Extremum Seeking for Dead-Zone Compensation and Its Application to a Two-Wheeled Robot

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***Abstract*—In this work, we apply discrete-time extremum seeking control with a moving average filter for tuning dead-zone compensation to cancel the input dead-zone property of the controlled plant. This tuning is accomplished to optimize a control performance in real time. Unstable plants with a dead-zone property such as a two-wheeled robot show vibration motion. The original extremum seeking control is proposed by Haring *et al.* in which the periodic steady-state output is treated.**

***Index Terms*—dead-zone compensation, discrete-time extremum seeking control, two-wheeled robot**

I. INTRODUCTION

Most actuators have nonlinearities that deteriorate control system performance. One of such typical nonlinearities is an input dead-zone property. For example, it can be seen in two-wheeled robot which is desired to be stabilized motion and impedes balancing in both standing and moving then vibration motion occurs. Many works have been done for dead-zone compensation, e.g. a FRIT method [1], the adaptive fuzzy scheme [2]. In practical use, real time rejecting the dead-zone property is important. Therefore, we propose a discrete-time extremum seeking control (ESC) for periodic steady-state to tune dead-zone parameter compensation that optimizes a performance in real time.

ESC is an adaptive control method which automatically optimizes an unknown objective function of a performance measure in real time. When we apply ESC for tuning, we do not need to know the detailed relation between the plant dynamics and the objective, but we only observe the performance measure of the plant [3], [4]. ESC commonly uses a perturbation signal, a low-pass filter, a high-pass filter and an integrator [5], [6], [7] (for the discrete-time case, see [8], [10]). So recently, ESC is developed to treat periodic steady-state, which use a moving average filter to estimate a gradient of cost function, see [3], [4], [9] but this ESC in continuous-time. We extend ESC by moving average filter in discrete-time for periodic steady-state. The contribution of this paper is to reject dead-zone property in real time by using discrete-time ESC.

The paper is organized as follows. Section 2 describes a problem formulation. Section 3 is about discrete-time ESC for periodic steady-state. Section 4 explains application discrete-time ESC for tuning dead-zone compensation to two-wheeled robot dynamic model from physical equation model, simulation results and discussion. Section 5 is conclusion.

II. PROBLEM FOR MULATION

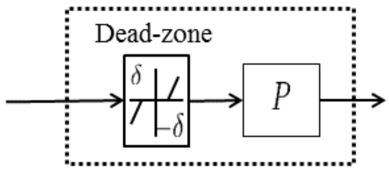

Figure 1. A system with an input dead-zone

We consider a single-input and multi-output system which consists of a linear time-invariant part P and an input dead-zone D_δ as shown in Fig. 1. We assume that the input-output relation of the dead-zone D_δ can be described as

$$D_\delta(u) = \begin{cases} u-\delta & \text{if} \quad u>\delta \\ 0 & \text{if} \quad |u| \le \delta \\ u+\delta & \text{if} \quad u<-\delta \end{cases} \tag{1}$$

With a dead-zone interval $[-\delta, \delta]$ $(\delta > 0)$. As in [1], when we know the exact value of δ, we can eliminate the dead-zone nonlinearity D_δ by using its right inverse as

$$\widehat{D}_\delta(\hat{u}) = \begin{cases} \hat{u}+\delta & \text{if} \quad \hat{u}>0 \\ 0 & \text{if} \quad \hat{u}=0 \\ \hat{u}-\delta & \text{if} \quad \hat{u}<0 \end{cases} \tag{2}$$

In this paper, we consider a feedback control system to use $\widehat{D}_\delta$ as depicted in Fig. 2. In the control system, a feedback controller C is designed to stabilize P.

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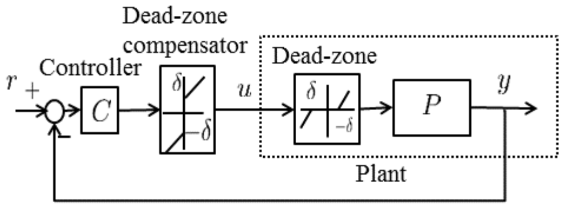


Figure 2. Configuration of a feedback control system with dead-zone compensation

In Fig. 2, r is the reference input, u is the control input, y is the measured output, respectively. Unlike the ideal case where the exact value of δ is available, it is difficult to cancel D_δ by $\widehat{D}_\delta$completely in practical application. The cancelation error causes the steady-state vibration in the control system when P is unstable. Then, we need to determine an appropriate value δ in $\widehat{D}_\delta$ to suppress the steady-state periodic motion in the control system.

III. Discrete-Time Extremum Seeking Control for Periodic Steady-State

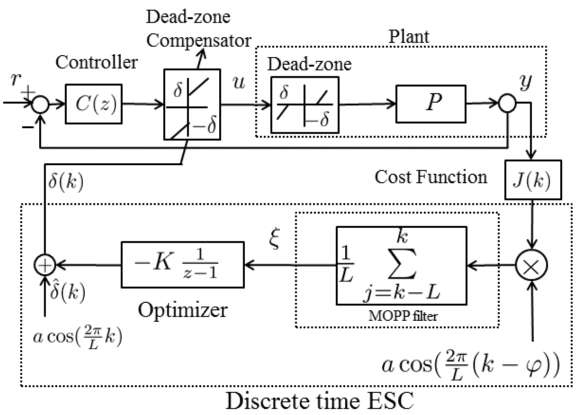


Figure 3. Overall the discrete-time ESC dead-zone compensation tuning scheme

Extremum seeking control is known as a powerful adaptive method to optimize control performance. It is mainly used to optimize a constant steady-state output. In [3], an extremum seeking scheme for periodic steady-state outputs was proposed in the non-equilibrium case. In this paper, we consider a discrete-time version of [3] which is summarized in Fig. 3. The overall closed loop system in Fig. 2 is connected with discrete-time extremum seeking control as in Fig. 3. The extremum seeking control aims to tune the dead-zone parameter δ in $\widehat{D}_\delta$ to minimize the cost function of performance output [3] by given as

$$J(\delta(k)) = \left[\frac{1}{N}\sum_{i=k-N}^{k} y(i)^2\right]^{\frac{1}{2}} \quad (3)$$

where N is the period of the steady-state out put y.

The extremum seeking scheme uses a perturbation signal (dither signal)

$$d_1(k) = a\cos\frac{2\pi}{L}k \quad (4)$$

With the period $L \in \mathbb{Z}$ and an estimate $\hat{\delta}$ of an optimal value $\delta*$ by applying

$$\delta(k) = \hat{\delta}(k) + d_1(k) \quad (5)$$

To the system, we denote the estimation error by

$$\tilde{\delta}(k) = \delta^* - \hat{\delta}(k) \quad (6)$$

To use (5) and (6), we have

$$\delta(k) = \delta^* - \tilde{\delta}(k) + d_1(k) \quad (7)$$

This perturbed signal affects (3). By applying the Tayler expansion to (3), we have

$$J(\delta(k)) = J\left(\delta^* - \tilde{\delta}(k) + d_1(k)\right)$$
$$\cong J(\delta^*) + \frac{\partial J}{\partial \delta}(\delta^*)(d_2(k) - \tilde{\delta}(k))$$
$$+\frac{1}{2}\frac{\partial^2 J}{\partial \delta^2}(\delta^*)(d_2(k) - \tilde{\delta}(k))^2 \quad (8)$$

where $d_2(k)$ denotes the time delayed signal of $d_1(k)$due to the dynamics in the closed-loop system as

$$d_2(k) = a\cos\frac{2\pi}{L}(k-\varphi)\,,\ \varphi \in \mathbb{Z} \quad (9)$$

Since $J(\delta)$ is optimal at δ^*, $\frac{\partial J}{\partial \delta}(\delta^*) = 0$.Hence,

$$J(\delta(k))$$
$$\cong J(\delta^*) + \frac{1}{2}\frac{\partial^2 J}{\partial \delta^2}(\delta^*)(d_2(k) - \tilde{\delta}(k))^2 \quad (10)$$

This cost function is multiplied by the demodulation signal $d_2(k)$, and applied into a moving-average filter, also called a mean-over-perturbation-period(MOPP)filter, over the period of $d_2(k)$. Then the output is

$$\xi(k) = \frac{1}{L}\sum_{j=k-L}^{k} d_2(j)(J(\delta^*)$$
$$+\frac{1}{2}\frac{\partial^2 J}{\partial \delta^2}(\delta^*)\left(d_2(j) - \tilde{\delta}(j)\right)^2\Big) \quad (11)$$

By simple calculation, we have

$$\sum_{j=k-L}^{k} d_2(j) = 0,\ \sum_{j=k-L}^{k} {d_2}^2(j) = \frac{a^2 L}{2}$$
$$\sum_{j=k-L}^{k} {d_2}^3(j) = 0. \quad (12)$$

Hence, when we can assume that $\tilde{\delta}(j)$ is constant over the period L, we have

$$\xi(k) = -\frac{a^2}{2}\frac{\partial^2 J}{\partial \delta^2}(\delta^*)\tilde{\delta}(k) \quad (13)$$

The signal $\xi(k)$is used to generate the estimate $\hat{\delta}$ by using the optimizer (the discrete-time integrator) as

$$\hat{\delta}(k) = -K\frac{1}{z-1}\,\xi(k) \quad (14)$$

Equivalently,

$$\hat{\delta}(k+1) = \hat{\delta}(k) - K\,\xi(k) \tag{15}$$

To use (6) and (13), we can rewrite (15) as

$$\tilde{\delta}(k+1) = \tilde{\delta}(k) + K\,\xi(k) = \left(1 - K\frac{a^2}{2}\frac{\partial^2 J}{\partial \delta^2}(\delta^*)\right)\tilde{\delta}(k) \tag{16}$$

Hence, we conclude next theorem.

Theorem 1.

If

$$\left|1 - K\frac{a^2}{2}\frac{\partial^2 J}{\partial \delta^2}(\delta^*)\right| < 1 \tag{17}$$

Then an estimate $\hat{\delta}$ converges to the optimal value δ^* by extremum seeking.

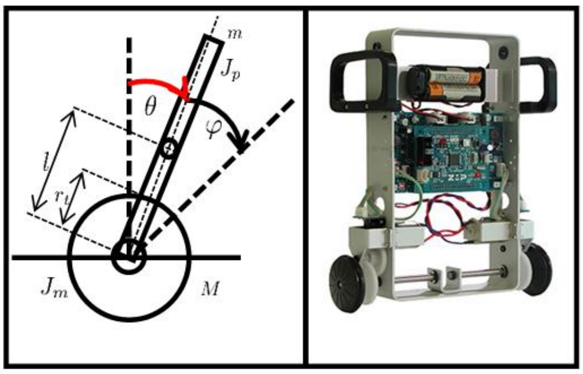
The convergence rate to the optimal value depends on the amplitude a of the perturbation signal d_1 and d_2, and the gain K of the optimizer. Since the Hessian $\frac{\partial^2 J}{\partial \delta^2}(\delta^*)$ of J is unknown, we should start with small values for a and K to find an appropriate values. Moreover, the following underlying assumptions are also required [3], [9].

Assumption1. For all fixed parameter δ over the range for tuning, the stabilized closed-loop system has a unique globally asymptotically stable steady-state solution with a constant period.

Assumption2. The cost function $J(\delta)$ has a unique global minimum at δ^* for steady-state performance.

IV. Application to Two-Wheeled Robot

In this section, we use the discrete-time extremum seeking control discussed in the previous section to optimize a dead-zone compensation for two-wheeled robot which is a commercial product called e-nuvo Wheel shown in Fig. 4.



(a) Approximation as an inverted pendulum

(b) e-nuvo WHEEL

Figure 4. Modeling of the two-wheeled robot

The feedback controller C for the two-wheeled robot is initially designed to use the model based on dynamic equations and the catalog parameters, and secondly done to use the model obtained by closed loop identification.

A. Model of Two-Wheeled Robot

As in [13], the state space continuous-time model of the two-wheeled robot P in Fig. 4 can be derived from physical equations as

$$\dot{x} = A_c x + B_c\, u \tag{18}$$

$$y = C_c\, x \tag{19}$$

TABLE I. Parameters of Two-Wheeled Robot [13], [14]

Mass of the cart(tire, draft shaft, gear) [Kg]	M	0.071
Mass of the body [Kg]	m	0.5392
Moment of inertia of the body [Kgm^2]	J_p	2.160×10^{-3}
Moment of inertia of the cart [Kgm^2]	J_t	8.632×10^{-5}
Moment of inertia of motor rotor [Kgm^2]	J_m	1.30×10^{-7}
Length between the wheel axle and gravity center of the body [m]	l	0.1073
Radius of the wheel [m]	r_t	0.02485
Friction of the wheel axle [Kgm^2/s]	c	1×10^{-4}
Torque constant of the motor [Nm/A]	K_t	2.79×10^{-3}
Reduction ratio of the gear	i	30
Efficiency drive system	η	0.75

where $x = [\theta \quad \varphi \quad \dot{\theta} \quad \dot{\varphi}]^T$ consists of the angle of the body θ, the relative angle of the wheel to the body φ, the angular velocity of the body $\dot{\theta}$ and the relative angular velocity of the wheel to the body $\dot{\varphi}$. Together with Table I, we have A_c, B_c as

$$A_c = \begin{bmatrix} 0_{2\times2} & I_{2\times2} \\ -E^{-1}G & -E^{-1}F \end{bmatrix} = \begin{bmatrix} 0 & 01 & 0 \\ 0 & 00 & 1 \\ 104.0500 & 0.06 & \\ -341.6400 & -0.37 & \end{bmatrix},\ B_c = \begin{bmatrix} 0_{2\times2} \\ -E^{-1}\zeta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 37.8 \\ -232.7 \end{bmatrix},$$

where

$$E = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} + ((M+m)rt^2 + Jt)I2,$$

$$F = \begin{bmatrix} 0 & c \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 0 \\ -mgl & 0 \end{bmatrix}, \zeta = \begin{bmatrix} \eta\, i\, K_t \\ 0 \end{bmatrix}$$

$$e_{11} = e_{22} = mlr_t + iJ_m$$

$$e_{12} = i^2 J_m$$

$$e_{21} = 2mlr_t + ml^2 + J_p + J_m.$$

When φ and $\dot{\theta}$ are measured by sensors, $C_c = \begin{bmatrix} 0100 \\ 0010 \end{bmatrix}$.

B. Design of a Stabilizing Controller

To discretize the continuous-time model (18) by zero-order hold we obtain the discrete-time model

$$x(k+1) = Ax(k) + Bu(k) \tag{20}$$

$$y(k) = C\,x(k) \tag{21}$$

When we use the sampling period $T_s = 0.01$sec, we have

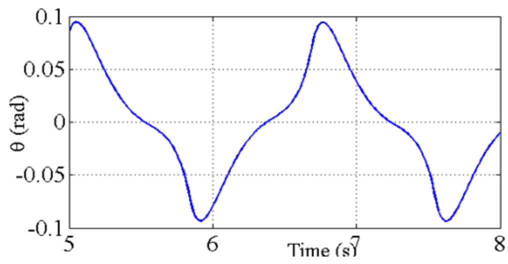
$$A = \begin{bmatrix} 1 & 0 & 0.01 & 0 \\ -0.021 & -0.0001 & 0.01 & \\ 1.040 & 1 & 0 & \\ -3.420 & -0.02 & 1 & \end{bmatrix}, B = \begin{bmatrix} 0.002 \\ -0.01 \\ 0.38 \\ -2.32 \end{bmatrix}$$

$$C = C_c$$

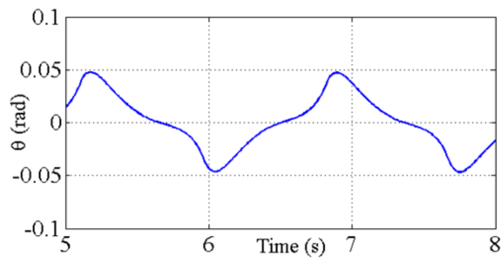
The discrete-time model is used to design the discrete-time LQG controller [11], [12] which minimizes

$$E\left\{\lim_{\tau\to\infty}\frac{1}{\tau}\sum_{k=0}^{\tau}[x^T(k)\,Q\,x(k) + u^T(k)\,R\,u(k)]\right\} \quad (22)$$

Under the existence of the process noise and the measurement noise, we assumed weight matrices $Q=I_4$, $R=1$ and covariance of the process noise $W=1$ and the measurement noise $V=0.01^2 I_2$ which means that 1% root mean square noise exists on each sensor channel.



(a)$\hat{D}_\delta$with$\hat{\delta}$=0and $D_\delta$with δ=2



(b)$\hat{D}_\delta$with$\hat{\delta}$=1and $D_\delta$with δ=2

Figure 5. Simulation result of the closed-loop system without extremum seeking

C. Dead-Zone Compensation for Two-Wheeled Robot

In the following, we set δ=1as the actual dead-zone parameter for the numerical simulations. When we use do not use the dead-zone compensator (this corresponds to $\hat{\delta}$=0), the angle of the body θ shows periodic steady periodic steady-state motion with amplitude 0.1rad(5.7degree)as shown in Fig. 5 (a).

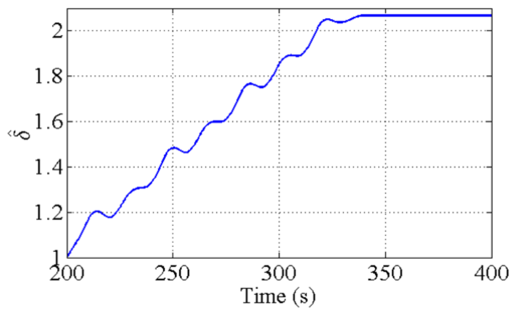
On the other hand, when we use the dead-zone compensator$\hat{D}_\delta$ with$\hat{\delta}$=1, the amplitude of the periodic steady-state motion of the angle of the body θ is 0.05 rad (2.8 degree)as showing Fig. 5 (b). Although the periodic steady-state motion is much reduced by the dead-zone compensator, it still remains due to the gap between the actual δ and $\hat{\delta}$ in the dead-zone compensator. Hence, it is important to tune $\hat{\delta}$ to suppress the periodic steady-state motion completely.

D. Extremum Seeking for Dead-Zone Parameter

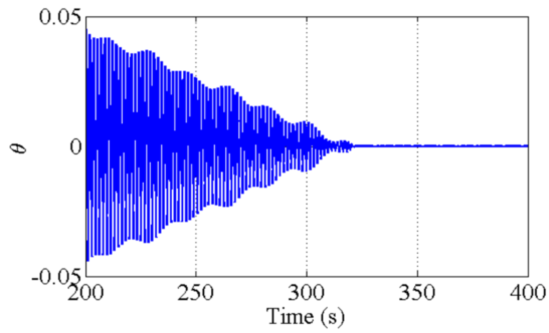
For simplicity, we use $y=\theta$ for the output for extremum seeking control and the cost function.

$$J(k) = \left[\frac{1}{N}\sum_{i=k-N}^{k}\theta(i)^2\right]^{\frac{1}{2}}$$

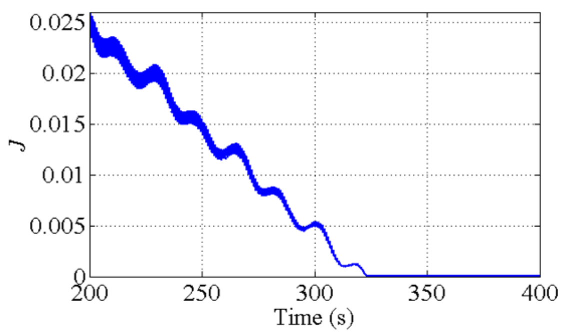
whereas the output for feedback control is $y = [\varphi \quad \dot{\theta}]^T$.The parameters for extremum seeking control are as follows; the amplitude and the period of the perturbation signal are a=1/16 and L=1800, the gain of the optimizer is K=3, the time delay of the perturbation signal in$d_2$is φ=100, the period of cost function is N=180. The mean-over-perturbation-period can be implemented by an FIR filter.



(a) The tuned value of dead-zone compensator



(b) The angle of the body



(c)Cost function

Figure 6. A simulation result when the discrete-time extremum seeking control is applied

A simulation result where tuning dead-zone compensation by the discrete-time ESC starts at t=200 sec is shown in Fig. 6 where $x\ [0]=[0.01000]^T$as the initial variable. The dead-zone compensation parameter $\hat{\delta}$ converges to δ=2.06 as shown in Fig. 6(a). Although this final value is not the actual value, the periodic steady-state motion in the body θ is sufficiently suppressed as shown in Fig. 6(b). Indeed, the cost function J decreases to sufficiently small value as shown in Fig. 6(c). This result shows that the gap between the dead-zone compensation and the actual dead-zone is acceptable.

V. Conclusion

In this paper, we have proposed discrete-time extremum seeking control to tune input dead-zone compensation in real time and applied it to the stabilized two-wheeled robot with the dead-zone compensation. The effectiveness is illustrated by numerical simulations. In the simulations, the compensation parameter converges to the optimal value minimizing the cost function of the performance output.

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