Direct MRAS Based an Adaptive Control System for a Two-Wheel Mobile Robot

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Abstract—In this paper, a Model Reference Adaptive Systems (MRAS) based an Adaptive System is proposed to a Two-Wheel Mobile Robot (TWMR). The TWMR is an open-loop unstable, non-linear and multi output system. The main task of this design is to keep the balance of the robot while moving toward the desired position. Firstly, the nonlinear equations of motion for the robot are derived in the Lagrange form. Next, these equations are linearized to obtain two separate linear equations. Finally, two separate adaptive controllers are designed for controlling a balancing angle, and a position. By applying Lyapunov stability theory the adaptive law that is derived in this study is quite simple in its form, robust and converges quickly. Simulation results and analysis show that the proposed adaptive PID controllers have better performance compared to the conventional PID controllers in the sense of robustness against internal and/or external disturbances.

Index Terms—model reference adaptive systems (MRAS), two-wheel mobile robot (TWMR); inverted pendulum system.

I. INTRODUCTION

An inverted pendulum is a classic problem in dynamics and control theory since it is a single-input multiple-output system and has a nonlinear characteristic [1]. The objective of the control system is to balance the inverted pendulum by applying suitable internal forces. Controlling the balancing angle of the inverted pendulum is challenging issue due to mentioned dynamics [2].

A two-wheel mobile robot works on the principle of the inverted pendulum [1], [2]. Physically, this system consists of the inverted pendulum which is mounted on a moving cart. Commonly, servomotors are used to control the translation motion of the cart through a belt mechanism. The inverted pendulum logically tends to fall down from the top vertical position, which is an unstable position. This causes the TWMR to be unstable, and it will quickly fall over if without any help [2]. Therefore, in this case, the goal of the control system is to stabilize the inverted pendulum by applying forces to the cart in order to remain upright on the top vertical. Although the TWMR is inherently unstable, it has several advantages since it has only two wheels which require less space and easy navigation on various terrains, turning sharp corners.

The TWMR is a common mechatronics case-study and is widely used as a standard setup for testing control algorithms, for example, PID control, full state feedback, neural networks, fuzzy control, genetic algorithms, etc… [3]. Conventional PID controllers could be applied to the position control for the TWMR. In general, fixed parameters in a PID controller do not have robust performance for control systems with parametric uncertainties and internal and/or external disturbances. Linear control techniques such as the full-state feedback was tested but had no success in controlling both a balancing angle and a position of the TWMR [4], [5].

Intelligent control techniques such as neural networks have shown that they are capable of identifying complex nonlinear systems. They have applied to the TWMR as an additional controller to support main feedback linear controllers for compensating the disturbances [5]. Fuzzy controllers are also a good candidate of intelligent tools that can perform better than linear controllers since they function as a nonlinear controller with infinite gains [6]. However, both neural networks and fuzzy logic need a time-consuming process to find optimal rules, which is considered as a negative point [5], [6].

In this study, design of MRAS-based adaptive control systems is developed for the TWMR which acts on the errors to reject system disturbances, and to cope with system parameter changes. In the model reference adaptive systems the desired closed loop response is specified through a stable reference model. The control system attempts to make the process output similar to the reference model output [7], [8]. The proposed controller is expected to improve the balancing performance and increase the robustness under the effects of disturbances and parameter changes. Two separate adaptive controllers are designed based on the Lyapunov’s stability theory for controlling a balancing angle and a position. Controlling a heading angle is not addressed in this paper.

This paper is organized as follows. Design of MRAS based an adaptive controller is introduced in Section II. In Section III, the dynamics of the two-wheel mobile robot is shown. The design of the proposed controller is...
introduced in Section IV. The simulation results are presented and discussed. At the end of this paper, summary of the paper is given.

II. DESIGN OF DIRECT MRAS

The general idea behind Model Reference Adaptive System (MRAS) is to create a closed loop controller with parameters that can be updated to change the response of the system. The output of the system is compared to a desired response from a reference model. The control parameters are update based on this error. The goal is for the parameters to converge to ideal values that cause the plant response to match the response of the reference model [7], [8].

Figure. 1. Adaptive system designed with Lyapunov

![Diagram of adaptive system designed with Lyapunov]

The structure depicted in Fig. 1 can be used as an adaptive PD controlled system. A second-order process is controlled with the aid of a PD-controller. The parameters of this controller are \( K_p \) and \( K_d \). Variations in the process parameters \( a_p \) and \( b_p \) can be compensated for by variations in \( K_p \) and \( K_d \). We are going to find the form of the adjustment laws for \( K_p \) and \( K_d \). The following steps are thus necessary to design an adaptive controller with the method of Lyapunov [7], [8]:

**Step 1: Determine the differential equation for \( \varepsilon \)**

The description of the process is:

\[
\dot{x}_{1p} = x_{2p}
\]

\[
\dot{x}_{2p} = -b_p K_p x_{1p} - (a_p + b_p K_d) x_{2p} + b_p K_p x_p
\]

Aid the state variables \( \varepsilon \) and \( x_{2p} \), where

\[
\varepsilon = R - x_{1p}
\]

The process in Fig 1 can be described in state variables:

\[
\dot{x}_p = A_p x_p + B_p u
\]

where

\[
x_p = [x_{2p}^T ; A_p = \begin{bmatrix} 0 & -1 \\ b_p K_p & -(a_p + b_p K_d) \end{bmatrix} ; B_p = \begin{bmatrix} 0 \end{bmatrix}]
\]

The desired performance of the complete feedback system is described by the transfer function:

\[
\frac{x_{1m}}{R} = \frac{\omega_m^2}{s^2 + 2\zeta \omega_m s + \omega_m^2}
\]

By the same way, the description of the reference model is:

\[
\dot{x}_m = A_m x_m + B_m u
\]

where

\[
x_m = [\varepsilon_m^T ; A_m = \begin{bmatrix} 0 & -1 \\ -2\zeta \omega_m & -2\zeta \omega_m \end{bmatrix} ; B_m = \begin{bmatrix} 0 \end{bmatrix}]
\]

and \( \varepsilon, x_{1p}, x_{2p}, R, \varepsilon_m, x_{1m}, x_{2m}, k_p, u, \omega_m, z \), and \( k_d \) are defined in Fig 1.

Subtracting Eq. (4) from Eq. (6) yields

\[
\dot{e} = \dot{x}_m - \dot{x}_p
\]

\[
e = x_m - x_p
\]

\[
e^T = [e_1 & e_2], e_1 = x_{1m} - x_{1p}, e_2 = x_{2m} - x_{2p}.
\]

**Step 2: Choose a Lyapunov function \( V(e) \)**

Simple adaptive laws are found when we use the Lyapunov function

\[
V(e) = e^T P e + a^T a + b^T b
\]

where \( P \) is an arbitrary definite positive symmetrical matrix; \( a \) and \( b \) are vectors which contain the non-zero element of the \( A \) and \( B \) matrices; \( a \) and \( b \) are diagonal matrices with positive elements which determine the speed of adaptation.

**Step 3: Determine the conditions under which \( V(e) \) is definite negative**

\[
\dot{V} = e^T P e + e^T P \dot{e} + 2a^T a e + 2b^T b e
\]

\[
= (A_m e + A x_p + B u)^T P e + e^T P (A_m e + A x_p + B u)
\]

\[
+ 2a^T a e + 2b^T b e
\]

Let:

\[
A_m^T P + PA_m = -Q
\]

where \( Q \) is a definite positive matrix.

After some mathematical manipulations, this yields [8]:

\[
K_p = \frac{1}{a_{11}} \int (P_{21} e_1 + P_{22} e_2) e dt + K_p(0)
\]

\[
K_d = -\frac{1}{a_{22}} \int (P_{21} e_1 + P_{22} e_2) x_{2p} dt + K_d(0)
\]

**Step 4: Solve \( P \) from \( A_m^T P + PA_m = -Q \)**

![Diagram of a system with Lyapunov stability]

Figure. 2. An extension of the adaptive control scheme presented in Fig. 1.
Based on Eq. (11) and Eq. (12) the adaptive system designed with Lyapunov in Fig. 1 is redrawn as in Fig. 2. In the adjustment laws the derivative of the error is needed. This derivative can be obtained by means of a second-order state variable filter (SVF).

III. TWO–WHEEL MOBILE ROBOT

In order to design a controller for the balancing robot, a dynamical model is first required. The mechanical system of the two-wheel mobile robot can be divided into sub systems of the wheels and the upper body, as indicated in Fig. 3 and Fig. 4, respectively [9].

The results are based on the project as introduced by [9]. Only the final non-linear equations of the robot are given in here

\begin{align}
\ddot{x} + \dot{\theta} + \frac{2k_m k_e}{Rr} \dot{x} + M_p g \sin \theta_p &= -\frac{2k_m}{R} V_a \\
2M_w + 2I_w + M_p \ddot{x} + M_p \cos \theta_p \ddot{\theta} + \frac{2k_m k_e}{Rr^2} \dot{x} &= 2k_m V_a \\
-M_p l_p \dot{\theta}^2 - M_p g \sin \theta_p &= \frac{2k_m}{r} V_a
\end{align}

with

\begin{align}
x &= \text{horizontal displacement}, & [\text{m}] \\
\dot{x} &= \text{velocity}, & [\text{m/s}] \\
\theta &= \text{tilt angle}, & [\text{rad}] \\
\dot{\theta} &= \text{angular rate}, & [\text{rad/s}] \\
V_a &= \text{applied terminal voltage}, & [\text{V}] \\
k_m &= \text{motor’s torque constant}, & [\text{N.m/A}] \\
k_e &= \text{back EMF constant}, & [\text{Vs/rad}] \\
R &= \text{nominal terminal resistance}, & [\text{Ohms}] \\
l &= \text{distance between the center of the wheel and the robot’s center of gravity}, & [\text{m}] \\
g &= \text{gravitational constant}, & [\text{m/s}^2] \\
M_p &= \text{mass of the robot’s chassis}, & [\text{kg}] \\
r &= \text{wheel radius}, & [\text{m}] \\
I_p &= \text{moment of inertia of the robot’s chassis}, & [\text{kg.m}^2] \\
I_w &= \text{moment of inertia of the wheels}, & [\text{kg.m}^2] \\
M_w &= \text{mass of the wheel connected to both sides of the robot}. & [\text{kg}] \\
H_L, H_R, P_L, P_R &= \text{reaction forces between the wheel and chassis.} \\
C_L, C_R &= \text{applied torque from the motors to the wheels.} \\
H_{fL}, H_{fR} &= \text{friction forces between the ground and the wheels.}
\end{align}

The above equations are obtained with assumption that the motor inductance and friction on the motor armature is neglected; the wheels of the robot will always stay in contact with the ground; there is no slip at the wheels; and cornering forces are also negligible [9]. These equations are then used to make a simulink model of the robot and design controllers.

In order to make simpler the design of the controller, the nonlinear equations of motion have to be linearized. Since we assume that robot is symmetrical, the desired balancing angle is set to zero. The above equations can be linearized by assuming \( \theta_p = \pi + \phi \), where \( \phi \) represents a small angle from the vertical upward direction.

Therefore,

\begin{align}
cos \theta_p &= -1, \sin \theta_p = -\phi, \text{ and } \dot{\theta}_p^2 = 0.
\end{align}

The linearized system dynamics can be written in terms of the system states and the input as

\begin{align}
(L_p + M_p l_p^2) \ddot{\theta} + M_p l \ddot{x} + 2k_m k_e \dot{x} + M_p g l \phi &= -\frac{2k_m}{R} V_a \\
2M_w + 2I_w + M_p \ddot{x} + M_p \cos \theta_p \ddot{\theta} + \frac{2k_m k_e}{Rr^2} \dot{x} &= 2k_m V_a \\
-M_p l_p \dot{\theta}^2 - M_p g \sin \theta_p &= \frac{2k_m}{r} V_a
\end{align}

If coupling terms in Eq. (16) and Eq. (17) are ignored, the independent linear equations of motion are

\begin{align}
(L_p + M_p l_p^2) \ddot{\theta} + M_p g l \phi &= -\frac{2k_m}{R} V_a
\end{align}

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The state space representation of the system is obtained
\[
\begin{align*}
\dot{\begin{bmatrix} \dot{\theta} \\ \dot{x} \end{bmatrix}} &= \begin{bmatrix} -\frac{M_p g l}{(l_p + M_p r)^2} & 1 \\ -\frac{2 k_m}{(2 M_w + \frac{2 k_m}{k_m}) r^2} \end{bmatrix} \begin{bmatrix} \theta \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2 k_m}{(2 M_w + \frac{2 k_m}{k_m}) r^2} \end{bmatrix} V_a \\
\dot{\begin{bmatrix} \theta \\ x \end{bmatrix}} &= \begin{bmatrix} 0 & 0 \\ -\frac{k_m}{(2 M_w + \frac{2 k_m}{k_m}) r^2} \end{bmatrix} \begin{bmatrix} \theta \\ x \end{bmatrix} + \begin{bmatrix} \frac{2 k_m}{(2 M_w + \frac{2 k_m}{k_m}) r^2} \end{bmatrix} V_a
\end{align*}
\] (19)

\[\begin{align*}
\dot{\theta} &= -\frac{M_p g l}{(l_p + M_p r)^2} \theta + \frac{2 k_m}{(2 M_w + \frac{2 k_m}{k_m}) r^2} V_a \\
\dot{x} &= -\frac{2 k_m}{(2 M_w + \frac{2 k_m}{k_m}) r^2} x + \frac{2 k_m}{(2 M_w + \frac{2 k_m}{k_m}) r^2} V_a
\end{align*}
\] (20)

IV. DESIGN CONTROL SYSTEM

A. PID Control System with Fixed Parameters

The PID control algorithm is mostly used in the industrial applications since it is simple and easy to implement when the system dynamics is not available. For the TWMR control variables are a balancing angle \( \theta \) and position \( x \) such that two separate controllers are required. In this study, the proportional – derivative (PD) controller is used for the balancing angle control because the integral (I) action has unfriendly effects on the case of balancing control of TWMR due to the accumulated errors and the PID controller is used for the position control. The angular position and position errors are regulated through the parameters for each controller. The control signals \( V_{a1} \) and \( V_{a2} \) as shown in Fig. 5 can be represented as in Eq. (22) and Eq. (23), respectively

\[
V_{a\theta} = \left( K_{p\theta} + K_{d\theta} s \right) \left( \theta_d - \theta_p \right)
\] (22)

\[
V_{ax} = \left( K_{px} + \frac{K_{ix}}{s} + K_{dx} s \right) \left( x_d - x_p \right)
\] (23)

\[
V_a = V_{a\theta} + V_{ax}
\] (24)

where \( s \) is the Laplace variable. There are many methods of choosing suitable values of the three gains to achieve the satisfied system performance. In this study, the Ziegler – Nichols approach is used to design both PD and PID controller to achieve a desired system performance.

Reference Model

Explicit position, velocity, and acceleration set point signals are created using the reference model, which is described by the transfer function

\[
H_{\text{ref}} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\] (25)

The parameters of the reference model are chosen such that the higher order dynamics of the system will not be excited [7]. This leads to the choice of \( \omega_n = 10 \text{ [rad/s]} \) and \( \zeta = 0.7 \), such that:

\[
A_m = \begin{bmatrix} 0 & 1 \\ -2\zeta \omega_n & -\omega_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -100 & -20 \end{bmatrix}
\] (26)

\[
B_m = \begin{bmatrix} 0 \\ \omega_n \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \end{bmatrix}
\] (27)

B. Adaptive PID Control System

For purposes of comparison, the process is repeated using an adaptive control structure, as shown in Fig. 6. The balancing angle and the position of the TWMR are controlled separately by two adaptive controllers by replacing two corresponding linear controllers indicated in Fig. 5.

Reference Model

State Variable Filter

As mentioned in Section II, the derivative of the error can be created using a state variable filter. The parameters of this state variable filter are chosen in such a way that the parameters of the reference model can vary without the need to change the parameters of the state variable filter every time. The parameters are chosen as: \( \omega_e = 100 \text{ [rad/s]} \), and \( \zeta = 0.7 \), then
Adaptive Controllers based on MRAS

Follow Eq. (11) and Eq. (12) the complete adaptive laws in integral form for the balancing angle controller are

\[ K_{p\theta} = \alpha_{p\theta} \int [(P_{21\theta} e_{1\theta} + P_{22\theta} e_{2\theta}) e_{\theta}] dt + K_{p\theta}(0) \]  (30)
\[ K_{d\theta} = \alpha_{d\theta} \int [(P_{21\theta} e_{1\theta} + P_{22\theta} e_{2\theta}) \dot{\theta}_p] dt + K_{d\theta}(0) \]  (31)

For the position controller

\[ K_{px} = \alpha_{px} \int [(P_{21x} e_{1x} + P_{22x} e_{2x}) e_{x}] dt + K_{px}(0) \]  (32)
\[ K_{dx} = \alpha_{dx} \int [(P_{21x} e_{1x} + P_{22x} e_{2x}) \dot{x}_p] dt + K_{dx}(0) \]  (33)
\[ K_{lx} = \alpha_{lx} \int [(P_{21x} e_{1x} + P_{22x} e_{2x})] dt + K_{lx}(0) \]  (34)

In the form of the adjustment laws, \( P_{21\theta}, P_{22\theta}, P_{21x}, P_{22x} \) are elements of the \( P_{\theta} \) and \( P_{x} \) matrices, obtained from the solution of the Lyapunov equations indicated in Eq. (35) and Eq. (36), respectively

\[ A_{m\theta} P_{\theta} + P_{\theta}^T A_{m\theta} = -Q_{\theta} \]  (35)
\[ A_{mx} P_{x} + P_{x}^T A_{mx} = -Q_{x} \]  (36)

where \( Q_{\theta} \) and \( Q_{x} \) are positive definite matrices and \( A_{m\theta} \) and \( A_{mx} \) are taken from Eq. (26), this yields

\[ A_{m\theta} = \begin{bmatrix} 0 & 1 \\ -\omega_{n\theta}^2 & -2z\omega_{n\theta} \end{bmatrix} \]  (37)
\[ A_{mx} = \begin{bmatrix} 0 & 1 \\ -\omega_{nx}^2 & -2z\omega_{nx} \end{bmatrix} \]  (38)

with \( \varepsilon_{\theta} = \theta - \theta_p \), \( \varepsilon = x - x_p \), \( \alpha_{p\theta}, \alpha_{d\theta}, \alpha_{px}, \alpha_{dx} \) are adaptive gains.

C. Simulation Results

The parameter values of the considered two-wheel mobile robotic are as follows: \( k_m = 0.28 \) [N.m/A], \( k_e = 0.67 \) [Vs/rad], \( R = 5.25 \) [Ohms], \( l = 0.1 \) [m], \( M_p = 1.2 \) [kg], \( r = 0.05 \) [m], \( I_p = 0.01 \) [kg.s²], \( J_w = 0.015 \) [kg.s²], \( M_w = 0.1 \) [kg], \( g = 9.81 \) [m/s²]. The simulation model of the proposed control structure is shown in Fig. 7.

The PD controller with fixed parameters for controlling the balancing angle is archived by setting \( K_{p\theta} = 21 \) and \( K_{d\theta} = 5 \) while for the PID controller for position with \( K_{px} = 29 \), \( K_{dx} = 20 \) and \( K_{dxz} = 20 \).

The adaptive PD controller for the balancing angle is archived by setting \( \alpha_{p\theta} = 3000 \), \( \alpha_{d\theta} = 1500 \), \( Q_{\theta} = [350 \ 100 \ 100 \ 250] \) while the adaptive PID controller for position with \( \alpha_{px} = 1800 \), \( \alpha_{dx} = 1600 \) and \( \alpha_{dxz} = 1500 \), \( Q_{x} = [250 \ 100 \ 100 \ 150] \).

The parameters of reference models are: \( \omega_{n\theta} = \omega_{nx} = \omega_n = 10 \) and \( z = 0.7 \).

Figure. 8. (a) Responses of the PID control system without disturbance

Figure. 8. (b) Responses of the adaptive PID control system without disturbance

Figure. 9. (a) Responses of the PID control system with disturbance
Comparison of balancing control simulation between the conventional PID controller and the adaptive PID controller based on MRAS is presented. Without disturbance both controllers are able to balance the system (see Fig. 8.a, Fig. 8.b). Responses of both controllers are almost the same. However, it is clearly that the tracking errors for the linear PID controllers due to a large enough disturbance are larger than those by the adaptive PID controllers (see Fig. 9.a and Fig. 9.b). The adaptive gains automatically reach stationary values (see Fig. 10.a and Fig. 10.b).

V. CONCLUSION

In this paper, the conventional PID controller and the adaptive PID controllers are successfully designed to balance the two-wheel mobile robot based on the inverted pendulum model under disturbances. The simple adaptive control schemes based on Model Reference Adaptive Systems (MRAS) algorithm are developed for the asymptotic output tracking problem with changing system parameters and disturbances under guaranteeing stability. Simulations have been carried out to investigate the effect of changing the external disturbance forces on the system. Based on the simulation results and the analysis, a conclusion has been made that both conventional and adaptive controllers are capable of controlling the angular and position of the non-linear robot. However, the adaptive PID controller has better performance compared to the conventional PID controller in the sense of robustness against disturbances.

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