# Adaptive Wavelet Backstepping Control for a Class of MIMO Underactuated Systems

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***Abstract*—This paper presents an adaptive backstepping controller for a class of multi input multi output (MIMO) underactuated system, with uncertain dynamics, in its original nonsquare form. The proposed controller combines a wavelet based backstepping controller and a robust control term to obtain the desired tracking performance. Proposed scheme utilizes the concept of regularized inverse for effective decoupling of subsystems. Uncertain system dynamics is estimated by using wavelet network. The wavelet parameters are tuned online using Lyapunov approach. The overall control scheme guarantees the ultimate upper boundedness of all closed loop signals. Finally a simulation study is carried out to demonstrate the effectiveness of proposed control scheme.**

***Index Terms*—MIMO systems, underactuated systems, backstepping control, adaptive control, wavelet network**

## I. Introduction

In recent years, significant development has been reported in the field of adaptive controller design for nonlinear multivariable systems. However, most of these control schemes are proposed for conventional multivariable systems where number of output is equal to number of inputs [1]-[3]. Recently, the researchers are inclined towards to the controller design for underactuated systems, which are characterized by the fact that they have lesser number of actuators than the degrees of freedom to be controlled [4]. This underactuation property has been displayed by several real time systems like spacecraft, underwater vehicle, twin rotor systems etc. Due to their underactuation property these systems are associated with complicated controller design and due to lesser number of actuators the controller schemes of fully actuated systems can't be applied directly to this class. Researchers have developed control schemes for this class of nonlinear systems [4], [5]. Few research findings on adaptive control schemes for uncertain underactuated systems have also been cited in the literature [6]-[8].

Back stepping design offers a systematic framework to design tracking and regulation control schemes for a wide class of state feedback linearizable nonlinear systems [9]. One of the distinguishable feature of this scheme is to shape performance. The conventional backstepping approach can suitably extended to uncertain nonlinear systems via adaptive backstepping [10]-[12].

Due to their universal approximation property neural networks have been proved as a promising tool for identification and control of dynamical systems. Application of neural network as system identification tool has greatly relaxed the constraint applied on nonlinearities to be linear in parameter thereby broadening the class of the uncertain nonlinear systems which can be effectively dealt by adaptive controllers. Several researchers have combined the conventional feedback control approaches with adaptive neural networks so as to achieve the desired performance by effective mitigation of uncertain dynamics [13].

During last decade, some researchers have proposed wavelet neural network as system identification tool. These networks use translated and dilated versions of some wavelet as activation function in the frame of a single layer feedforward neural network. Theses wavelet bases due to their space and frequency localization properties posses superior learning capability and training algorithm for wavelet network convergence rapidly in comparison to conventional neural networks. Furthermore, orthonormality of wavelet bases assures a unique and most efficient wavelet network [14], [15]. A wavelet network thus combines the learning ability of neural network with wavelet decomposition for identification and offers a better performance than conventional neural network [16].

In this paper, a wavelet adaptive backstepping controller is proposed for a class of uncertain multiple input multiple output underactuated systems. The tracking control scheme presented in this paper consists of a wavelet based adaptive backstepping controller and a robust control term. Proposed control strategy combines the advantages of wavelet networks for approximating unknown system dynamics with

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conventional backstepping control. To deal with the singularity issues of input gain matrix regularized matrix inversion is used. A robust controller is added for effective attenuation of the uncertainties inducted by wavelet approximators and error introduced by regularized inverse.

This paper is organized as follows. Problem under consideration is presented in section II. Design issues of wavelet adaptive backstepping controller and robust control term are discussed in first two subsections of section III, whereas approximation properties of wavelet network and tuning laws for online estimation wavelet parameters along with stability issues are discussed in next two subsections. A simulation study, to illustrate the effectiveness of proposed scheme is carried out in Section IV, whereas Section V gives the conclusion on the work developed in this paper.

## II. Problem Statement

Consider the following underactuated system of the form [8]

$$\begin{cases} \dot{\eta}_1 = \zeta_1 \\ \dot{\eta}_2 = \zeta_2 \\ \vdots \\ \dot{\eta}_n = \zeta_n \\ \dot{\zeta}_1 = f_1(x) + g_{11}(x)u_1 + g_{12}(x)u_2 + \cdots + g_{1p}(x)u_p \\ \dot{\zeta}_2 = f_2(x) + g_{21}(x)u_1 + g_{22}(x)u_2 + \cdots + g_{2p}(x)u_p \\ \vdots \\ \dot{\zeta}_n = f_n(x) + g_{n1}(x)u_1 + g_{n2}(x)u_2 + \cdots + g_{np}(x)u_p \end{cases} \quad (1)$$

$$\begin{cases} y_1 = \eta_1 \\ y_2 = \eta_2 \\ \vdots \\ y_n = \eta_n \end{cases}$$

where $x = [\eta, \zeta]^T \in \Re^{2n}$ are system states with $\eta = [\eta_1, \eta_2, \cdots \eta_n]^T$ and $\zeta = [\zeta_1, \zeta_2, \cdots \zeta_n]^T$ while $u = [u_1, u_2, \cdots, u_p]^T \in \Re^p$ and $y(t) = [y_1(t), y_2(t), \cdots, y_n(t)] \in \Re^n$ are input and output vectors respectively. System under consideration belongs to the class of underactuated systems with $n > p$.

Here $f(x) = [f_1(x), f_2(x), \cdots f_n(x)] \in \Re^n \to \Re^n$ is a vector of smooth unknown nonlinear functions of states whereas and $g(x) = [g_1(x), g_2(x), \cdots, g_n(x)]^T \in \Re^n \to \Re^{nxp}$ with $g_i(x) = [g_{i1}(x), g_{i2}(x), \cdots, g_{ip}(x)]^T$ represents the nonlinear gain matrix and is assumed to be partially unknown and satisfying following assumptions:

### A. Assumption 1

Function $g_{ij}(x)$ is always bounded away from zero with a known bound so that

$$|g_{ij}(x)| \geq |\underline{g}_{ij}| > 0 \quad \forall x \in S_x; t \geq 0; i = 1, 2, \cdots, n; j = 1, 2, \cdots, p$$

where $S_x \in \Re^{2n}$ is some compact set of allowable state trajectories.

It implies that $g_{ij}(x)$ is strictly positive or negative.

### B. Assumption 2

Sign of function $g_{ij}(x)$ is known.

Thus the system (1) can be expressed as

$$\begin{cases} \dot{\eta} = \zeta & (2a) \\ \dot{\zeta} = f(x) + g(x)u & (2b) \end{cases}$$

Objective is to utilize backstepping technique so that the system states $\eta$ are able to track the desired trajectories $\eta_d \in \Re^n$ with tracking errors converging to small neighborhood of origin. The desired trajectories $\eta_d$ assumed to be smooth, continuous and available for measurement.

## III. Wavelet Adaptive Backstepping Controller

### A. Pseudo Control Design

From (2a), the dynamics of the tracking error $\dot{e} = \dot{\eta} - \dot{\eta}_d$ $e \triangleq \eta - \eta_d \triangleq [e_1, e_2, \cdots e_n]^T$ is given by

$$\dot{e} = \dot{\eta} - \dot{\eta}_d \quad (3)$$

From (3), defining the pseudo control inputs $\zeta_d$ as

$$\zeta_d = -ke + \dot{\eta}_d \quad (4a)$$

$$\lambda = \zeta - \zeta_d \in \Re^n \quad (4b)$$

where $k = diag[k_1, k_2, \cdots, k_n]$; $k_i > 0, i = 1, \cdots, n$

Then (3) becomes

$$\dot{e} = -ke + \lambda \quad (5)$$

Choosing the Lyapunov function as

$$V_e = \frac{1}{2} e^T P e \quad (6)$$

where $P \in \Re^{nxn}$ is a positive definite symmetric matrix. From (5) the time derivative of $V_e$ is given by

$$\dot{V}_e = -e^T Pke + e^T P\lambda \quad (7)$$

### B. Backstepping Controller Design

From (2b) and (4b), the time derivative of $\lambda$ is given by

$$\dot{\lambda} = f(x) + g(x)u - \dot{\zeta}_d \quad (8)$$

Considering $f(x)$ and $g(x)$ to be the unknown dynamics of the system (1). Let $\hat{f}(x)$ and $\hat{g}(x)$ be the estimates of $f(x)$ and $g(x)$ respectively. To tackle the problem of singularity for $\hat{g}\hat{g}^T$ concept of regularized inverse [2, 3, and 8] is used in this work.

$$\hat{g}^{-1} = \hat{g}^T(\varepsilon I + \hat{g}\hat{g}^T)^{-1} \tag{9}$$

where $\varepsilon$ is a small positive constant and $I$ is a $nxn$ identity matrix.

Then control law is defined as

$$u = \hat{g}^T(\varepsilon I + \hat{g}\hat{g}^T)^{-1}(u_b + u_r) \tag{10}$$

where $u_b \in \Re^n$ is the principle control component derived using backstepping methodology [9] while $u_r \in \Re^n$ is the robust term incorporated for effective attenuation of uncertainties. These control terms are defined as

$$u_b = -\hat{f}(x) - Q\lambda - R^{-1}Pe + \dot{\zeta}_d \tag{11}$$

where $Q \in \Re^{nxn}$ and $R \in \Re^{nxn}$ are positive definite symmetric design matrices.

$$u_r = -\frac{\lambda(\rho^2+1)}{2\rho^2} \tag{12}$$

where $\rho$ is the prescribed attenuation[17].

*C. Wavelet Approximator and Tuning Laws*

To estimate the nonlinear functions $f(x)$ and $g(x)$ wavelet networks are used. Output of an *n* dimensional wavelet network with *m* nodes is [12]

$$z = \alpha^T \varphi(x, w, c) \tag{13}$$

where $x = [x_1, x_2, ..., x_n]^T \in R^n$ is the input vector, $\varphi = [\varphi_1, \varphi_2, ..., \varphi_m]^T \in \Re^m$ are wavelet functions while; $w = [w_1, w_2, ..., w_m]^T \in R^{mxn}$ and $c = [c_1, c_2, ..., c_m]^T \in R^{mxn}$ are dilation and translation parameters and $\alpha = [\alpha_1, .., \alpha_m]^T \in R^m$ are weights of wavelet and bias function respectively.

Let $z^*$ be the optimal function approximation using an ideal wavelet approximator then

$$z = z^* + \Delta = \alpha^{*T}\varphi^* + \Delta \tag{14}$$

where $\varphi^* = \varphi(x, w^*, c^*)$ and $\alpha^*, w^*, c^*$ are the optimal parameter vectors of $\alpha, w, c$ respectively and $\Delta$ denotes the approximation error and is assumed to be bounded by $|\Delta| \le \Delta^*$,in which $\Delta^*$ is a positive constant.

Optimal parameter vector is constant and is needed for best approximation of the function, however it is difficult to obtain optimal function approximation so defining an estimate function as

$$\hat{z} = \hat{\alpha}^T \hat{\varphi} \tag{15}$$

where $\hat{\varphi} = \varphi(x, \hat{w}, \hat{c})$ and $\hat{\alpha}, \hat{w}, \hat{c}$ are the estimates of $\alpha^*, w^*, c^*$ respectively. Defining the estimation error as:

$$\tilde{z} = z - \hat{z} = z^* - \hat{z} + \Delta = \alpha^T\tilde{\varphi} + \hat{\alpha}^T\tilde{\varphi} + \tilde{\alpha}^T\hat{\varphi} + \Delta \tag{16}$$

where $\tilde{\alpha} = \alpha^* - \hat{\alpha}, \tilde{\varphi} = \varphi^* - \hat{\varphi}$

By properly selecting the number of nodes, the estimation error $\tilde{z}$ can be made arbitrarily small on the compact set so that the bound $\|\tilde{z}\| \le \tilde{z}_m$ holds for all $x \in \Re$ [12].

Applying Taylor expansion linearization technique for transforming (16) to partially linear form,

$$\tilde{\varphi} = A_1^T \tilde{w} + B_1^T \tilde{c} + h_1 \tag{17}$$

where $\tilde{w} = w^* - \hat{w}, \tilde{c} = c^* - \hat{c}$ and $h_1$ is a vector of higher order terms and

$$A_1 = \left[\frac{d\varphi_1}{dw}, \frac{d\varphi_2}{dw}, ..., \frac{d\varphi_m}{dw}\right]\Bigg|_{w=\hat{w}} \quad B_1 = \left[\frac{d\varphi_1}{dc}, \frac{d\varphi_2}{dc}, ..., \frac{d\varphi_m}{dc}\right]\Bigg|_{c=\hat{c}}$$

with

$$\frac{d\hat{\varphi}_i}{dw} = \left[0...0\frac{d\hat{\varphi}_i}{dw_{1i}}, \frac{d\hat{\varphi}_i}{dw_{2i}}, ..., \frac{d\hat{\varphi}_i}{dw_{ni}}, 0...0\right]^T$$

$$\frac{d\hat{\varphi}_i}{dc} = \left[0...0\frac{d\hat{\varphi}_i}{dc_{1i}}, \frac{d\hat{\varphi}_i}{dc_{2i}}, ..., \frac{d\hat{\varphi}_i}{dc_{ni}}, 0...0\right]^T$$

Substituting (16) into (17)

$$\tilde{z} = \left(\tilde{\alpha}^T\left(\hat{\varphi} - A_1^T\hat{w} - B_1^T\hat{c}\right) + \tilde{w}^T A_1\hat{\alpha} + \tilde{c}^T B_1\hat{\alpha} + \varepsilon\right) \tag{18}$$

where the uncertain term is given by following expression

$$\varepsilon = \left(\alpha^{*T}h_1 + \tilde{\alpha}^T A_1^T w^* + \tilde{\alpha}^T B_1^T c^*\right) \tag{19}$$

Approximation of $f(x)$ and $g(x)$: Using wavelet neural network uncertainties $f(x)$ and $g(x)$ can be approximated as

$$\hat{f}(x) = \left[\hat{f}_1(x), \hat{f}_2(x), \cdots \hat{f}_n(x)\right]^T = \hat{\alpha}_f^T \hat{\varphi}_f \tag{20}$$

where $\hat{\alpha}_f = [\hat{\alpha}_{f1}, \hat{\alpha}_{f2} \cdots \hat{\alpha}_{fn}]$ with $\hat{\alpha}_{fi} = [\hat{\alpha}_{fi1}, \hat{\alpha}_{fi2} \cdots \hat{\alpha}_{fim}]^T$ so $\hat{f}_i(x) = \hat{\alpha}_{fi}^T \hat{\varphi}_f$, $i = 1, \cdots, n$

$$\hat{g}(x) = \begin{bmatrix} \hat{g}_{11}(x) & \cdots & \hat{g}_{1p}(x) \\ \vdots & \vdots & \vdots \\ \hat{g}_{n1}(x) & \cdots & \hat{g}_{np}(x) \end{bmatrix} = \hat{\alpha}_g^T \Theta \tag{21}$$

where $\hat{\alpha}_g = \begin{bmatrix} \hat{\alpha}_{g_{11}} & \cdots & \hat{\alpha}_{g_{n1}} \\ \vdots & \vdots & \vdots \\ \hat{\alpha}_{g_{1p}} & \cdots & \hat{\alpha}_{g_{np}} \end{bmatrix}$ with

$\hat{\alpha}_{gab} = [\hat{\alpha}_{gab1}, \hat{\alpha}_{gab2} \cdots \hat{\alpha}_{gabm}]^T$ and $\Theta = \begin{bmatrix} \hat{\varphi}_g & 0 & \cdots & 0 \\ 0 & \hat{\varphi}_g & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\varphi}_g \end{bmatrix}$

so $\hat{g}_{ab}(x) = \hat{\alpha}_{gab}^T \hat{\varphi}_g$, $a = 1, \cdots, n;$ $\mathrm{b} = 1, \cdots, p$

 

Proposed tuning laws for wavelet networks used to approximate $f_i(x)$ and $g_i(x)$ are:

*1) Adaptation laws for the wavelet networks used to approximate $f(x)$ will be:*

$$\begin{aligned}\dot{\hat{\alpha}}_f &= -\dot{\tilde{\alpha}}_f = \gamma_1\left(\hat{\varphi}_f - A_f^T\hat{w}_f - B_f^T\hat{c}_f\right)\lambda^T R \\ \dot{\hat{w}}_f &= -\dot{\tilde{w}}_f = \gamma_2\left[A_f\hat{\alpha}_{f_1}\ A_f\hat{\alpha}_{f_2}\ \cdots\ A_f\hat{\alpha}_{f_n}\right]R\lambda \\ \dot{\hat{c}}_f &= -\dot{\tilde{c}}_f = \gamma_3\left[B_f\hat{\alpha}_{f_1}\ B_f\hat{\alpha}_{f_2}\ \cdots\ B_f\hat{\alpha}_{f_n}\right]R\lambda\end{aligned} \tag{22}$$

*2) Adaptation laws for the wavelet networks used to approximate $g(x)$ will be:*

$$\dot{\hat{\alpha}}_g = -\dot{\tilde{\alpha}}_g = \gamma_5\begin{bmatrix}\psi & 0 & \cdots & 0\\ 0 & \psi & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \psi\end{bmatrix}u\lambda^T R$$

where $\psi = \left(\hat{\varphi}_g - A_g^T\hat{w}_g - B_g^T\hat{c}_g\right)$

$$\begin{aligned}\dot{\hat{w}}_g &= -\dot{\tilde{w}}_g = \gamma_5\left[\gamma_1\ \cdots\ \gamma_n\right]R\lambda \\ \dot{\hat{c}}_g &= -\dot{\tilde{c}}_g = \gamma_6\left[\nu_1\ \cdots\ \nu_n\right]R\lambda\end{aligned} \tag{23}$$

where

$$\gamma_i = A_g\hat{\alpha}_{gi1}u_1 + A_g\hat{\alpha}_{gi2}u_2 + \cdots\ A_g\hat{\alpha}_{gip}u_p\ ;i=1\cdots n$$

$$\nu_i = B_g\hat{\alpha}_{gi1}u_1 + B_g\hat{\alpha}_{gi2}u_2 + \cdots\ B_g\hat{\alpha}_{gip}u_p\ ;i=1\cdots n$$

here $\gamma_1, \gamma_2, \ldots, \gamma_6$ are the learning rates with positive constants.

In next subsection the proposed control law is examined.

## D. Stability Analysis

Choosing the final Lyapunov function as [9]

$$V = V_e + \frac{1}{2}\lambda^T R\lambda + \frac{\tilde{w}_f^T\tilde{w}_f}{2\gamma_2} + \frac{\tilde{c}_f^T\tilde{c}_f}{2\gamma_3} + \frac{\tilde{w}_g^T\tilde{w}_g}{2\gamma_5} + \frac{\tilde{c}_g^T\tilde{c}_g}{2\gamma_6} + tr\left[\frac{\tilde{\alpha}_f^T\tilde{\alpha}_f}{2\gamma_1}\right] + tr\left[\frac{\tilde{\alpha}_g^T\tilde{\alpha}_g}{2\gamma_4}\right] \tag{24}$$

From (7) and (8), the time derivative of (24) is given as

$$\begin{aligned}\dot{V} &= \dot{V}_e + \lambda^T R\dot{\lambda} + \frac{\tilde{w}_f^T\dot{\tilde{w}}_f}{\gamma_2} + \frac{\tilde{c}_f^T\dot{\tilde{c}}_f}{\gamma_3} + \frac{\tilde{w}_g^T\dot{\tilde{w}}_g}{\gamma_5} + \frac{\tilde{c}_g^T\dot{\tilde{c}}_g}{\gamma_6} + tr\left[\frac{\tilde{\alpha}_f^T\dot{\tilde{\alpha}}_f}{\gamma_1}\right] + tr\left[\frac{\tilde{\alpha}_g^T\dot{\tilde{\alpha}}_g}{\gamma_4}\right] \\ &= -e^T Pke + e^T P\lambda + \lambda^T R(f(x) + g(x)u - \dot{\zeta}_d) + \frac{\tilde{w}_f^T\dot{\tilde{w}}_f}{\gamma_2} + \frac{\tilde{c}_f^T\dot{\tilde{c}}_f}{\gamma_3} + \frac{\tilde{w}_g^T\dot{\tilde{w}}_g}{\gamma_5} + \frac{\tilde{c}_g^T\dot{\tilde{c}}_g}{\gamma_6} + tr\left[\frac{\tilde{\alpha}_f^T\dot{\tilde{\alpha}}_f}{\gamma_1}\right] + tr\left[\frac{\tilde{\alpha}_g^T\dot{\tilde{\alpha}}_g}{\gamma_4}\right]\end{aligned} \tag{25}$$

Substitution of control law (10) in (25) results in

$$\dot{V} = -e^T Pke + e^T P\lambda + \lambda^T R(f(x) + g(x)\hat{g}^T(\varepsilon I + \hat{g}\hat{g}^T)^{-1}(u_b + u_r) - \dot{\zeta}_d) + \frac{\tilde{w}_f^T\dot{\tilde{w}}_f}{\gamma_2} + \frac{\tilde{c}_f^T\dot{\tilde{c}}_f}{\gamma_3} + \frac{\tilde{w}_g^T\dot{\tilde{w}}_g}{\gamma_5} + \frac{\tilde{c}_g^T\dot{\tilde{c}}_g}{\gamma_6} + tr\left[\frac{\tilde{\alpha}_f^T\dot{\tilde{\alpha}}_f}{\gamma_1}\right] + tr\left[\frac{\tilde{\alpha}_g^T\dot{\tilde{\alpha}}_g}{\gamma_4}\right]$$

Which can be further simplified as:

$$\dot{V} = -e^T Pke + e^T P\lambda + \lambda^T R(f(x) + \hat{g}(x)\hat{g}^T(\varepsilon I + \hat{g}\hat{g}^T)^{-1}(u_b + u_r) + \tilde{g}u + \varepsilon_g u - \dot{\zeta}_d) + \frac{\tilde{w}_f^T\dot{\tilde{w}}_f}{\gamma_2} + \frac{\tilde{c}_f^T\dot{\tilde{c}}_f}{\gamma_3} + \frac{\tilde{w}_g^T\dot{\tilde{w}}_g}{\gamma_5} + \frac{\tilde{c}_g^T\dot{\tilde{c}}_g}{\gamma_6} + tr\left[\frac{\tilde{\alpha}_f^T\dot{\tilde{\alpha}}_f}{\gamma_1}\right] + tr\left[\frac{\tilde{\alpha}_g^T\dot{\tilde{\alpha}}_g}{\gamma_4}\right]$$

$$\dot{V} = -e^T Pke + e^T P\lambda + \lambda^T R(f(x) + (u_b + u_r) - \varepsilon(\varepsilon I + \hat{g}\hat{g}^T)^{-1}(u_b + u_r) + \tilde{g}u + \varepsilon_g u - \dot{\zeta}_d) + \frac{\tilde{w}_f^T\dot{\tilde{w}}_f}{\gamma_2} + \frac{\tilde{c}_f^T\dot{\tilde{c}}_f}{\gamma_3} + \frac{\tilde{w}_g^T\dot{\tilde{w}}_g}{\gamma_5} + \frac{\tilde{c}_g^T\dot{\tilde{c}}_g}{\gamma_6} + tr\left[\frac{\tilde{\alpha}_f^T\dot{\tilde{\alpha}}_f}{\gamma_1}\right] + tr\left[\frac{\tilde{\alpha}_g^T\dot{\tilde{\alpha}}_g}{\gamma_4}\right]$$

Substitution of backstepping control term (11) in above equation results in

$$\dot{V} = -e^T Pke + \lambda^T R(\tilde{f} - Q\lambda + u_r - \varepsilon(\varepsilon I + \hat{g}\hat{g}^T)^{-1}(u_b + u_r) + \tilde{g}u) + \frac{\tilde{w}_f^T\dot{\tilde{w}}_f}{\gamma_2} + \frac{\tilde{c}_f^T\dot{\tilde{c}}_f}{\gamma_3} + \frac{\tilde{w}_g^T\dot{\tilde{w}}_g}{\gamma_5} + \frac{\tilde{c}_g^T\dot{\tilde{c}}_g}{\gamma_6} + tr\left[\frac{\tilde{\alpha}_f^T\dot{\tilde{\alpha}}_f}{\gamma_1}\right] + tr\left[\frac{\tilde{\alpha}_g^T\dot{\tilde{\alpha}}_g}{\gamma_4}\right] \tag{26}$$

Expanding $\tilde{f}$ and $\tilde{g}$ using (18) and substituting adaptation laws (22) and (23) in (26) results in

$$\dot{V} = -e^T Pke + \lambda^T R(\varepsilon_f - Q\lambda + u_r - \varepsilon(\varepsilon I + \hat{g}\hat{g}^T)^{-1}(u_b + u_r) + \varepsilon_g u) \tag{27}$$

Substituting the robust control term (12) in above equation

$$\dot{V} = -e^T Pke - \lambda^T RQ\lambda + \lambda^T R(\delta - \frac{\lambda(\rho^2 + 1)}{2\rho^2}) \tag{28}$$

where $\delta = \varepsilon_f - \varepsilon(\varepsilon I + \hat{g}\hat{g}^T)^{-1}(u_b + u_r) + \varepsilon_g u$

Equation (28) can be further simplified as

$$\dot{V} \le -e^T Pke - \lambda^T RQ\lambda - \lambda^T R\lambda + \frac{\rho^2}{2}\delta^T R\delta \tag{29}$$

Assuming that $\delta$ is bounded on a compact set so that the bound $\delta \le \delta_{\max}$ holds for all $x \in \Re$.

Stability is assured as long as

$$\frac{\rho^2}{2}\delta^T R\delta \le e^T Pke + \lambda^T RQ\lambda + \lambda^T R\lambda \tag{30}$$

Therefore $\dot{V}$ is negative outside a compact set which implies the convergence of all the tracking error signals to small neighborhood of origin and thus assures the uniform boundedness of all the closed loop signals of the system under consideration [9].

## IV. SAIMULATION RESULTS

To demonstrate the applicability of the proposed scheme, it is applied to a MIMO underactuated system with following system dynamics

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= 0.1x_3 \sin x_1 + \frac{0.2}{1+\cos^2 x_2}u_1 + \frac{0.1}{1+\cos^2 x_1}u_2 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= 0.5x_1x_4 \cos x_2 + \frac{0.2}{1+x_4^2}u_1 + \frac{0.2}{1+x_1^2x_2^2}u_2 \\
\dot{x}_5 &= x_6 \\
\dot{x}_4 &= 0.2x_1x_5 \cos x_2x_4 + \frac{0.5}{1.5-\sin^2 x_1}u_1 + \frac{0.1}{1.5-\sin^2 x_3}u_2
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_2
\end{aligned}
$$

System belongs to the class of the system described by (1) with $n=3$ and $p=2$ with following nonlinearities

$$
f = \begin{bmatrix} 0.1x_3 \sin x_1 \\ 0.5x_1x_4 \cos x_2 \\ 0.2x_1x_5 \cos x_2x_4 \end{bmatrix}
$$

$$
g = \begin{bmatrix} \frac{0.2}{1+\cos^2 x_2} & \frac{0.1}{1+\cos^2 x_1} \\ \frac{0.2}{1+x_4^2} & \frac{0.2}{1+x_1^2x_2^2} \\ \frac{0.5}{1.5-\sin^2 x_1} & \frac{0.1}{1.5-\sin^2 x_3} \end{bmatrix}
\tag{32}
$$

In this simulation, these nonlinearities are treated as system uncertainties.

Proposed controller (10) with backstepping (11) and robust control term (12) is applied to this system with an objective to force the system states $x_1$, $x_3$ and $x_5$ to track the desired trajectories $x_{1d} = 0.5\sin t + 0.1\cos .5t + .3$, $x_{2d} = 0.5\sin .5t + 0.1\cos t + .1$ and $x_{3d} = 0.5\sin .5t + 0.5\cos .5t - .3$ respectively.

For identification of uncertain system dynamics wavelet networks are synthesized using Mexican hat wavelet as mother wavelet function. Parameters for these wavelet networks are estimated online using the proposed adaptation laws.

Initial conditions are taken as $x(0) = [0.1, 0, 0.1, 0.0.5, 0]^T$ while initial conditions for wavelet parameters are set to zero. Attenuation level for robust controller is taken as 0.01.Controller parameters chosen to perform this simulation are

$$
k = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 5 \end{bmatrix}, P = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix}
$$

$$
Q = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

Fig. 1 and Fig. 2 show the performance to the system (31) under the action of proposed control scheme, while Fig. 3 shows the applied control effort. Effectiveness of proposed scheme is reflected by the rapid convergence of desired trajectories by system states in presence of almost smooth control efforts.

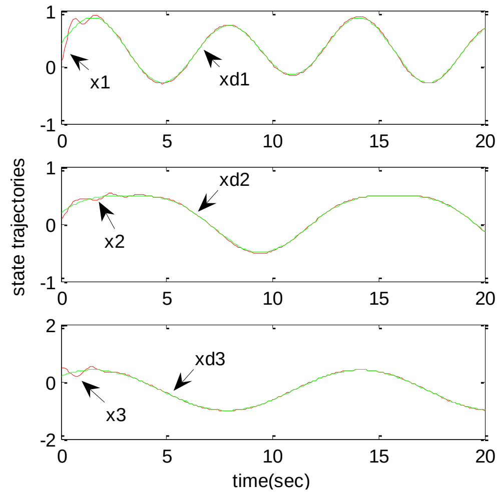


Figure 1. Desired and system trajectories

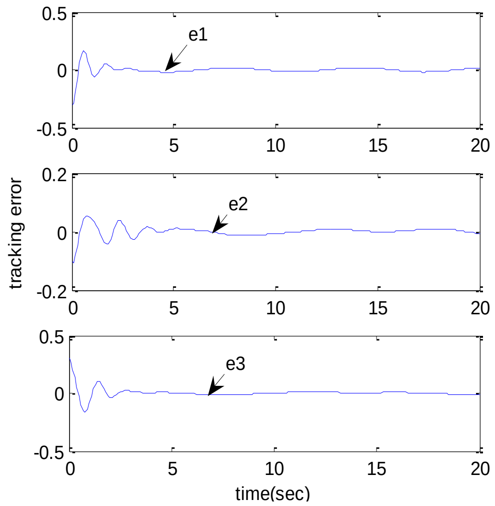


Figure 2. Tracking error.

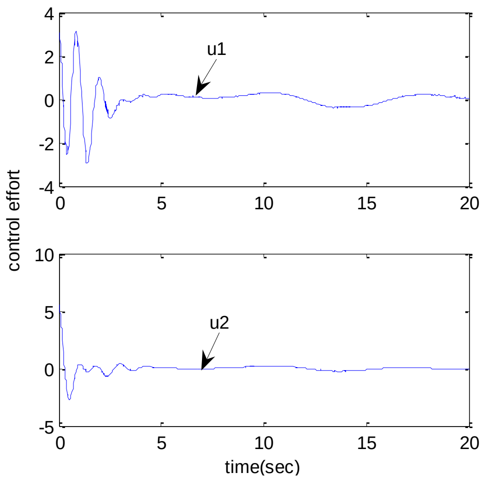


Figure 3. Control inputs

## V. Conclusion

In this work, an adaptive backstepping control law has been proposed to solve the tracking control of a class of MIMO underactuated systems with unknown system dynamics. Proposed controller guarantees the ultimate upper boundedness of all the closed loop signals. Adaptive wavelet networks are used for approximating the unknown system dynamics of the system. A robust control component is incorporated to mitigate the uncertainties due to wavelet networks and perturbations due pseudo inverse term used in control term. A simulation is carried out to validate the effectiveness of adaptive control law.

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