman Filter with Applications to Bearings Only Tracking

Yuan Huang and Taek Lyul Song
Department of Electronic Systems Engineering, Hanyang University, Republic of Korea
Email: hy4335657@hotmail.com, tsong@hanyang.ac.kr

***Abstract*—A nonlinear filter called the iterated modified gain extended Kalman filter (IMGEKF) is presented in this paper. This filter uses bearings only measurements to estimate the target state in passive target tracking scenario. This work combines the MGEKF and the iteration method. The filter utilizes the updated state to re-linearize the measurement equation. Then the proposed work is tested in a two dimensional scenario. The simulation study compares the IMGEKF and some other filters to show the improvement.**

***Index Terms*—surveillance, target tracking, nonlinear estimation, bearings only, iteration method**

I. Introduction

Target tracking problem arises in a variety of practical applications, such as antimissile, aircraft surveillance and GPS. The single and multiple target tracking algorithms are proposed to solve the tracking problem. The target tracking problem considers both linear and non-linear measurements.

The bearings only target tracking is broadly used in many passive tracking applications. Typical examples are submarine tracking using a passive sonar or satellite to satellite passive tracking using a radar in passive mode [1], [2]. The bearings only target tracking is an inherent non-linear state estimation problem.

The basic problem in bearings only target tracking is to estimate the target state (usually position and velocity) from noise corrupted angle data. For a single sensor tracking scenario, the angle data are obtained from a single moving observer. The problem of observability of the target parameter in passive localization is demonstrated in [3]. To make the target observable, careful designed maneuver must be applied to the observer. Some good principles for generating the observer maneuver are given in [4] and [5]. So far, most articles in the bearings only tracking field assumed the target moved with a constant velocity [6], [7], [8]. As for tracking a maneuvering target, very limited research has been published in the open literature.

Due to the inherent nonlinearity and observability problem, it's not easy to construct an optimal Bayesian filter. But a lot of nonlinear filters have been put forward to solve the nonlinearity problem involved in bearings only measurements. The most widely used in practice is the Extended Kalman filter (EKF). This filter linearizes nonlinearities through the Taylor series expansion at the predicted target state. The EKF requires evaluation of the Jacobians of the nonlinear dynamics and measurement equations. The iterated extended Kalman filter (IEKF) computes the updated state not as an approximate conditional mean but as a maximum a posteriori (MAP) estimate [3]. The unscented Kalman filter (UKF) samples and propagates the probability density function using a small number of deterministically chosen samples [9]. The particle Filter (PF) samples non-linear probability density function by a set of random (Monte Carlo) samples [10].

The MGEKF is designed for target motion analysis (TMA) with nonlinear bearings only measurements [11]. It requires that the measurement function is 'modifiable' which implies a universal linearization can be achieved. The concept of "modifiable" also applies to nonlinear dynamic systems in [12]. The "modifiable" function plays an important role in developing the structure of the filter and this will be presented in section III.

Based on the MGEKF framework, the iterated modified gain extended Kalman filter (IMGEKF) is developed in this paper. This filter uses the updated state to re-linearize the measurement function to get the new updated state and the corresponding covariance. The modified function is used to obtain an approximation in linearizing the measurement function. The IMGEKF and the IEKF are very similar to each other as both of them focus on the linearization and use the iteration procedure. But the IEKF uses the Taylor expansion and the IMGEKF uses the modified function to carry out the universal linearization. So, performance comparison is made among the EKF, the IEKF, the MGEKF and the IMGEKF in the simulation study part.

This paper first defines the single sensor bearings only target tracking model in section II. In section III, the modified function and the structure of the IMGEKF are demonstrated in detail. In section IV, the IMGEKF is applied to a two dimensional bearings only measurement problem to show the improvement.

In this paper, we evaluate the state estimation capability of the IMGEKF used in single sensor bearings only tracking scenario. The target exists and is detected at every scan. To summarize, the contributions of this paper

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are: (1) it presents a solution of single sensor bearings only tracking, using the IMGEKF developed in this paper, (2) this solution is compared with some other methods, and with the Cramer-Rao lower bound.

II. Mathematical Model

A two dimensional bearings only measurement case is defined in the Cartesian coordinates. A single moving observer measures the direction of target emissions at known times indexed by *k*. A sample relative measurement geometry relationship is illustrated in Fig. 1. θ is the measurement and c is the target course at current time. In order to make the target observable, we need to make our observer motion model (at least in some interval) at least one derivative higher than the target motion [4]. The target moves at a constant velocity, so the observer has to have at least one maneuver to make the target observable.

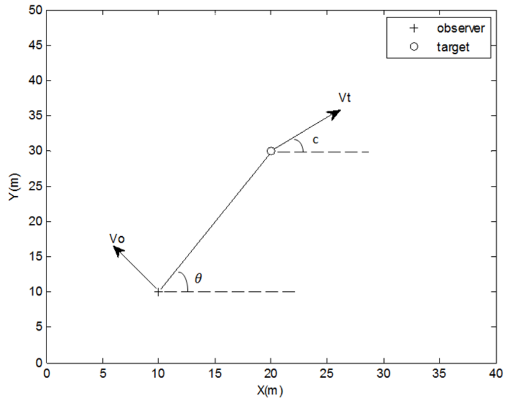

Figure 1. Relative measurement geometry

The target motion is modelled as

$$x_k = f_{k-1}\left(x_{k-1}\right) + v_{k-1} \tag{1}$$

where x_{k-1} is target state vector at time $k-1$, v_{k-1} is a zero mean, white Gaussian process noise with the known variance

$$E[v_k] = 0 \tag{2}$$

$$E[v_k v_j^T] = Q_k \delta\left(k, j\right) \tag{3}$$

where $\delta\left(k, j\right)$ is the Kronecker delta function and Q_k is the process noise covariance matrix.

The measurement model is

$$z_k = h_k\left(x_k\right) + w_k \tag{4}$$

where $h(x) = \tan^{-1}(y/x)$ and w_k is a zero mean, white Gaussian measurement noise with the known variance

$$E[w_k] = 0 \tag{5}$$

$$E[w_k w_j^T] = R_k \delta\left(k, j\right) \tag{6}$$

where R_k is the sensor additive noise covariance matrix.

It is assumed that the measurement noise and the process noise sequence are uncorrelated.

III. The Interated Modified Gain Extended Kalman Filter (IMGEKF)

In this section, the IMGEKF is developed on the basis of the MGEKF and the iteration method. As stated in [12], the gain algorithm of the MGEKF is altered from

that of the modified gain extended Kalman observer (MGEKO) in order to reduce the biases due to direct correlations between the gain and residual in stochastic environments. This thought is kept in developing the IMGEKF structure.

The stochastic case given in section II where the system dynamic is linear and measurement is non-linear.

$$x_k = F x_{k-1} + v_{k-1} \tag{7}$$

$$z_k = h\left(x_k\right) + w_k \tag{8}$$

Assume that the process noise and measurement noise have a constant covariance *Q* and *R*, respectively.

Definition 1: A time-varying function $h: R^m \to R^n$ is modifiable if there exists an $n \times m$ time-varying matrix of functions $g: R^n \times R^m \to R^{n\times m}$ so that for any x, $\underline{x} \in R^m$ and $k \in Z_+$,

$$h\left(x\right) - h\left(\underline{x}\right) = g\left(z^*, \underline{x}\right)\left(x - \underline{x}\right) \tag{9}$$

where $z^* = h\left(x\right)$.

The difference $h\left(x\right) - h\left(\underline{x}\right)$ is equal to $g\left(z^*, \underline{x}\right) \cdot \left(x - \underline{x}\right)$ without any approximations. $h\left(x\right)$ means the angle is generated from the true target state x. However, the true target state is not available in a practical estimation problem due to the process and measurement noise. So, the measurement z will be used instead of z^* in linearizing the measurement function.

The main idea of the IMGEKF is to re-linearize the predict measurement $\hat{z}_{k|k-1}$ around the updated state. This thought consents that the updated state contains more effective information of the target. During the unobservable period, the iteration depends on the updated state which may have a big error compared with the true target state. This condition leads the tracking to a wrong direction and finally makes the filter divergent. So, a good suggestion is that using the iteration method after the target become observable.

The IMGEKF structure is derived here. At time *k* the updated state $\hat{x}_{k-1|k-1}$ and the corresponding covariance $P_{k-1|k-1}$ are given.

In the iteration, the superscript "*i*" is the iterative number where $i = 1,2,3,...,n\left(n \in Z_+\right)$. "*n*" is decided either a priori or based on a convergence criterion. When i=1, $\hat{x}_{k|k}^{i-1}$ is the predicted state $\hat{x}_{k|k-1}$.

Then the state update step is

$$\hat{x}_{k|k-1} = F\hat{x}_{k-1|k-1} \tag{10}$$

$$\hat{x}_{k|k}^{i} = \hat{x}_{k|k-1} + K^{i}\left(z_k - \hat{z}_{k|k-1}^{i}\right) \tag{11}$$

where $\hat{x}_{k|k}^{i}$ and $\hat{z}_{k|k-1}^{i}$ are the updated state and measurement prediction in the ith iteration.

$$\begin{aligned}\hat{z}_{k|k-1}^{i} &= E\left[z_k|Z^{k-1}\right] = E\left[h\left(x_k\right)+w_k|Z^{k-1}\right]\\ &= E\left[h\left(\hat{x}_{k|k}^{i-1}\right)+g\left(z_k^{*},\hat{x}_{k|k}^{i-1}\right)\left(x_k-\hat{x}_{k|k}^{i-1}\right)|Z^{k-1}\right]\\ &\approx h(\hat{x}_{k|k}^{i-1})+g(z_k,\hat{x}_{k|k}^{i-1})(\hat{x}_{k|k-1}-\hat{x}_{k|k}^{i-1})\end{aligned} \tag{12}$$

To get the final form of $\hat{z}_{k|k-1}^{i}$ in (12), the assumption that w_k is zero mean and the modifiable function (13) are used.

$$h\left(x_k\right)-h\left(\hat{x}_{k|k}^{i-1}\right) = g\left(z_k^{*},\hat{x}_{k|k}^{i-1}\right)\left(x_k-\hat{x}_{k|k}^{i-1}\right) \tag{13}$$

$g\left(z_k,\hat{x}_{k|k}^{i-1}\right)$ is calculated as

$$g\left(z_k,\hat{x}_{k|k}^{i-1}\right) = \varepsilon^{i}\left[\frac{\sin z_k, -\cos z_k, 0, 0}{\hat{y}^{i-1}\cos z_k - \hat{x}^{i-1}\sin z_k}\right] \tag{14}$$

where

$$\varepsilon^{i} = z_k - h\left(\hat{x}_{k|k}^{i-1}\right) \tag{15}$$

$$\hat{y}^{i-1} = \hat{y}_t^{\,i-1} - \hat{y}_o \tag{16}$$

$$\hat{x}^{i-1} = \hat{x}_t^{\,i-1} - \hat{x}_o \tag{17}$$

$\hat{y}_t^{\,i-1}$ and $\hat{x}_t^{\,i-1}$ are the elements of target state $\hat{x}_{k|k}^{i-1}$ in y and x direction, respectively. $\hat{y}_o$ is the element of observer state in y direction and $\hat{x}_o$ is the element of observer state in x direction.

In the iteration, the measurement z_k is always used to approximate the z_k^{*} in the modified function.

The iterated state update is

$$\begin{aligned}\hat{x}_{k|k}^{i} &= \hat{x}_{k|k-1} + K^{i}((z_k - h\left(\hat{x}_{k|k}^{i-1}\right)\\ &-g\left(z_k,\hat{x}_{k|k}^{i-1}\right)\left(\hat{x}_{k|k-1}-\hat{x}_{k|k}^{i-1}\right))\end{aligned} \tag{18}$$

The K^{i} is the gain sequence calculated as

$$P_{k|k-1} = FP_{k-1|k-1}F^{T} + Q \tag{19}$$

$$S^{i} = H\left(\hat{x}_{k|k}^{i-1}\right)P_{k|k-1}H\left(\hat{x}_{k|k}^{i-1}\right)^{T} + R \tag{20}$$

$$K^{i} = P_{k|k-1}H\left(\hat{x}_{k|k}^{i-1}\right)^{T}\left(S^{i}\right)^{-1} \tag{21}$$

where

$$H\left(\hat{x}_{k|k}^{i-1}\right) = \frac{\partial h\left(x\right)}{\partial x}\Big|_{x=\hat{x}_{k|k}^{i-1}} \tag{22}$$

Is the Jacobian calculated at the iterated state $\hat{x}_{k|k}^{i-1}$.

To calculate the covariance of the updated state, the measurement noise is taken into consideration to make it more accurate.

The measurement residual error is calculated as

$$\begin{aligned}z_k - \hat{z}_{k|k-1}^{i} &= h\left(x_k\right) + w_k - h\left(\hat{x}_{k|k}^{i-1}\right)\\ &-g\left(z_k,\hat{x}_{k|k}^{i-1}\right)\left(\hat{x}_{k|k-1}-\hat{x}_{k|k}^{i-1}\right)\end{aligned} \tag{23}$$

The modified function is used here to make an approximation

$$h\left(x_k\right)-h\left(\hat{x}_{k|k}^{i-1}\right) = g\left(z_k,\hat{x}_{k|k}^{i-1}\right)\left(x_k-\hat{x}_{k|k}^{i-1}\right) \tag{24}$$

after that, the iterative form of the measurement residual is

$$z_k - \hat{z}_{k|k-1}^{i} = g\left(z_k,\hat{x}_{k|k}^{i-1}\right)\left(x_k-\hat{x}_{k|k-1}\right)+w_k \tag{25}$$

The updated state can be expressed as

$$\hat{x}_{k|k}^{i} = \hat{x}_{k|k-1} + K^{i}g\left(z_k,\hat{x}_{k|k}^{i-1}\right)\left(x_k-\hat{x}_{k|k-1}\right)+K^{i}w_k \tag{26}$$

The updated state estimate error is defined as

$$\begin{aligned}\hat{e}_{k|k}^{i} &\triangleq x_k - \hat{x}_{k|k}^{i}\\ &=\left(x_k-\hat{x}_{k|k-1}\right)-K^{i}g\left(z_k,\hat{x}_{k|k}^{i-1}\right)\left(x_k-\hat{x}_{k|k-1}\right)-K^{i}*w_k\end{aligned} \tag{27}$$

The covariance is given by

$$\begin{aligned}P_{k|k}^{i} &= E[\hat{e}_{k|k}^{i}\hat{e}_{k|k}^{i\,T} \mid Z^{k}]\\ &=\left(I-K^{i}g\left(z_k,\hat{x}_{k|k}^{i-1}\right)\right)P_{k|k-1}\left(I-K^{i}g\left(z_k,\hat{x}_{k|k}^{i-1}\right)\right)^{T}+K^{i}RK^{iT}\end{aligned} \tag{28}$$

where

$$Z^{k} \triangleq \left\{z_n, n=1,\ldots,k\right\} \tag{29}$$

Denotes the measurement sequence up to time k.

Equation (18) is used to obtain the updated state and (26) is just used to calculate the updated covariance. TABLE I demonstrates the difference between the MGEKF and the IMGEKF.

TABLE I. STRUCTURE COMPARISON OF MGEKF AND IMGEKF

Structure Comparison	Approach	
	MGEKF	IMGEKF
Structure	State update: $\hat{x}_{k\|k-1} = F\hat{x}_{k-1\|k-1}$ $\hat{x}_{k\|k} = \hat{x}_{k\|k-1} + K\left(z_k - h(\hat{x}_{k\|k-1})\right)$ Gain sequence: $S = H\left(\hat{x}_{k\|k-1}\right)P_{k\|k-1}H\left(\hat{x}_{k\|k-1}\right)^T + R$ $K = P_{k\|k-1}H\left(\hat{x}_{k\|k-1}\right)^T S^{-1}$ Covariance update: $P_{k\|k-1} = FP_{k-1\|k-1}F^T + Q$ $P_{k\|k} = \left(I - Kg\left(z_k, \hat{x}_{k\|k-1}\right)\right)P_{k\|k-1} \cdot \left(I - Kg\left(z_k, \hat{x}_{k\|k-1}\right)\right)^T + KRK^T$	State update: $\hat{x}_{k\|k-1} = F\hat{x}_{k-1\|k-1}$ $\hat{x}^i_{k\|k} = \hat{x}_{k\|k-1} + K^i((z_k - h\left(\hat{x}^{i-1}_{k\|k}\right) - g\left(z_k, \hat{x}^{i-1}_{k\|k}\right)\left(\hat{x}_{k\|k-1} - \hat{x}^{i-1}_{k\|k}\right))$ Gain sequence: $S^i = H\left(\hat{x}^{i-1}_{k\|k}\right)P_{k\|k-1}H\left(\hat{x}^{i-1}_{k\|k}\right)^T + R$ $K^i = P_{k\|k-1}H\left(\hat{x}^{i-1}_{k\|k}\right)^T (S^i)^{-1}$ Covariance update: $P_{k\|k-1} = FP_{k-1\|k-1}F^T + Q$ $P^i_{k\|k} = \left(I - K^i g\left(z_k, \hat{x}^{i-1}_{k\|k}\right)\right)P_{k\|k-1} \cdot \left(I - K^i g\left(z_k, \hat{x}^{i-1}_{k\|k}\right)\right)^T + K^i R K^{iT}$

IV. SIMULATION STUDY

The scenario is illustrated in Fig. 2. A single target follows a constant velocity. A single observer tracks the target and performs a maneuver to make the target observable in the tracking period. The target speed and initial target to sensor distance are 10m/s and 10km, respectively. The observer scan time is 5s and the total surveillance time is 1200s. The measurements are the angles corrupted with a white zero mean Gaussian noise with standard deviation $\sigma_\theta = 1^\circ$. The initialization method which can be found in [13] is used for all filters.

The initial observer state is given by

$$x_{ob} = 0, y_{ob} = 0, v_{xob} = 5, v_{yob} = 0 \tag{30}$$

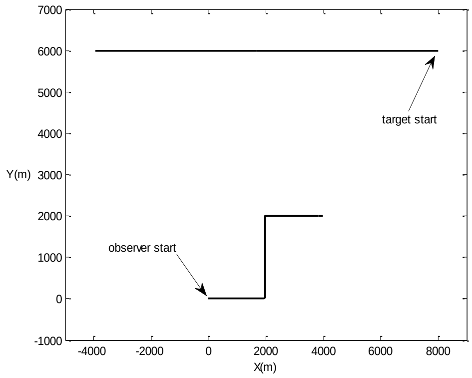

Figure 2. Note how the caption is centered in the column.

From scan 81 to 160 the velocity of observer is changed to $v_{xob} = 0, v_{yob} = 5$. After 160 scan, the velocity is changed to $v_{xob} = 5, v_{yob} = 0$.

The process noise covariance is given by

$$Q = q\begin{bmatrix} 0.25T^4 I_2 & 0.5T^3 I_2 \\ 0.5T^3 I_2 & T^2 I_2 \end{bmatrix} \tag{31}$$

For the stochastic environment, the results of 500 runs of Monte Carlo simulations are presented in Fig. 3 and Fig. 4. Each run contains 240 scans. The scan time is 5s during the surveillance time interval 0s to 1200s.

The measurement in the first scan is used for initialization in each run, coupled with the prior range pdf which is assumed Gaussian, with a mean of 15620m, and standard deviation of 6000m. The prior target speed is assumed Gaussian, with a mean of 11m/s and standard deviation of 1.2m/s. The prior target course is also assumed Gaussian, with a mean of π and standard deviation of $\pi / \sqrt{12}$. The q is equal to $0.00001 m^2 / s^3$ in this scenario which means that the process noises are the same in x and y direction.

In this simulation, at each scan we just do 2 iteration which means i=1,2 in (10)-(28). If iteration method is used in all the scans, the tracking results show us the divergent problem. This problem partly due to the target is not observable in the front scans. So we will take use of the iteration method after some scans that the observer performed maneuver which can reduce the influence of target non-observable. In this scenario, the observer performs maneuver form 81 to 160 scan. So, first the MGEKF is used in the front 100 scans. After 100 scan, the IMGEKF is used. The same process is applied to the EKF and the IEKF.

The IMGEKF is compared with the EKF, the IEKF and the MGEKF. The Cramer-Rao lower bound (CRLB) is presented here as a lower limit. Root mean square (RMS) errors over time and the CRLB are demonstrated in Fig. 3. In Fig. 4, the RMS errors in last 90 scans and the CRLB are presented. In front 80 scans the error progressively reduced as the observer get the information of the target. At scan 80, the observer turned a ninety degree bend which made the target observable. Then, the IMGEKF and the IEKF method are applied after 100 scan. The IMGEKF and the IEKF both had improvement on the MGEKF and the EKF, respectively. In this scenario, the IMGEKF had the best performance.

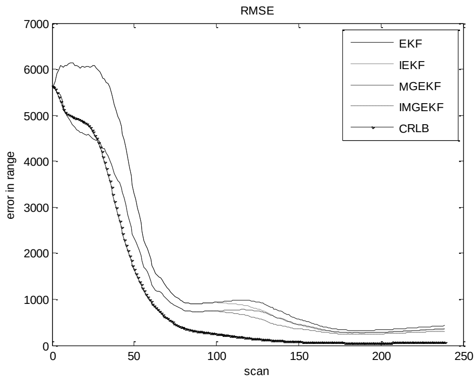

Figure 3. RMS estimation error

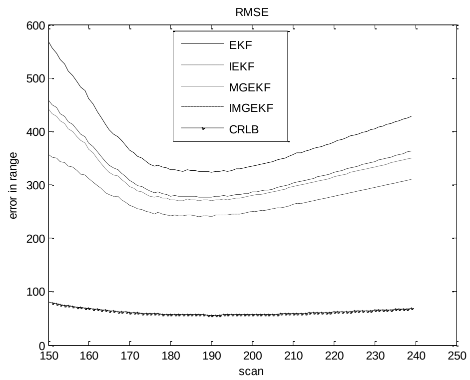

Figure 4. RMS estimation error

V. Conclusions

This paper presents an iterative method based on the MGEKF to track the target using bearings only measurements. It linearizes the measurement function in a new way compared with the EKF and MGEKF. The updated state is used to re-linearize the nonlinear measurement function. This process intended to use more information of the target state in each scan.

The IMGEKF is applied to the two dimensional bearing only measurements scenario. Both the IEKF and the IMGEKF face the divergent problem which should be processed carefully. The simulation results verify the IMGEKF and compare it with the EKF, the IEKF and the MGEKF. One can't draw definitive conclusions after a limited set of experiments. But the simulation results give us a good suggestion when we face the similar problem.

As future work we consider to extend the proposed work to the three dimensional scenario which is more practical. Another thing is to find some method to deal with the divergence problem.

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Taek Lyul Song (M'83) was born in Andong, Korea, in 1952. He received the B.Sc. degree in nuclear engineering from Seoul National University, Seoul, Korea, in 1974, and both the M.Sc. and Ph.D. degrees in aerospace engineering from the University of Texas at Austin, in 1981 and 1983, respectively. From 1974 to 1994, he was with the Agency for Defense Development where his major research topics included missile guidance and control and control systems design, target tracking, and navigation systems analysis. He has been a Professor at the Electronic Systems Engineering Department, Hanyang University, since 1995. His research interests include target state estimation, guidance, navigation, and control.

Yuan Huang was born in HuBei, China, in 1990. He received the B.Sc. degree in mechanical engineering and automation from Wuhan University of Science and Technology, Wuhan, China, in 2013. He is currently working toward a Ph.D. degree in Hanyang University. His research interests include target state estimation.

