

Target Allocation Study for Formation Ship-To-Air Missile System Based on the Missile Fire Zone Division

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Abstract—Based on the proposed definition method of single ship-to-air missile fire zone, horizontal sector division methods of formation ship-to-air missile fire zone in typical orders are proposed, the single missile fire zone can be divided and determined. Afterwards, the formation target allocation methods based on the missile fire zone division are proposed, including the formation target allocation method based on the single type missile fire zone division, the formation target allocation method based on the multi-types missile fire zone division, which can be used to assist the formation missile target allocation decision support.

Index Terms—ship-to-air missile, fire zone division, formation, target allocation

I. INTRODUCTION

Target allocation for formation ship-to-air missile system is the decision-making process of assigning arriving targets to the missile system. Because the air defense capability of the formation missile system is determined by the formation missile fire zone, target allocation for the formation missile system should be based on the missile fire zone.

Prior methods are used for target allocation for the formation missile system, such as genetic algorithms¹, improved hybrid genetic algorithm², adaptive chaotic particle swarm optimization method³, distribution estimation algorithm⁴, hybrid particle swarm optimization method⁵. the formation missile fire zone isn't considered by the above methods.

In this paper, definition method of the single missile fire zone is proposed, the formation target allocation methods based on the missile fire zone division are proposed, which can be used to assist the formation missile target allocation decision support.

II. DEFINITION METHOD OF SINGLE SHIP-TO-AIR MISSILE FIRE ZONE

Single ship-to-air missile fire zone is a three-dimensional airspace, whose parameters are as follows.

A. Horizontal Sector of Single Ship-To-Air Missile Fire Zone

The parameters of horizontal sector of the single missile fire zone are the sector number, and the starting side angle, termination side angle, maximum distance of the sector. Based on the horizontal sector division methods of the formation missile fire zone in typical orders, the parameters of horizontal sector of the single missile fire zone in typical orders can be determined. Assume that r be the maximum distance of the sector, L_{ys} be the maximum horizontal projection value of the slope distance of the missile kill zone, T_f be the missile system combat reaction time, V_m be the maximum target speed. r can be calculated by

$$r = L_{ys} + V_m T_f \quad (1)$$

B. Parameters of Single Ship-To-Air Missile Kill Zone⁶

The first parameter is the average slope distance of near boundary of the single missile kill zone, it is defined by D_s . The second parameter is the maximum elevation angle of near boundary of the missile kill zone, it is defined by β . The third parameter is the lower bound of the missile kill zone, it is defined by H_d . The fourth parameter is the higher bound of the missile kill zone, it is defined by H_g .

The vertical plane projection of the single missile kill zone is shown in Fig. 1, the missile kill zone can be obtained by rotating Fig. 1 in the horizontal sector.

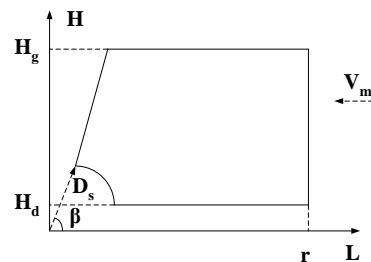


Figure 1. Vertical plane projection of single ship-to-air missile kill zone

III. HORIZONTAL SECTOR DIVISION METHODS OF FORMATION SHIP-TO-AIR MISSILE FIRE ZONE IN TYPICAL ORDERS

The formation missile system must be of the same type when the horizontal sector division is on. Assume that the number of the ship of the formation is three.

A. Single Column Order

There is overlapping region between the two adjacent missile fire zones for the two missile systems onboard two adjacent ships in single column order, the rest region is the nonoverlapping region for anyone of the two adjacent missile systems. Following the principle of being responsible for the half of the overlapping region for the single missile system, the horizontal sector of the single missile fire zone for the single missile system is its nonoverlapping region plus the half of the overlapping region, which is shown in Fig. 2 with starboard case. The portside case is the same as that of the starboard case. In the end, the horizontal sector of the formation missile fire zone in single column order can be divided.

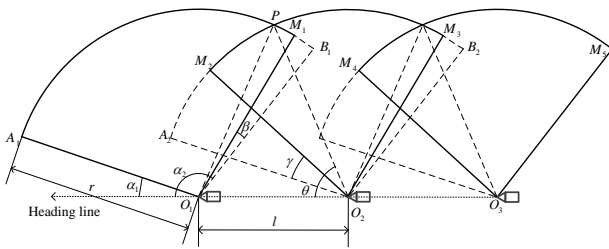


Figure 2. Horizontal sector division figure of formation ship-to-air missile fire zone in single column order

Assume that α_1 and α_2 be the minimum starboard angle and maximum starboard angle of the single missile fire boundary, and l be ship spacing, and the far arcs of the two adjacent ships (defined by O_1 and O_2) are A_1B_1 and A_2B_2 respectively, the two far arcs intersect at point P. The angle $\angle PO_1B_1$ is bisected by a straight line (defined by O_1M_1), the angle $\angle PO_2A_2$ is bisected by a straight line (defined by O_2M_2). Assume that $\angle M_1O_1B_1 = \beta$, $\angle M_2O_2A_2 = \gamma$, $\angle PO_1O_2 = \theta$.

In the triangle ΔPO_1O_2 , $\theta = \arccos \frac{l}{2r}$.

Then, $\beta = \frac{1}{2}(\arccos \frac{l}{2r} - \pi + \alpha_2)$, $\gamma = \frac{1}{2}(\arccos \frac{l}{2r} - \alpha_1)$.

The horizontal sector of the missile fire zone for each missile system of the formation is shown in the solid line of the figure 2.

The horizontal sector of the missile fire zone for the missile system onboard ship O_1 is the sector $A_1O_1M_1$ whose angle interval is $[\alpha_1, \alpha_2 - \beta]$. The horizontal sector of the missile fire zone for the missile system onboard ship O_2 is the sector $M_2O_2M_3$ whose angle interval is $[\alpha_1 + \gamma, \alpha_2 - \beta]$. The horizontal sector of the missile fire zone for the missile system onboard ship O_3

is the sector $M_4O_3M_5$ whose angle interval is $[\alpha_1 + \gamma, \alpha_2]$.

B. Single Transverse Order

There is overlapping region between the two adjacent missile fire zones for the two missile systems onboard the two adjacent ships in single transverse order, the rest region is the nonoverlapping region for anyone of the two adjacent missile systems. Following the principle of being responsible for larger right sector for the missile system onboard the rightmost ship, and larger left sector for the missile system onboard the leftmost ship, the horizontal sector of the formation missile fire zone in single transverse order can be divided, which is shown in the solid line of the Fig. 3 with starboard case. The portside case is the same as that of the starboard case.

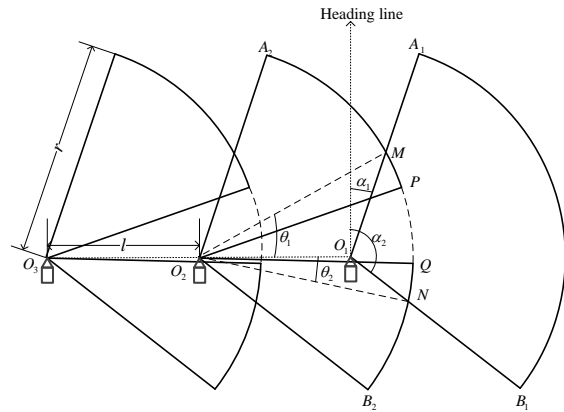


Figure 3. Horizontal sector division figure of formation ship-to-air missile fire zone in single transverse order

The missile system onboard the rightmost ship O_1 is responsible for larger right sector, the horizontal sector of its fire zone is keeping the original unchanged, it is the sector $A_1O_1B_1$ whose angle interval is $[\alpha_1, \alpha_2]$.

The horizontal sector of the missile fire zone for the missile system onboard ship O_2 is the nonoverlapping region plus the fourth of the overlapping region between the two adjacent missile fire zones for the two missile systems onboard ship O_1 and ship O_2 . The far arc of the horizontal sector for the missile system onboard ship O_2 (defined by A_2B_2) intersects with the horizontal sector for the missile system onboard ship O_1 at point M and point N. Then, $\angle MO_2P = \angle QO_2N = \frac{1}{4}\angle MO_2N$.

The horizontal sector for the missile system onboard ship O_2 is the sector A_2O_2P plus sector QO_2B_2 . Assume that $\angle MO_2O_1 = \theta_1$, and $\angle NO_2O_1 = \theta_2$, $\angle MO_2N = \theta_1 + \theta_2$.

Coordinate system of the pole is established, the position of the ship O_2 is the coordinate pole, the

positive direction is to the right. Assume that ρ be the polar radius of any point in the coordinate system, the linear equation of the straight line O_1A_1 can be expressed as⁷

$$\frac{\rho}{\sin[\pi - (\frac{\pi}{2} - \alpha_1)]} = \frac{l}{\sin(\frac{\pi}{2} - \alpha_1 - \theta_1)} \quad (2)$$

The $\odot O_2$ equation can be expressed as $\rho = r$, $\theta_1 = \arccos \frac{l \cos \alpha_1}{r} - \alpha_1$.

The linear equation of the straight line O_1B_1 can be expressed as

$$\frac{\rho}{\sin(\pi - \alpha_2 + \frac{\pi}{2})} = \frac{l}{\sin(\pi - \alpha_2 + \frac{\pi}{2} - \theta_2)} \quad (3)$$

$\theta_2 = -(\arccos \frac{l \cos \alpha_2}{r} - \alpha_2)$. Then, the angle interval

of the sector A_2O_2P is $[\alpha_1, \alpha_1 + \frac{\pi}{2} + \frac{-3\theta_1 + \theta_2}{4}]$, the angle interval of the sector QO_2B_2 is $[\frac{\pi}{2} - \frac{\theta_1 - 3\theta_2}{4}, \alpha_2]$.

The horizontal sector of the missile fire zone for the missile system onboard ship O_3 is the nonoverlapping region plus the fourth of the overlapping region between the two adjacent missile fire zones for the two missile systems onboard ship O_3 and ship O_2 , which can be determined by the same method as that of ship O_2 .

C. Orientation Order

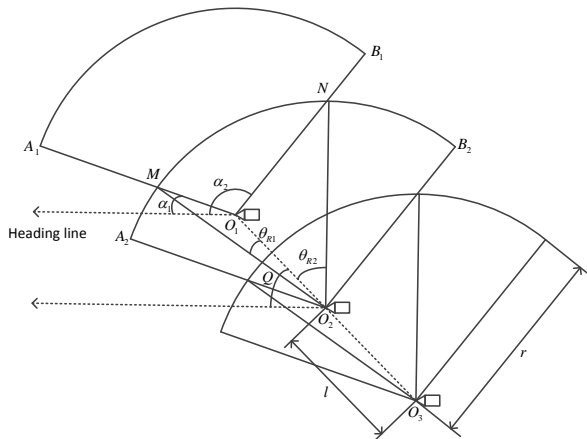


Figure 4. Horizontal sector division figure of formation ship-to-air missile fire zone in orientation order

There is overlapping region between the two adjacent missile fire zones for the two missile systems onboard

two adjacent ships in orientation order, the rest region is the nonoverlapping region for anyone of the two adjacent missile systems. Following the principle of being responsible for larger right sector for the missile system onboard the rightmost ship, and larger left sector for the missile system onboard the leftmost ship, the horizontal sector of the formation missile fire zone in orientation order can be divided, which is shown in the solid line of the Fig. 4 with starboard case. The portside case is the same as that of the starboard case.

The missile system onboard the rightmost ship O_1 is responsible for larger right sector, the horizontal sector of its fire zone is keeping the original unchanged, it is the sector $A_1O_1B_1$ whose angle interval is $[\alpha_1, \alpha_2]$.

The far arc of the horizontal sector for the missile system onboard ship O_2 (defined by A_2B_2) intersects with the horizontal sector for the missile system onboard ship O_1 at point M and point N . Then, the horizontal sector for the missile system onboard ship O_2 is the sector A_2O_2M plus sector NO_2B_2 .

Coordinate system of the pole is established, the position of the ship O_2 is the coordinate pole, the positive direction is $\overrightarrow{O_2O_1}$. Assume that the side angle of ship O_2 looking at ship O_1 is Q , $\angle MO_2O_1 = \theta_{R1}$. The linear equation of the straight line O_2M can be expressed as

$$\frac{\rho}{\sin(Q - \alpha_1)} = \frac{l}{\sin(Q - \alpha_1 - \theta_{R1})}.$$

$$\theta_{R1} = Q - \alpha_1 - \arcsin \frac{l \sin(Q - \alpha_1)}{r} \quad (4)$$

The angle interval of the sector A_2O_2M is $[\alpha_1, Q - \theta_{R1}]$.

Assume that $\angle O_1O_2N = \theta_{R2}$, The linear equation of the straight line O_2N can be expressed as

$$\frac{\rho}{\sin(\alpha_2 - Q)} = \frac{l}{\sin(\alpha_2 - Q - \theta_{R2})}.$$

$$\theta_{R2} = \alpha_2 - Q - \arcsin \frac{l \sin(\alpha_2 - Q)}{r} \quad (5)$$

The angle interval of the sector NO_2B_2 is $[Q - \theta_{R2}, \alpha_2]$.

The horizontal sector of the missile fire zone for the missile system onboard ship O_3 can be determined by the same method as that of ship O_2 .

D. Herringbone Order

The herringbone order can be seen as left orientation order and right orientation order. Following the principle of being responsible for the right sector in the right

orientation order, and for the left sector in the left orientation order, the horizontal sector of the formation missile fire zone in herringbone order can be divided.

IV. FORMATION TARGET ALLOCATION METHODS BASED ON SHIP-TO-AIR MISSILE FIRE ZONE DIVISION

A. Formation Target Allocation Method based on Single Type Ship-To-Air Missile Fire Zone Division

When the formation is of the single type missile system, the formation target allocation method based on the single type missile fire zone division is as follows. Based on the definition method of the single missile fire zone and the horizontal sector division methods of the formation missile fire zone in typical orders, the missile fire zone for any missile system of the formation can be determined. When any target is within the missile fire zone for any missile system, the target is allocated to the missile system. In the end, the formation target allocation results based on the single type missile fire zone division can be obtained.

B. Formation Target Allocation Method based on Multi-Types Ship-To-Air Missile Fire Zone Division

The formation target allocation method based on the multi-types missile fire zone division is as follows. The first step is to get the formation target allocation results for the formation single type missile system through the formation target allocation method based on the single type missile fire zone division. The second step is to classify the formation missile system. Based on the missile range, the formation missile systems include the terminal range missile system, the short range missile system, the short medium range missile system, the medium range missile system, the medium long range missile system, the long range missile system. From the front principle to back principle, the formation target allocation results based on the multi-types missile fire zone division can be obtained.

(1) Priority fire principle of terminal range ship-to-air missile system gives to terminal range target

For terminal range single target, it should be intercepted by the terminal range missile system, it shouldn't be intercepted by the short range missile system, the short medium range missile system, the medium range missile system, the medium long range missile system, the long range missile system. When the target can be intercepted by the multiple terminal range missile systems, it should be fired by the terminal range missile system with least total number of the fired targets.

(2) Priority fire principle of short range ship-to-air missile system gives to short range target

For short range single target, it should be intercepted by the short range missile system, it shouldn't be intercepted by the short medium range missile system, the medium range missile system, the medium long range missile system, the long range missile system. When the target can be intercepted by the multiple short range missile systems, it should be fired by the short range missile system with least total number of the fired targets.

(3) Priority fire principle of short medium range ship-to-air missile system gives to short medium range target

For short medium range single target, it should be intercepted by the short medium range missile system, it shouldn't be intercepted by the medium range missile system, the medium long range missile system, the long range missile system. When the target can be intercepted by the multiple short medium range missile systems, it should be fired by the short medium range missile system with least total number of the fired targets.

(4) Priority fire principle of medium range ship-to-air missile system gives to medium range target

For medium range single target, it should be intercepted by the medium range missile system, it shouldn't be intercepted by the medium long range missile system, the long range missile system. When the target can be intercepted by the multiple medium range missile systems, it should be fired by the medium range missile system with least total number of the fired targets.

(5) Priority fire principle of medium long range ship-to-air missile system gives to medium long range target

For medium long range single target, it should be intercepted by the medium long range missile system, it shouldn't be intercepted by the long range missile system. When the target can be intercepted by the multiple medium long range missile systems, it should be fired by the medium long range missile system with least total number of the fired targets.

(6) Priority fire principle of long range ship-to-air missile system gives to long range target

For long range single target, it should be intercepted by the long range missile system. When the target can be intercepted by the multiple long range missile systems, it should be fired by the long range missile system with least total number of the fired targets.

V. EXAMPLE

Assume that formation includes three ships equipped ship-to-air missile system, the ships are numbered as the 1st ship, the 2nd ship, the 3rd ship. Assume that the formation aligning angle is 60° and the formation course is 0° and the longitude and latitude of the formation center are 33.4° N and 123.4° E respectively, and the formation spacing is 6 km, and $r=30\text{km}$, and $\alpha_1=15^\circ$, and $\alpha_2=150^\circ$.

Assume that the formation is of the single type missile system, based on the proposed horizontal sector division methods of the formation missile fire zone in typical orders, the horizontal sector division figures of the formation missile fire zone in single column order, single transverse order, orientation order and herringbone order are shown in Fig. 5, Fig. 6, Fig. 7 and Fig. 8 respectively.

Through the proposed formation target allocation method based on the single type missile fire zone division, when any target is within the missile fire zone for anyone of the three missile systems, the target is allocated to the missile system.

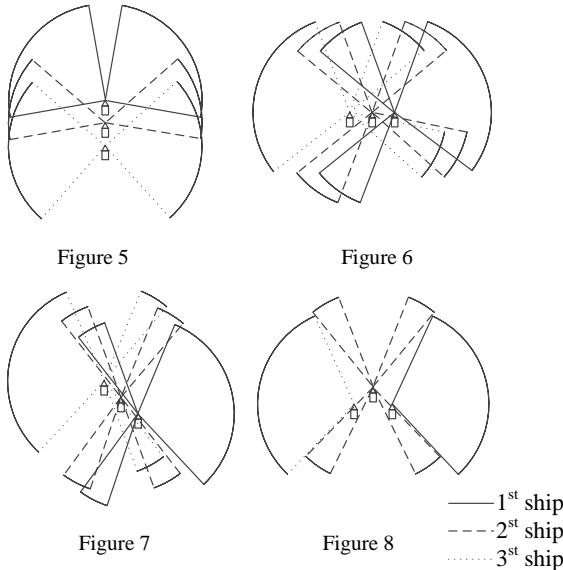


Figure 5-8. Horizontal sector division figures of formation ship-to-air missile fire zone in single column order, single transverse order, orientation order and herringbone order respectively

VI. CONCLUSION

Based on the proposed definition method of single ship-to-air missile fire zone, and the proposed horizontal sector division methods of the formation missile fire zone in typical orders, the single missile fire zone can be divided and determined. Afterwards, the proposed formation target allocation method based on the single type missile fire zone division can be used to get the formation target allocation results when the formation is of the single type missile system, the formation target allocation method based on the multi-types missile fire

zone division can be used to get the formation target allocation results when the formation is of the multi-types missile systems.

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