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Feedforward and Feedback Kinematics Controller for Wheeled Mobile Robot Trajectory Tracking

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***Abstract*—In this paper, trajectory tracking of a differential drive nonholonomic mobile robot is presented. In addition to the complex relations of the control system, the nonholonomic system adds complexity to the system which has been solved using the feed-forward and feedback fuzzy logic controllers. An innovative scheme has been developed to track the reference trajectory in the presence of model uncertainties and disturbances. The performance comparison of the proposed controller is done with the standard backstepping controller and the simulation results show that the developed controller is best suited for the tracking trajectory problems.**

***Index Terms*—WMR; trajectory tracking; kinematics; fuzzy logic**

I. INTRODUCTION

In the recent years, the tracking and control of wheeled mobile robots (WMR) has received lots of attentions from the robotic community because of many applications includes security, transportation, inspection and exploration. Due to the mechanical design and configuration, the WMR is classified into two categories: holonomic and nonholonomic. Holonomic robots are those in which the controllable degree of freedom is equal to total degree of freedom, whereas nonholonomic robots have less controllable degree of freedom compare to total degree of freedom and have restricted mobility [1]-[3]. Therefore, the controlling and trajectory tracking of nonholonomic WMR has been considered more challenging problem and many researchers proposed various controllers [3].

In case of trajectory tracking, the WMR has to follow a reference path either predefined or obtained from the other WMR. The standard technique to solve this problem is to design a kinematics controller which generates desired velocities based on position errors compare to reference position. Various researchers have proposed the kinematic controller for trajectory tracking based on “perfect velocity tracking” using e.g. fuzzy control [4], neural network [5], adaptive feedback [6], input-output feedback linearization [7], [8] and back-stepping [1], and so on. However, the presented works have poor transient and steady state characteristics in the presence of disturbance and model uncertainties. In addition, most of the works use simple trajectories with constant linear and angular velocities. In this research work, an innovative kinematic controller is proposed for trajectory following to generate the kinematic velocities. The controller structure uses feed-forward and feedback controllers. The feed-forward controller is designed based on reference positions and velocities along with error propagation model, whereas the feedback controller is designed using fuzzy logic. The effectiveness of the deigned controller is validated by computer simulations. In addition, the performance and effectiveness of the proposed scheme is compared with standard kinematic controller. The lamniscate curve reference trajectory is used for non-constant linear and angular velocities.

The main motivation of this work is to design a controller which can model the system with poor transient and steady state error in the presence of disturbances and model uncertainties. An innovative feed-forward and feedback controller is designed to cater above mentioned issues. Most importantly a difficult trajectory is chosen to test the controller so that the effectiveness of the controller may be verified. A comprehensive program using MATLAB (Simulink) is developed to test the feasibility of entire work which would be presented later. In this paper, a Fuzzy logic controller is used which is more dynamic and can model uncertainties in a better fashion. The novelty of this work is its comparison with the back stepping controller and the results shows the effectiveness of the proposed controller.

The breakup of the further work is structured as follows: Section II discusses the modeling of the WMR. Section III presents the designing of feed-forward and feedback kinematic controller. The simulation results are presented in Section IV. Finally Section V provides the conclusion.

II. KINEMATIC MODELING OF WMR

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The first order kinematic model of unicycle mobile robot as shown in Fig. 1, with the presence of uncertainties, disturbances and non-holonomic constrained can be defined as [9]

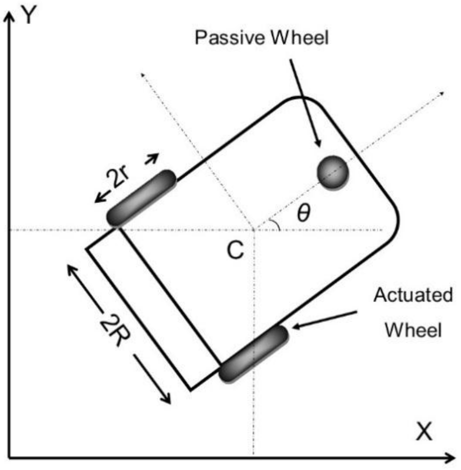

Figure. 1. Nonholonomic model of unicycle mobile robot

$$\dot{\boldsymbol{q}}(t)=\begin{bmatrix}\dot{x}\ (t)\\ \dot{y}\ (t)\\ \dot{\theta}\ (t)\end{bmatrix}=\begin{bmatrix}\cos\theta(t) & 0\\ \sin\theta\ (t) & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v(t)\\ w(t)\end{bmatrix}+\begin{bmatrix}\Delta\cos\theta(t) & 0\\ \Delta\sin\theta\ (t) & 0\\ 0 & \Delta\end{bmatrix}\begin{bmatrix}v(t)\\ w(t)\end{bmatrix}+d(t)\quad t\geq 0 \qquad (1)$$

$$\dot{\boldsymbol{q}}(t)=\boldsymbol{S}(\boldsymbol{q})\boldsymbol{v_k}(t)+\Delta\boldsymbol{S}(\boldsymbol{q})\boldsymbol{v_k}(t)+d(\mathrm{t})\quad t\geq 0$$

where $x(t)$ and $y(t)$ provides the centre point position of vehicle at time t, $\theta(t)$ is the orientation of the robot, $v(t)$ is the linear velocity, $w(t)$ is the angular velocity, $\boldsymbol{S}(\boldsymbol{q})$ is a Jacobian matrix which transforms the input velocities to robot velocities, $\Delta\boldsymbol{S}(\boldsymbol{q})$ representing the unmodeled dynamics or structural variations and d(t) is the disturbances and $\boldsymbol{q}(t)$ is a vector as shown below.

$$\boldsymbol{q}(t)=[x(t)\quad y(t)\quad \theta(t)]^T \qquad (2)$$

In general the uncertainties and disturbances are assumed to be bounded, and define as $\|\Delta\boldsymbol{S}(\boldsymbol{q})\|\leq\delta_1$ and $\|\mathrm{d(t)}\|\leq\delta_2$, where δ_i is the positive constant.

The kinematic control design objective is to follow the time varying reference trajectory defined as

$$\boldsymbol{q_r}(t)=\ [x_r(t)\quad y_r(t)\quad \theta_r(t)]^T \qquad (3)$$

The kinematic control is designed in such a way so as bring the position and orientation error to zero as $t\rightarrow\infty$ with arbitrary initial error. The posture tracking error of the mobile robot q_e is defined as follows

$$\boldsymbol{q_e}(t)=\begin{bmatrix}e_x(t)\\ e_y(t)\\ e_\theta(t)\end{bmatrix}=\begin{bmatrix}cos\,\theta(t) & sin\,\theta(t) & 0\\ -\,sin\,\theta\ (t) & cos\,\theta\ (t) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_r(t)-x(t)\\ y_r(t)-y(t)\\ \theta_r(t)-\theta(t)\end{bmatrix} \qquad (4)$$

The derivative of eq. (1) gives the posture tracking error as

$$\dot{\boldsymbol{q}}_{\boldsymbol{e}}(t)=\begin{bmatrix}\dot{e}_x(t)\\ \dot{e}_y(t)\\ \dot{e}_\theta(t)\end{bmatrix}=\begin{bmatrix}-v(t)+w(t)e_y(t)+v_r(t)\,cos\,e_\theta(t)\\ -w(t)e_x(t)+v_r(t)\,sin\,e_\theta(t)\\ -w(t)+w_r(t)\end{bmatrix} \qquad (5)$$

where $[v_r(t)\quad w_r(t)]^T$ are the reference linear and angular velocities and define as

$$v_r(t)=(\sqrt{\dot{x}_r^2(t)+\dot{y}_r^2(t)}\,) \qquad (6)$$

and
$$w_r(t)=\frac{\dot{x}_r(t)\ddot{y}_r(t)-\dot{y}_r(t)\ddot{x}_r(t)}{\dot{x}_r^2(t)+\dot{y}_r^2(t)} \qquad (7)$$

III. FEED-FORWARD CONTROLLER DESIGN

After defining the posture tracking error, next step is to design a controller that forces the posture tracking error to zero. Therefore, the kinematic controller for precisely following the reference trajectory is defined

$$\boldsymbol{v_k}(t)=\begin{bmatrix}v_k(t)\\ w_k(t)\end{bmatrix}=\begin{bmatrix}v_{ff}(t)\\ w_{ff}(t)\end{bmatrix}+\begin{bmatrix}v_{fb}(t)\\ w_{fb}(t)\end{bmatrix} \qquad (8)$$

where $[v_k(t)\quad w_k(t)]^T$ are the linear and angular velocities generated by kinematic controller. Whereas $[v_{fb}(t)\quad w_{fb}(t)]^T$ are the feed backward linear and angular velocities and $[v_{ff}(t)\quad w_{ff}(t)]^T$ are the feed-forward linear and angular velocities.

Feed-forward control velocities are calculated using reference velocities and error propagation mechanism. The calculated value of the desired feed-forward control action is used along with the delayed velocity error as

$$\boldsymbol{v_{ff}}(t)=\begin{bmatrix}v_{ff}(t)\\ w_{ff}(t)\end{bmatrix}=\begin{bmatrix}v_r(t)\,cos\,e_\theta(t)+\gamma_1 v_e(t-1)\\ w_r(t)+\gamma_2 w_e(t-1)\end{bmatrix} \qquad (9)$$

where $\gamma_i$is the positive constant,$v_e(t-1)=v_k(t-1)-v_{fb}(t-1)$ and $w_e(t-1)=w_k(t-1)-w_{fb}(t-1)$. The feed-forward control action is computed based on velocity error $\boldsymbol{v_e}(t)$ generated by the previous incorrect control signal.

IV. FEEDBACK FUZZY CONTROLLER DESIGN

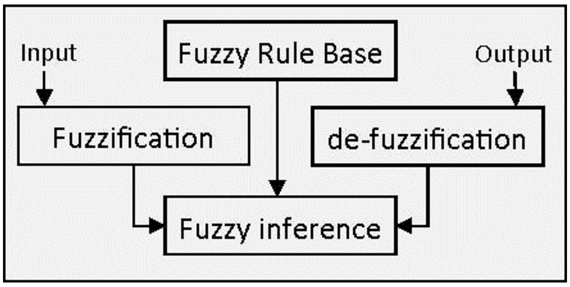

Figure 2 Typical fuzzy logic control system

Fuzzy control is very useful and effective tool for approximating unknown system dynamics [10]. It offers human reasoning abilities to reject external disturbances and to capture model uncertainties which is difficult to model using precise mathematical models. The main goal of Fuzzy Logic Controller (FLC) is to generate the feedback linear and angular velocities in such a way that the error between the reference trajectory and the robot trajectory is forced to zero so that the robot precisely tracks the desired trajectory.

The general configuration of the fuzzy logic controller (FLC), which is divided into four main parts [10]: fuzzifier, fuzzy inference engine, fuzzy rule base and defuzzifier as shown in Fig. 2. In this paper, Multiple Input Single Output (MISO) fuzzy systems are consider which is expressed as $f(x): U \subset \mathbb{R}^n \rightarrow V \subset \mathbb{R}$, where U is the system input and V is the system output as follows

$$v_{fb}(t) = k_v(FLC(e_x(t), \int e_x dx)) = k_v \tilde{v}_{fb}(t) \quad (10)$$

and

$$w_{fb}(t) = k_w(FLC(e_y(t), e_\theta(t))) = k_w \tilde{w}_{fb}(t) \quad (11)$$

where k_v and k_v are the output gains. The fuzzifier maps the observed crisp system input $U \subset \mathbb{R}^n$ to the fuzzy sets define in U, whereas the defuzzifier performs the mapping from fuzzy sets to the crisp output space V. The mapping decision is based on the rule base and membership functions. The membership functions are defined in linguistic terms such as positive large (PL), positive medium (PM), positive small (PS), zero (Z), negative small (NS), negative medium (NM), and negative large (NL) to inputs and the output variable. The fuzzy rule table for both linear and angular velocities are shown in Table I and Table II respectively. The fuzzy rule base works on the mechanism of IF – THEN rules. In this paper, two fuzzy logic controllers are designed to estimate the linear feedback velocity $\tilde{v}_{fb}(t)$ and angular feedback velocity $\tilde{w}_{fb}(t)$. The fuzzy rule base comprise on 49 rules defined as

$$R_i\text{: if } x_1 \text{ is } A_1^i \text{ and } x_2 \text{ is } A_2^i \text{ then } u \text{ is } B^i$$

where i = 1, 2,…M, M=49, x_1 and x_2 are input variables of FLC, u is the output variable of the FLC, and A_1^i, A_2^i *and* B^i are linguistic terms defined by the membership functions $\mu_{A_1^i}(x_1)$, $\mu_{A_2^i}(x_2)$ and $\mu_{B^i}(u)$ respectively. The FLC inferred the output on the basis of fuzzy rule base and can be defined as

$$f(x) = \frac{\sum_{i=1}^{M} \bar{u}^i \left(\prod_{j=1}^{2} \mu_{A_j^i}(x_i) \right)}{\sum_{i=1}^{M} \left(\prod_{j=1}^{2} \mu_{A_j^i}(x_i) \right)} = \emptyset^T \boldsymbol{\xi}(x) \quad (12)$$

where $f(x)$ is defined for product type of inference, centroid defuzzification and Gaussian membership functions, $x_i = [x_1\ x_2]^T$ is the input vector, $\bar{u}^i$ is the output at which $\mu_{A_j^i}(x_i)$ attain maximum value, $\emptyset = [\emptyset_1, \emptyset_2, \dots, \emptyset_M]^T$ is the adjustable parameter vector and $\boldsymbol{\xi}(x) = [\xi_1(x), \xi_2(x), \dots, \xi_M(x)]^T$ is the vector of the fuzzy basis function and $\mu_{A_j^i}(x_i)$ is the Gaussian membership function define as

$$\mu_{A_j^i}(x_i) = a_j^i \left[-\frac{1}{2} \left(\frac{x_i - \overline{x_j^i}}{\sigma_j^i} \right)^2 \right] \quad (13)$$

where a_j^i, $\overline{x_j^i}$ and $\overline{\sigma_j^i}$ are real defined in the range of $0 < a_j^i \leq 1$. The inputs of first FLC are position error e_x and its integral $\int e_x dx$ which outputs the linear feedback velocity $\tilde{v}_{fb}$. Second fuzzy controller uses the orientation errors e_y and e_θ as an input and generates the angular feedback velocity w_{fb} as an output. The complete control structure of the proposed controller is shown in Fig. 3.

TABLE I: FUZZY TABLE for FLC 1 ($\tilde{v}_{fb}$)

$e_x / \int e_X$	NL	N	NS	Z	PS	P	PL
NL	PL	PL	PL	P	P	PS	Z
N	PL	PL	P	P	PS	Z	NS
NS	PL	P	P	PS	Z	NS	N
Z	P	P	PS	Z	NS	N	N
PS	P	PS	Z	NS	N	N	NL
P	PS	Z	NS	N	N	NL	NL
PL	Z	NS	N	N	NL	NL	NL

TABLE II: FUZZY TABLE FOR FLC 1 ($\tilde{v}_{fb}$)

e_y / e_θ	NL	N	NS	Z	PS	P	PL
NL	PL	PL	PL	P	P	PS	Z
N	PL	PL	P	P	PS	Z	NS
NS	PL	P	P	PS	Z	NS	N
Z	P	P	PS	Z	NS	N	N
PS	P	PS	Z	NS	N	N	NL
P	PS	Z	NS	N	N	NL	NL
PL	Z	NS	N	N	NL	NL	NL

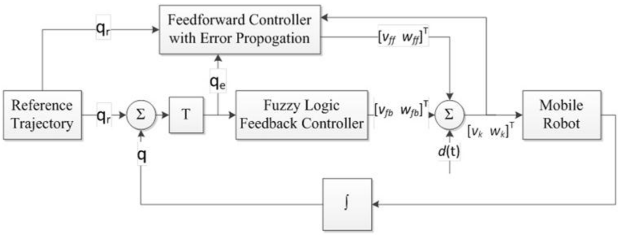

Figure 3. Block diagram of the proposed controller

V. RESULTS AND DISCUSSION

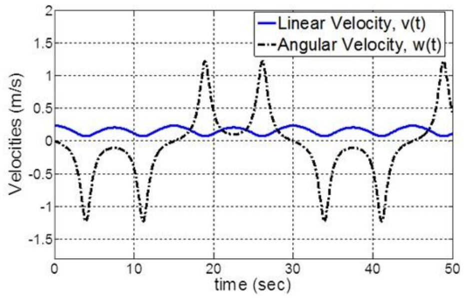

Figure 4. Reference linear and angular velocities

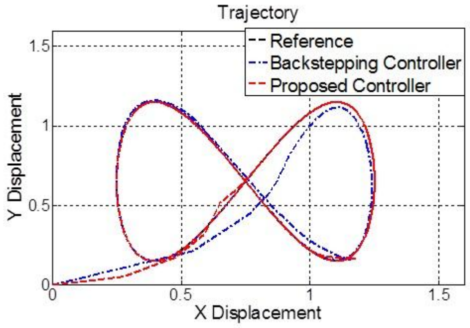

Figure 5. Tracking performance analysis lamniscate curve trajectory with disturbances between Backstepping and Fuzzy control system.

In this section, the performance of the proposed controller is evaluated on wheeled Mobile Robot (WMR) for trajectory tracking. The effectiveness of the proposed controller is also compared with the standard backstepping controller. The lamniscate curve is selected as reference trajectory which provides constantly changing linear and angular velocities as shown in Fig. 4. The trajectory tracking performance of WMR in the presence of output disturbances and model uncertainties using proposed controller and backstepping kinematic controller is shown in Fig. 5. Initial values for both controllers as assume as $\boldsymbol{q_r}(t) = [0 \quad 0 \quad 0]^T$ whereas the disturbance signals are shown in Fig. 6. As it can be seen, the proposed controller shows better tracking performance and precisely follows the reference trajectories. The x and y position errors plots for both controllers are shown in Fig. 7. The proposed controller exhibits better transient performance and shows fast error convergence which indicates the better responsiveness for WMR.

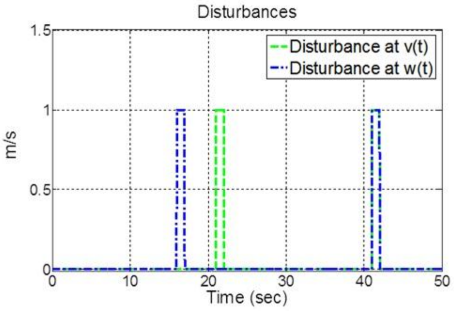

Figure 6. Disturbance signals

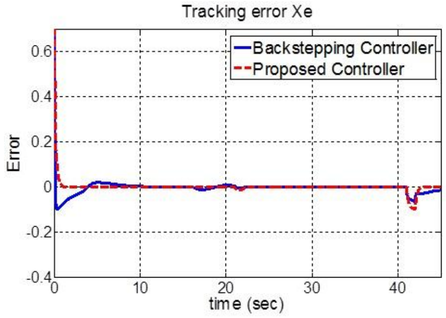

Figure 7a. Comparison of tracking error in X direction between backstepping controller and proposed controller

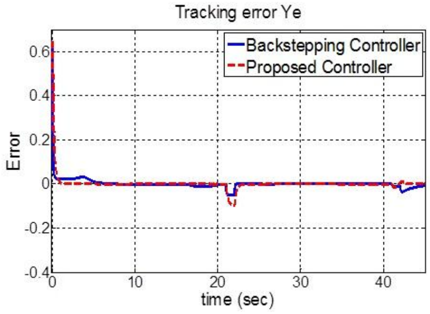

Figure 7b: Comparison of tracking error in Y direction between backstepping controller and proposed controller

VI. CONCLUSION

In this paper, feed-forward and feedback kinematic controller for unicycle mobile robot trajectory tracking is presented. Feed-forward control velocities are calculated using reference velocities and error propagation mechanism. The calculated value of the desired feed-forward control action is used along with the delayed velocity error. Whereas, fuzzy logic control is used to compute the feedback velocities. The performance comparison of the proposed controller is done with standard backstepping controller. It is shown in the simulation results that the proposed controller precisely follows the reference trajectory and shows better transient performance and zero steady state error.

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