Closed Loop Speed Control of Miniature Brushless DC Motors

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Abstract—Three-phase miniature brushless DC (BLDC) motors are very popular on small Unmanned Aerial Vehicles (UAVs). Due to the shape and size limitation, it is hard to install devices like optical encoders for implementing a closed loop speed control. It is important to have speed control of motors on UAVs since the system dynamics are related to the rotation speed of the motors. This paper presents a sensorless phase voltage detection scheme to measure the rotation speed. A PID controller is implemented to ensure the performance of the motors. A mathematical model of Delta-Connected BLDC motors is built for analyzing the relationship between phase voltage and rotation speed. The experimental results demonstrate a fast response time and accurate results.

Index Terms—BLDC motors, UAV, closed loop feedback, phase voltage, speed control.

I. INTRODUCTION

Miniature brushless DC (BLDC) motors are used on small UAVs because of their higher performance compared to DC motors [1]. However due to the out-running design of the motors and the limited space on UAVs, traditional feedback devices like hall-effect sensors and optical encoders are hard to install [2]. There are some high-end Electronic Speed Controllers (ESCs) able to output the current rotation speed of the motors but they are not affordable for the general public [3]. Most of the UAVs on the market are running without proper motor control.

It is known that the performance of motors and the performance of the UAVs they are installed on are closely related. For example there are four motors on a quadcopter in which the upward thrust is provided by the four motors. Variations in the rotation speed with the same input control signal will affect the magnitude of propeller thrust. Since the forces of the quadcopter are generated by turning the propellers, having better control of the motors will be beneficial to the stability of the machine.

There are a few similar works using current sensing techniques to control the motors. The work presented in [4] shows that there are quite a lot of limitations like the computational overhead and fluctuations in the readings.

A sensorless approach for implementing a closed loop control for miniature BLDC motors on UAVs is proposed in this paper. The phase voltage of the motors is passed to circuitry which is able to convert the fast changing phase voltage signals into rotation speed information. The circuitry can be divided into three parts: filtering, converting and measuring. After getting the current rotation speed of the motors, a PID controller is added to minimize the deviation of the speed with respect to the reference level.

This paper is organized as follows: the modeling of a Delta-Connected type BLDC motor is presented in Section II. In Section III, the methodologies of both speed measurement scheme and speed controller are presented. In Section IV, the implementation and experimental results of the sensorless speed measurement scheme will be covered, while Section V will present the implementation and corresponding results of speed. Finally the conclusion is presented in Section VI.

II. MODELING OF DELTA-CONNECTED BLDC MOTOR

A. Background Information of the Motor

Most BLDC motors used in industry are WYE-Connected (Y-Connected) because of their higher energy-power efficiency compared to Delta-Connected motors. However the Delta-Connected type is more often used on small UAVs owing to their higher top speed.

The motors on UAVs are out-runners which means the rotating part is in the outside position of the motor while the stationary part is in the inside position. The mechanical structure of BLDC motors can be divided into three main parts: the stator, the rotor and the winding.

Figure 1. Simplified schematic diagram of a three-phase half bridge circuit with a Delta-Connected motor

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The motors are driven by Electronic Speed Controllers (ESCs). The ESCs are responsible for supplying three-phase voltage to the motors in a special order for achieving electronic commutation. Fig. 1 shows a simplified schematic diagram of an ESC and a Delta-Connected BLDC motor. There will only be one high side switch and one low side switch being switched on for every switch cycle. For example the switching pattern can be A+B- → A+C- → B+C- → B+A- → C+A- → C+B- [5] given that ‘X+Y-’ means that the positive terminal is connected to Phase X while negative, or ground, terminal is connected to Phase Y. The commutation is controlled by the ESC and it is dependent on the position of the rotor only.

B. Single Phase Model

There are six commutation steps for one complete cycle [6]. To simplify the problem, one step will be modeled. The remaining steps are considered to be similar. Fig. 2 shows the simplified schematic diagram for a running three-phase BLDC motor for one commutation step.

![Simplified schematic diagram of one commutation step of a three-phase BLDC motor](image)

Assume that the commutation is in the A+B- step, the three-phase model can be split into several sets of simple DC equations. Denote $E_k$, $L_k$, $R_k$, $i_k$ as the back emf, self-inductance, internal resistance and current for phase k, respectively. The back emf is directly proportional to the rotational speed $\omega$ with the back emf constant ($K_{ek}$).

$$E_k = K_{ek}\omega$$  \hspace{1cm} (1)

Decomposing the potential difference across T1 and T2,

$$V_s = E_a + i_a R_a + L_a \frac{di_a}{dt}$$  \hspace{1cm} (2)

$$V_s = -i_c R_c + L_c \frac{di_c}{dt} - K_{ec}\omega$$

$$-i_b R_b + L_b \frac{di_b}{dt} - K_{eb}\omega$$  \hspace{1cm} (3)

Consider that the phase windings are balanced, which means the internal resistance and self-inductance for all the phases are equivalent. The phase current will become

$$i_a = \frac{2}{3} i_m$$  \hspace{1cm} (4)

$$i_b = i_c = \frac{-1}{3} i_m$$  \hspace{1cm} (5)

Substitute Eq. (4) & (5) into Eq. (2) & (3) and, by solving,

$$K_{ea} + K_{eb} + K_{ec} = 0$$  \hspace{1cm} (6)

Since the torque generated is directly proportional to phase current with torque constant ($K_t$), the total torque ($\tau$) generated is

$$\tau = K_{ea} i_a + K_{eb} i_b + K_{ec} i_c$$  \hspace{1cm} (7)

Given that SI units are used, torque constants are equivalent to back emf constants [7].

$$K_{eb} = K_{ec}$$  \hspace{1cm} (8)

Substitute Eq. (6) & (8) into Eq. (7), the resultant torque for the motor is

$$\tau = K_t i_m$$  \hspace{1cm} (9)

As the motor will continue the commutation process, the conditions will be almost the same except the direction of the currents passing through the three-phases. In the step of A+B-, Phase A is in parallel with series-connected Phase B and Phase C. For the next step A+C-, Phase C is in parallel with series-connected Phase A and Phase B. They should have the same result with Eq. (9).

C. Simple DC Motor Model

BLDC motors implement the commutation electronically instead of physically. They are very similar to each other in other aspects, for example, the rotator and stator are built with permanent magnets and current-carrying materials. Therefore, for the sake of simplicity, the BLDC motors are modeled as simple DC motors [5].

Define $L$, $R$, and $K_{ek}$ as the equivalent internal resistance, inductance and back emf constant of motor, respectively, for a BLDC-equivalent DC motor. The equation of the electrical properties is

$$V_s = K_{ek}\omega + i R + L \frac{di}{dt}$$  \hspace{1cm} (10)

The torque generated is directly proportional to phase current with torque constant,

$$\tau = K_t i$$  \hspace{1cm} (11)

The DC motor model is much simpler than the single phase model. In this paper, the DC motor model will be adopted.

III. PROPOSED SENSORLESS SPEED MEASUREMENT SCHEME AND CLOSED LOOP SPEED CONTROLLER

A. Conventional Speed Measurement Scheme

There are many ways to generate feedbacks for motors. Traditional feedback devices such as optical encoders, hall-effect sensors and current sensors are very popular. Fig. 3 shows the output signals of optical encoders and hall-effect sensors. Both kinds of signals are in quadrature form and the change in angular position can be obtained. The rotation speed can be found by taking the derivative of the angular displacement.
Another way to obtain the rotation speed is to measure the armature current. The equation of the mechanical properties is

\[ (J + J_L) \frac{da}{dt} + K_f \omega + \tau_L = iK_f \]  

(12)

where \( J, J_L, K_f \) and \( \tau_L \) denote the moment of inertia of the motor, moment of inertia of the load, the friction constant of the motor, and the load torque, respectively.

By solving Eq. (12), the rotation speed with respect to current is

\[ \omega = \frac{r_L - iK_f}{K_f} \sqrt{\frac{\tau_L}{r_L}} + \frac{K_f - r_L}{K_f} \]  

(13)

As a result, solving for the rotation speed requires very high computational power and instantaneous measurements of load torque.

Directly considering the linear current-torque relationship would be much simpler. However due to the inaccurate factors in obtaining the current value, such as errors in the analog-digital converter (ADC) and changing thermal resistance, the result will not be accurate enough.

**B. Proposed Speed Measurement Scheme**

1) Characteristics of phase voltage

The three-phase voltages are switched to implement the electronic commutation. The voltages for the three phases are the same as the others but with a phase shift of 120 degrees [10]. There will be periodic on, off and floating states in each phase and this will be dependent on the position of the rotor.

A schematic diagram of a four-pole-pair motor is shown in Fig. 4(a). To simplify the problem, only Phase A is considered for the following analysis. In order to turn the rotor for a one-fourth revolution (or turn one pole pair for one cycle), there are four states to go: (i) propel N1 & N3 and attract S4 & S2, (ii) hold, (iii) propel S4 & S2 and attract N4 & N2, and (iv) hold. By the right-hand rule of electromagnetism, the current through the windings should change accordingly, which corresponds to a change in voltage level as shown in Fig. 4(b).

![Figure 4. Running BLDC motor](image)

It takes one cycle of phase voltage to turn one pole pair. Therefore, it takes \( N \) cycles of phase voltage for a motor to turn one complete revolution with \( N \) pole pairs [12]. This proposed solution is to measure the period of phase voltage which is the time needed to turn one pole pair. The total time needed to turn the motor for one revolution is the period of phase voltage switch (\( T_{\text{phase}} \)) multiplied by \( N \). As a result the rotation speed can be found. The corresponding rotation speed of the motor with the unit rpm is

\[ \omega = \frac{60}{T_{\text{phase}}N} \]  

(14)

To successfully measure the phase voltage, the process can be divided into three parts, as shown at Fig. 5.

![Figure 5. Measurement flow chart](image)

2) Signal filtering

The coil inside the motor has a large amount of inductance. As the motor power supply scheme is controlled by the PWM, the high frequency change in the current through the coil will induce very large voltage spikes. Filtering is needed to ensure the signal is not distorted. A low pass filter is added to attenuate the voltage spikes. The input phase voltage (\( V_{\text{in}} \)) with frequency (\( f \)) first passes through a capacitor in order to remove the DC offset. Then an RC low pass filter is implemented to attenuate the voltage spikes. It acts as a voltage divider at which the filtered output voltage (\( V_f \)) is

\[ V_f = V_{\text{in}} \frac{1}{\sqrt{R_f^2 + \left(\frac{1}{2\pi fC_2}\right)^2}} \]  

(15)

3) Signal conversion

The signal after filtering looks like a sinusoidal wave. It has very small amplitude where the peak voltages can neither be distinguished by CMOS nor TTL logic. The coming step is to convert the filtered signal into a readable format.
An op-amp is used as a comparator to differentiate the high and low states. The simplest way is to divide the signals by the middle voltage level. The original signal is compared with half of the original signal.

\[
V_{out} = \begin{cases} 
V_{CC}, \ V_f > \frac{V_f}{2} \\
0, \ V_f < \frac{V_f}{2}
\end{cases}
\]  

(16)

4) Duration measurement

The last step is to measure the period of the output square wave. The signal is connected to a microcontroller (MCU) and interrupts will be generated if there is a rising edge trigger. A timestamp will be created for each interrupt where the period will be the difference in every two adjacent timestamps.

C. Proposed Closed Loop Speed Controller

1) PWM voltage supply scheme

The speed of the BLDC motor is directly proportional to the applied voltage. Since a dynamic voltage supply is hard to implement, a PWM scheme controlled by the ESC is usually adopted. Typically a 50 Hz PWM signal of pulse width 1 ms to 2 ms (PWM_{ESC}) is used to control the ESCs. The ESCs will generate corresponding PWM signals (PWM_{MOSFET}) to control the MOSFETs. As the ESCs are purchased on the market, the control strategy is hidden. Therefore the ESCs used will be considered as a black box which reads the control PWM signals and then drives the motors. The relationship between the duty cycle (d_{MOSFET}), voltage supply by the battery (V_{bat}) and the voltage supply to the motor (V_s) is

\[
V_s = V_{bat}d_{MOSFET}
\]  

(17)

The transfer function of the rotation speed under no load condition with respect to the supplied voltage G(s) is [5]

\[
G(s) = \frac{\omega(s)}{V(s)} = \frac{K_s}{(L+s)(R+jK_f)+K_aR}
\]  

(18)

2) PID controller

A PID controller is built and the details are presented in the block diagram in Fig. 6.

![PID controller block diagram](image)

Figure 6. PID speed controller

The error of rotation speed (ε) is defined as

\[
\epsilon(t) = \omega_{desired}(t) - \omega(t)
\]  

(19)

And the output of the PID controller is

\[
u(t) = K_P \epsilon(t) + K_I \int_0^t \epsilon(\tau) d\tau + K_D \frac{d\epsilon(t)}{dt}
\]  

(20)

The corresponding transfer function is

\[
G_{PID}(s) = \frac{\omega(s)}{V(s)} = K_P + \frac{K_I}{s} + K_Ds
\]  

(21)

IV. IMPLEMENTATION AND EXPERIMENTAL RESULTS OF SPEED MEASUREMENT UNIT

A. Implementation of the Speed Measurement Unit

1) Filtering and conversion circuitry

The input phase voltage is very noisy owing to the voltage spikes caused by the PWM control shown in Fig. 7.

![Phase voltage under PWM control](image)

Figure 7. Phase voltage under PWM control

Implementation of the proposed filtering and conversion circuitry is shown in Fig. 8.

![Filtering and conversion circuitry](image)

Figure 8. Filtering and conversion circuitry

Firstly the input signal is passed to a capacitor C1 in order to filter the DC offset, and then passed to a low pass filter consisting of a resistor R1 and capacitor C2 for attenuating the voltage spikes. The attenuation of the voltage spikes is affected by the ratings of R1 and C2. To simplify the problem, R1 is fixed at 10 k ohms. Analysis of the attenuation effects is shown in Fig. 9.
The voltage divider consists of two resistors R2 and R3 with the same rating which produces another signal stream with halved amplitude. The original signal stream and the halved signal stream are passed to V+ and V- of the op-amp. The zero-volt level is the threshold for differentiating the high and low states. A readable square wave is generated as shown at Fig. 10.

The output square wave is connected to an IO pin of an MCU with interrupt enabled. A timer is running at 84 MHz which corresponds to the shortest time unit 11.9 ns. Timestamps will be created for each rising edge trigger. According to Eq. (14), the rotation speed can be calculated.

**B. Experimental Results**

Theoretically the maximum error for the time measurement will be 23.8 ns. Since the period of the phase voltage varies from 750 us to 1000 us, the maximum percentage error is 0.0032%.

In order to verify the correctness of the proposed scheme, another measurement system is built. An IR sensor is installed to detect the blades, as shown in Fig. 11. The same algorithm mentioned above is applied to IR detection with the same shortest time unit.

The comparison of the speed reading from both the circuitry and the IR sensor is shown in Fig. 12.
The statistics for the readings from IR sensor and circuitry is shown in Table I. There are more fluctuations in the speed readings from the circuitry than from IR sensor. However this does not mean that there exist larger errors in the measurement of proposed scheme. It may be due to the difference in the update frequencies of two schemes. For the same rotation speed, there will be only one measurement for the IR sensor scheme while there will be N measurements for the proposed scheme. The fluctuations may reflect the real frequently changing speed due to different reasons, for example, friction exists when the rotor moving across the permanent magnets. The performance of the proposed scheme is almost equivalent to the actual readings.

<table>
<thead>
<tr>
<th></th>
<th>By circuitry minus by IR (RPM)</th>
<th>By circuitry with moving average minus by IR (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant Speed</strong></td>
<td>mean: 0.316364</td>
<td>0.041525</td>
</tr>
<tr>
<td></td>
<td>SD: 14.14065</td>
<td>2.909533</td>
</tr>
<tr>
<td><strong>Varying Speed</strong></td>
<td>mean: 0.320415</td>
<td>-0.35438</td>
</tr>
<tr>
<td></td>
<td>SD: 34.53977</td>
<td>8.130555</td>
</tr>
</tbody>
</table>

The response time of this scheme mainly depends on three parts: the response time of phase voltage, the propagation delay of the circuitry and the overhead of measurement. The switching of phase voltage is controlled by the ESC for doing the commutation and it reflects the actual commutation duration (time needed for turning one pole pair) with negligible delay. The propagation delay represents the length of time from which the phase voltage enters the circuitry to the square wave is outputted by the op-amp. The delay varies from 300us to 600us. The overhead of measurement means the time for measuring the period of the square wave by the MCU. As interrupts are used, the MCU will process immediately and it takes almost no time to finish the simple operations. Therefore the total delay should not exceed 600us and it demonstrates the fast response of the system.

### V. IMPLEMENTATION AND EXPERIMENTAL RESULTS OF SPEED CONTROLLER

#### A. Mapping of Control Signals to Rotation Speed

Since ESCs are considered black boxes, the actual relationship between the input control signals PWM\(_{ESC}\) and effective output voltage \(V_s\) is unknown. Recording of the duty cycle of the PWM\(_{ESC}\) (\(d_{ESC}\)) and rotation speed in the real situation is needed to find out the mappings from \(d_{ESC}\) to rotation speed \(f\) and rotation speed to \(d_{ESC}(f^{-1})\). By means of interpolation, \(f\) and \(f^{-1}\) can be found as shown in Fig. 13.

#### B. Results and Evaluations of the Proposed Speed Controller

The performance of the speed controller can be demonstrated by measuring the rotation speed for multi motors with same input \(d_{ESC}\), which is shown in Fig. 15.

### VI. CONCLUSION

The proposed feedback scheme of BLDC motors measure the period of the phase voltage and calculate the corresponding rotation speed. A PID controller is implemented to control the rotation speed of motors by using the feedback provided. The feedback information is accurate and has a fast response. The controller is able to...
correct the misbehaviors of different motors given the same control signals. This work can be applied to small UAV platforms to develop better control of the motors. This scheme does not require any installation of sensors because only one wire is needed to connect the motor and the circuitry. The whole circuitry contains few electronic components which are cheap and small in size (around 4cm$^2$). This work is able to establish closed loop speed control of the three-phase BLDC motors.

REFERENCES


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