Design of LQG Controller Using Operational Amplifiers for Motion Control Systems

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***Abstract*—This paper proposes additional steps to the traditional design procedures for continuous control systems using operational amplifiers. Designing an analog Linear Quadratic Gaussian (LQG) controller is selected as a case study. The controller in the s-domain is firstly designed based on the mathematical model of the plant to be controlled, and then simulated and adjusted. Next, the plant and the controller in the s-domain will be converted to equivalent corresponding continuous electronic circuits using operational amplifiers and continued for simulating. The main purpose of these proposed additional steps is to confirm that converting the controller from s-domain to corresponding analog electronic circuits using operational amplifiers is correct or not. After that, the controller will be implemented and applied to the real setup. For a good design, the simulation results of the resulting controlled system in s-domain, also in equivalent analog electronic circuits, and experimental results in the real setup are almost the same.**

***Index Terms*—linear quadratic gaussian (LQG), linear quadratic regulator (LQR), linear quadratic estimator (LQE), motion control systems, operational amplifiers (Op-Amp)**

I. Introduction

In analog control systems, the controllers use continuous devices and circuits. In digital control systems, the controllers use digital devices and circuits. The choice between analog or digital control systems depends on the application requirements. The most important advantage of analog control systems overs digital control systems is that in the analog control systems, any change in either reference inputs or system disturbances is immediately sensed, and the controllers adjust their outputs accordingly [1]. However, the analog controllers are recommended for use in the non-sophisticated systems. In practice, most analog control systems have relied on operational amplifiers as essential building blocks [2].

Operational Amplifiers are among the most widely used electronic devices today. An operational amplifier is a DC-coupled high-gain electronic voltage amplifier with a differential input and, usually, a single-ended output [2]. The instrumentation amplifiers using operational amplifiers provide great benefits to the designers. The mathematical operations such as inversion, addition, subtraction, integration, differentiation, and multiplication can be performed by using operational amplifiers [2]. Such that many practical continuous control systems can be constructed using operational amplifiers. Electronic circuits using operational amplifiers can be used to compare to most physical systems such that analog electronic simulation was effectively used in the research and development of electro-mechanical systems.

Model-based design is a mathematical and visual method of addressing problems associated with designing complex control, signal processing and communication systems [3]. In model-based design of control systems, development is constructed in these five main steps:

- Modeling the system;
- Analyzing and synthesizing a controller for the system;
- Simulating the resulting controlled system;
- Validating the simulation results;
- Implementing the controller.

The mathematical model is used to identify dynamic characteristics of the system model. A controller can then be synthesized based on these characteristics. The time response of the dynamic system is investigated through offline simulation and real-time simulation

It is clear that when Step 4 has done the resulting controlled system is in **s-**domain. However, in Step 5 the analog controller is implemented using operational amplifiers. The problem is that if correctly follow the above procedure, we cannot confirm converting the controller in s-domain to equivalent analog electronic circuits using operational amplifiers is correct or not.

In this paper we propose additional steps between Step 4 and Step 5. First of all the controller and also the plant are converted to analog electronic circuits using operational amplifiers. Next, the equivalent continuous electronic circuits are simulated. This step will be shown in Section IV. The same inputs, the response outputs in Step 3 and in the proposed steps are almost the same. Such that after doing with the proposed steps we can retest the final resulting continuous controller.

This paper is organized as follows: First, the dynamic characteristic of the setup is analyzed in Section II. In Section III, design of LQG controller is shown. Analog LQG controller using operational amplifiers is introduced in Section IV. Simulation and experimental tests are

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performed in Section V. Finally, conclusions are given in Section VI.

II. Mathematical Model of The Setup

The setup (see Fig. 1) is designed for the purpose of testing the results of the controller for linear and non-linear systems. It consists of a slider which can move back and forth over a rail. A DC motor, rail and slider are fixed on a frame. The parameters of this setup are shown in Table I [4].

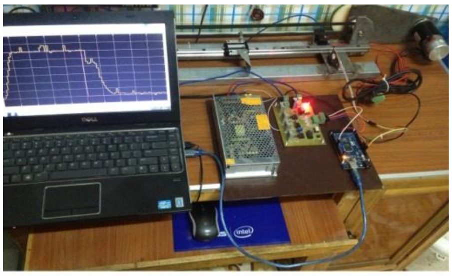

Figure 1. The configuration of the setup

The mechanical part of the setup is designed mimicking printer technology. For this process, a computer based control system has been implemented with software generated by MATLAB. This setup is also suitable for the analog electronic circuits based control systems.

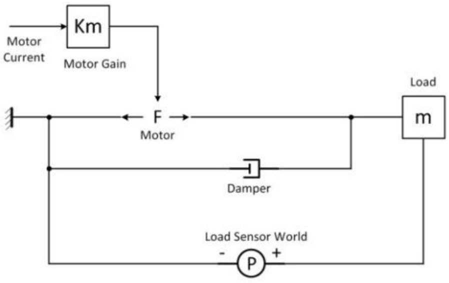

Figure 2. Second order model of the setup

TABLE I. Plant Parameters of The Setup

Elements	Parameters	Labels	Values
Motor-Gain	Motor constant	k_m	8.5 N/A
Motor-Inertia	Inertia of the motor	i_m	2e-5 kg
Load	Mass of the slider	m	0.35 kg
Belt-Flex	Spring constant	s_F	80 kN/m
	Damping in belt	d_E	1 Ns/m
Damper	Viscous friction	d	8 Ns/m
	Coulomb friction	d_C	0.75 N

The Damper component represents a viscous and Coulomb friction. Coulomb friction always opposes relative motion and is simply modeled as

$$F_c = d_c.tanh(1000.\dot{x}) \quad (1)$$

where d_c is the Coulomb parameter of the Damper element, $\dot{x}$ is the velocity of the load. Viscous friction is proportional to the velocity. It is normally described as

$$F_v = d.\dot{x} \quad (2)$$

where d is the viscous parameter of the Damper element.

The mathematical expression for the combination of viscous and Coulomb friction is

$$F = F_v + F_c = d.\dot{x} + d_c.tanh(1000.\dot{x}) \quad (3)$$

If the non-linear Coulomb friction part is disregarded, the model only contains linear components. In this case we get a linear process model. A second order approximation model is obtained with a state space description as given in (4) [4].

$$\begin{bmatrix}\dot{v}_L \\ \dot{x}_L\end{bmatrix} = \begin{bmatrix}-\frac{d}{m} & 0 \\ 1 & 0\end{bmatrix}\begin{bmatrix}v_L \\ x_L\end{bmatrix} + \begin{bmatrix}\frac{k_m}{m} \\ 0\end{bmatrix}F \quad (4)$$

$$y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}v_L \\ x_L\end{bmatrix} + [0]F$$

where v_L is the velocity of the load; x_L is the position of the load; and F is applied force on the process. When we mention the nonlinear friction term of the Damper element then:

$$\begin{bmatrix}\dot{v}_L \\ \dot{x}_L\end{bmatrix} = \begin{bmatrix}-\frac{d}{m} & 0 \\ 1 & 0\end{bmatrix}\begin{bmatrix}v_L \\ x_L\end{bmatrix} + \begin{bmatrix}-\frac{d_c}{m}sgn(v_L) \\ 0\end{bmatrix} + \begin{bmatrix}\frac{k_m}{m} \\ 0\end{bmatrix}F \quad (5)$$

The second order model of the setup is given in Fig. 2.

III. Design of LQG Controller

A. Linear Quadratic Regulator

In the theory of optimal control, the Linear Quadratic Regulator (LQR) is a method of designing state feedback control laws for linear systems that minimize a given quadratic cost function [5], [6].

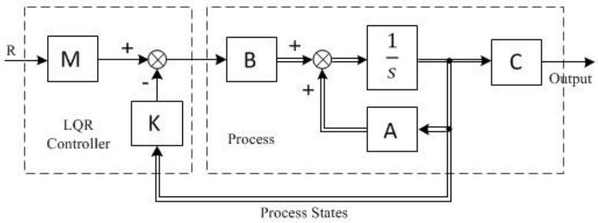

Figure 3. Principle of state feedback

In the so called Linear Quadratic Regulator, the term "Linear" refers to the system dynamics which are described by a set of linear differential equations and the term "Quadratic" refers to the performance index which is described by a quadratic functional. The aim of the LQR algorithm is finding an appropriate state-feedback controller. The design procedure is implemented by choosing the appropriate positive semi-definite weighting matrix Q_R and positive definite weighting matrix R_R. The advantage of the control algorithm is that it provides a robust system by guaranteeing stability margins.

An LQR however requires access to system state variables. A state feedback system is depicted in Fig. 3 [7]. The internal states of the system are fed back to the controller, which converts these signals into the control signal for the process. In order to implement the deterministic LQR, it is necessary to measure all the states of the system. This can be implemented by means of sensors in the system. However, these sensors have noise associated with them, which means that the

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measured states of the system are not clean. That is, controller designs based on LQR theory fail to be robust to measurement noise. In addition, it may be difficult or too expensive to measure all states.

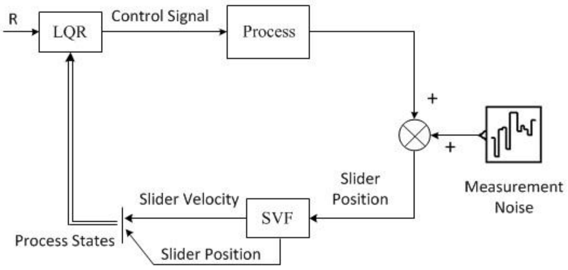

Figure 4. Clean state feedback of the process is obtained by using State Variable filter

State Variable Filters (SVFs) can be used to make a complete state feedback (see Fig. 4) [7]. When the noise spectrum is principally located outside the band pass of the filter, measurement noise can be suppressed by properly choosing ω of the filter. For example, in the experimental setup, information about the position is measured with a lot of noise at any time instant. The SVFs remove the effects of the noise and produce a good estimate of the positions and velocities

However, SVFs cause phase lags. The phase lags can be reduced by means of increasing the omega of the SVF. In practice, the choice of the omega is a compromise between the phase lag and the sensitivity for noise.

B. *Linear Quadratic Estimator*

Another way to estimate the internal state of the system is by using a Linear Quadratic Estimator (LQE) (see Fig. 5). In control theory, the LQE is most commonly referred to as a Kalman filter or an Observer [8], [9].

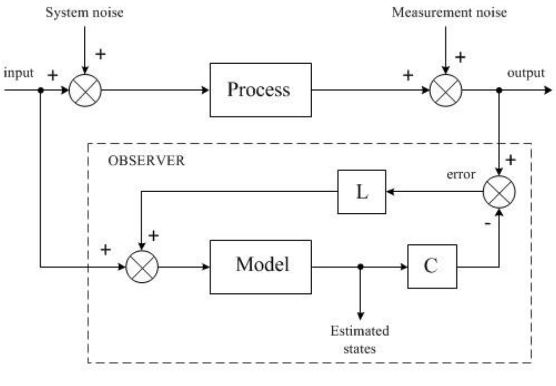

Figure 5. Principle of an observer

The Kalman filter is a recursive estimator. This implies that to compute the estimate for the current state, the estimated state from the previous time step and the current measurement are required. The Kalman filter is implemented with two distinct phases: i - The prediction phase, the estimate from the previous step is used to create an estimate of the current state; ii - The update phase uses measurement information from the current step to refine this prediction to arrive at a new estimate. A Kalman filter is based on a mathematical model of a process. It is driven by the control signals to the process and the measured signals. When we use Kalman filters or observers disturbances at the input of the process are mostly referred to as "system noise" as in Fig. 5.

Its output is an estimate of the states of the system including the signals that cannot be measured directly. The Kalman filter provides an optimal estimate of the states of the system in the presence of measurement noise and system noise. In order to obtain optimality the following conditions must be satisfied [10], [11]: i - Structure and parameters of process and model must be identical; ii - Measurement and system noise have average zero and known variance. The LQE design determines the optimal steady-state filter gain L based on linear parameters of the process, the system noise covariance Q_E and the measurement noise covariance R_E. The states of the model will follow the states of the plant, depending on the choice of Q_E and R_E.

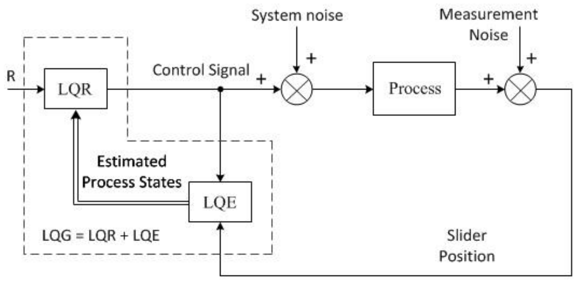

Figure 6. LQG explanation

C. *Linear Quadratic Gaussian*

Linear Quadratic Gaussian (LQG) is simply the combination of a Linear Quadratic Regulator (LQR) and a Linear Quadratic Estimator (LQE) [8]. This means that LQG is a method of designing state feedback control laws for linear systems with additive Gaussian noise that minimizes a given quadratic cost function. The control configuration is shown in Fig. 6. The design of the LQR and LQE can be carried out separately. LQG enables us to optimize the system performance and to reduce measurement noise. The LQE yields the estimated states of the process. The LQR calculates the optimal gain vector and then calculates the control signal.

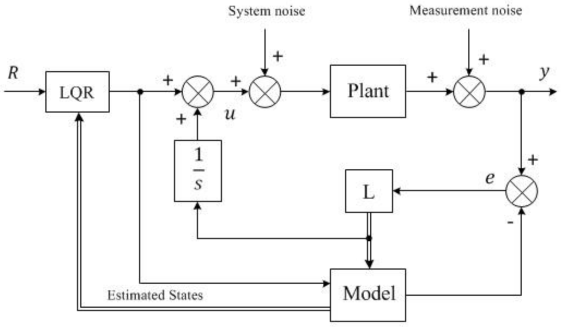

Figure 7. Addition of integrator to LQG

However, in state feedback controller designs reduction of the tracking error is not automatically realized. In motion control systems, Coulomb friction is the major non-linearity, which causes a static error. This problem can be solved, by introducing an additional integral action to the LQG control structure. The difference between process and model is integrated, instead of the error between reference and process output. Adding the integral term to the LQG control structure leads to the system indicated in Fig. 7.

LQG design

We consider the LQG design based on the 2nd order mathematical model of the plant to be controlled [4], [7].

Continuous LQR design

We consider a continuous-time linear plant described by

$$\begin{cases} \dot{x}_p = A_p x_p + B_p u \\ y_p = C_p x_p + D_p u \end{cases} \tag{6}$$

With a performance index defined as

$$j = \int_0^\infty (e^T Q_R e + u^T R_R u) dt \tag{7}$$

In (6) and (7) A_p and B_p are continuous state matrices of the plant to be controlled, x_p denotes the state of the plant, e is the tracking error, u is the control signal, Q_R and R_R are matrices in the optimization criterion (Q_R is positive semi-definite weighting matrix and R_R is positive definite weighting). The optimal state feedback controller will be achieved by choosing a feedback vector

$$K_{LQR} = R_R^{-1} B_p^T P_R \tag{8}$$

in which P_R is found by solving the continuous time algebraic Riccati equation

$$A_p^T P_R + P_R A_p - P_R A_p R_R^{-1} B_p^T P_R + Q_R = 0 \tag{9}$$

The output of the state feedback controller is

$$u = -K_{LQR} \hat{x}_p \tag{10}$$

where

$$\hat{x}_p = [\hat{x}_{p2} \quad \hat{x}_{p1}]^T, \tag{11}$$

$\hat{x}_{p1}$ and $\hat{x}_{p2}$ denote the states of the estimator (see Fig. 8).

The following parameters are used in the simulation:

$$A_p = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -22.9 & 0 \\ 1 & 0 \end{bmatrix}; B_p = \begin{bmatrix} 24.3 \\ 0 \end{bmatrix};$$

$$Q_R = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 1000 \end{bmatrix}; \; R_R = 0.1. \tag{12}$$

These values results in the following feedback controller gains

$$K_{LQR} = [K_d \quad K_p] = [2.1 \quad 100] \tag{13}$$

Continuous LQE design

The feedback matrix L_{LQE} yielding optimal estimation of the process states is computed as

$$L_{LQE} = C_p P_E R_E^{-1} \tag{14}$$

where P_E is the solution of the following matrix Riccati equation

$$A_p P_E + P_E A_p^T + Q_E - P_E C_p^T R_E^{-1} C_p P_E = 0 \tag{15}$$

in which A_p and C_p are continuous state matrices of the plant to be controlled, Q_E is the system noise covariance, and R_E the sensor noise covariance. The following settings were used:

$$A_p = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} -22.9 & 0 \\ 1 & 0 \end{bmatrix}; C_p = [0 \quad 1];$$

$$Q_E = \begin{bmatrix} 10 & 0.0 \\ 0.0 & 10 \end{bmatrix}; \; R_E = 10. \tag{16}$$

These values results in the following feedback controller gains

$$L_{LQE} = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 0.001 \\ 1.000 \end{bmatrix} \tag{17}$$

The results lead to the control structure shown in Fig. 8.

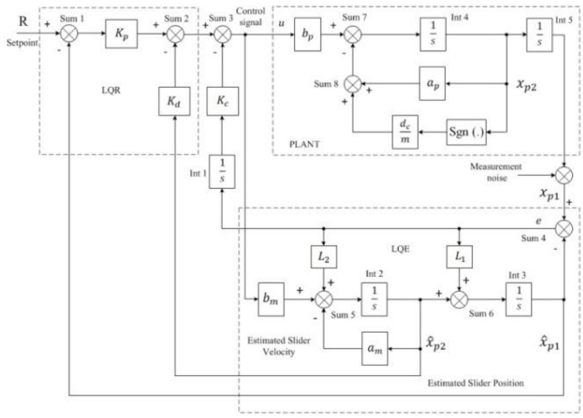

Figure 8. The setup with 2^{nd} order continuous LQG controller

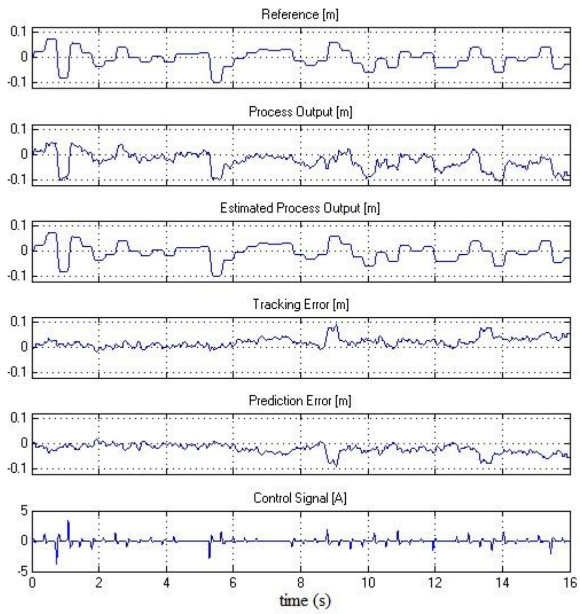

Figure 9. Control signal is insensitive for measurement noise

With the LQG regulator, noise on the measurements of the process has almost no influence on the system. This is illustrated in Fig. 9; the real position state (second line) and the position state error (fourth line) are corrupted by measurement noise, whereas, the estimated position state (third line) and the control signal (lowest line) are almost

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clean. The integral action is used to compensate the effect of the process disturbances (see Fig. 7 and Fig. 8). The gain of the integrator can be tuned manually, or it can be included in the solution of the Riccati equation [8]. By comparing two simulation results as indicated in Fig. 10, it is observed that when the integral action is used with $K_c = 1500$ the tracking error is decreased.

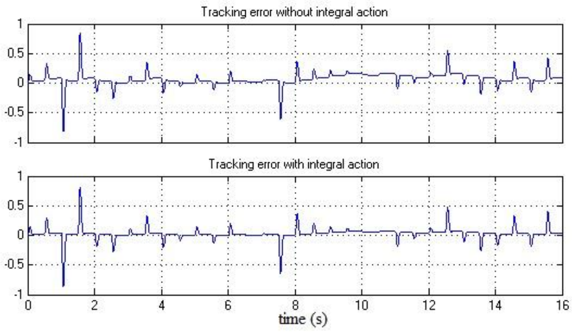

Figure 10. Tracking error without (first line) and with (second line) integral action

The estimator and feed-back controller may be designed independently. It enables us to compromise between regulation performance and control effort, and to take into account process and measurement noise. However, it is not always obvious to find the relative weights between state variables and control variables. Most real world control problems involve nonlinear models while the LQG control theory is limited to linear models. Even for linear plants, the mathematical models of the plants are subject to uncertainties that may arise from un-modeled dynamics, and parameter variations. These uncertainties are not explicitly taken into account in the LQG design.

IV. Design of Controlled System Using Operational Amplifiers

A. *Design* of *LQG Controller Using Operational Amplifiers*

Inverting and non-inverting amplifiers, weighted-sum adders, integrators, and differentiators are used to design analog controllers. From analog LQR and LQE blocks in s-domain as shown in Fig. 8 we can design corresponding analog electronic LQR and LQE circuits using operational amplifiers that indicated in Fig. 11 and Fig. 12, respectively. The controller and observer parameters with some explanations are listed in Table II and Table III.

TABLE II. Analog LQR Parameters

Symbols	Parameters	Notes
Sum 1	$R_1 = R_2 = R_3 = 10\ K$	Calculate the tracking error
K_p	$R_4 = 10\ K, WR_1$	Proportional gain $K_p = \frac{WR_1}{R_4}$
K_d	$R_7 = 10\ K, WR_2$	Derivative gain $K_d = \frac{WR_2}{R_7}$
Sum 2	$R_4 = R_5 = R_7 = 10\ K$	Calculate the control signal u

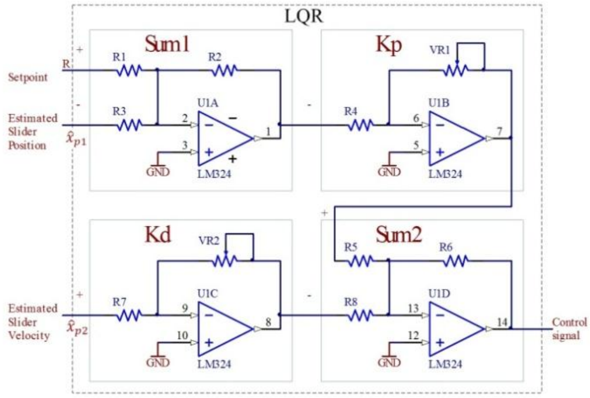

Figure 11. Analog LQR using operational amplifiers

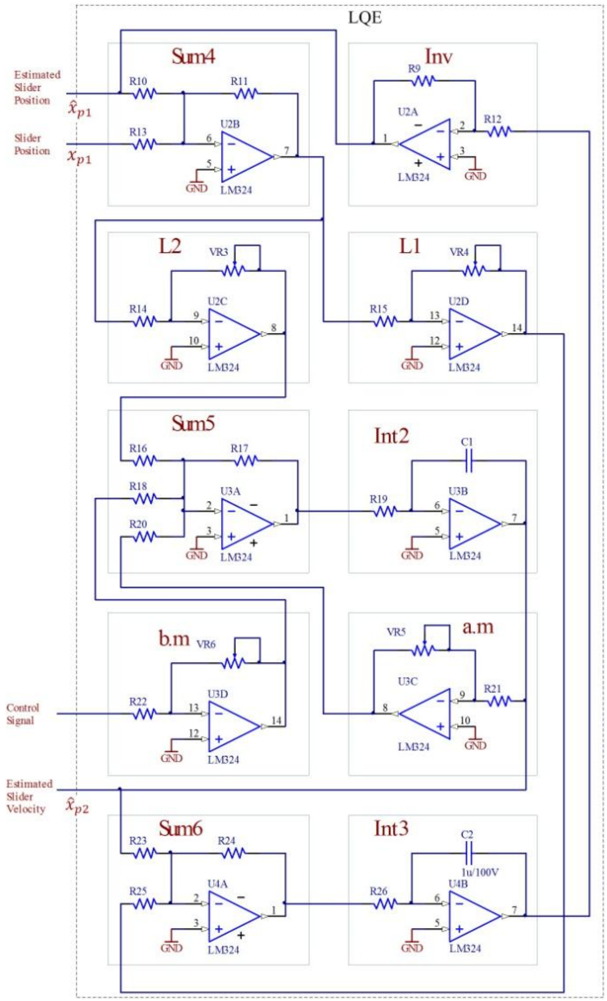

Figure 12. Analog LQE using operational amplifiers

In this step the LQG using operational amplifiers is obtained. However we do not sure that converting the controller in s-domain to corresponding analog electronic circuits using operational amplifiers is correct or not.

B. *Design of the Plant Using Operational Amplifiers*

The proposed step is shown in this section. From analog plant in s-domain is shown in Fig. 8 we can design corresponding analog electronic plant using operational

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amplifiers that shown in Fig. 13. The plant parameters with some explanations are listed in Table IV.

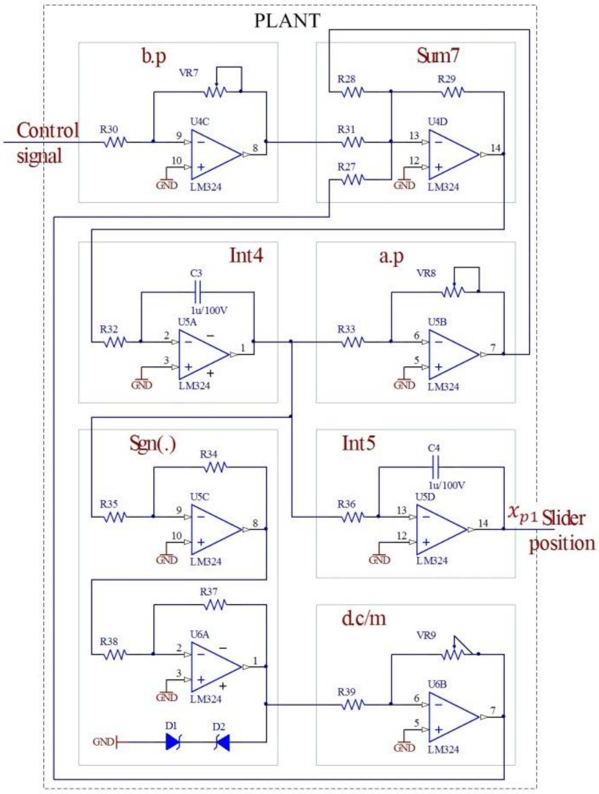

Figure 13. The plant model using operational amplifiers

TABLE III. ANALOG LQE PARAMETERS

Symbols	Parameters	Notes
Sum 4	$R_{10} = R_{11} = R_{13} = 10\ K$	Calculate the prediction error $e = x_{p1} - \hat{x}_{p1}$
Inv	$R_9 = R_{12} = 10\ K$	Create suitable sign for estimated slider position
L_1	$R_{15} = 10\ K, WR_4$	Optimal steady-state filter gain $L_1 = \frac{WR_4}{R_{15}}$
L_2	$R_{14} = 10\ K, WR_3$	Optimal steady-state filter gain $L_2 = \frac{WR_3}{R_{14}}$
Sum 5	$R_{17} = R_{23} = R_{20} = 10\ K;$	Weighted-sum adder
Int 2	$R_{19} = 10\ K, C_1$	Create estimated slider velocity $\hat{x}_{p2}$
b_m	$R_{22} = 10\ K, WR_6$	Model parameter $b_m = \frac{k_m}{m} = \frac{WR_6}{R_{22}}$
a_m	$R_{21} = 10\ K, WR_5$	Model parameter $a_m = \frac{d}{m} = \frac{WR_5}{R_{21}}$
Sum 6	$R_{23} = R_{24} = R_{25} = 10\ K$	Weighted-sum adder
Int 3	R_{26}, C_2	Create estimated slider position $\hat{x}_{p1}$

TABLE IV. PLANT PARAMETERS

Symbols	Parameters	Notes
b_p	R_{10}, WR_7	Plant parameter $b_p = \frac{k_m}{m} = \frac{WR_7}{R_{30}}$
Sum 7	$R_{27} = R_{28} = R_{29} = R_{31} = 10\ K$	Create slider velocity x_{p2}
Int 4	R_{32}, C_3	Integrator
a_p	R_{33}, WR_8	Plant parameter $a_p = \frac{d}{m} = \frac{WR_5}{R_{21}}$
Int 5	R_{36}, C_4	Integrator
dc/m	R_{39}, WR_9	Invertor

V. SIMULATION AND EXPERIMENTAL TESTS

In this section the resulting LQG controller in s-domain is firstly simulated using the Matlab Simulik. Next the equivalent analog electronic LQG circuit is performed and simulated using the Multisim software. The simulation results in this step confirm converting LQG in s-domain to equivalent electronic circuits is correct or not. Finally, the analog electronic LQG circuit is implemented and tested in the real setup.

We address the problem relating to the precision control of the DC motor which requires the needs of mechanical transmission from the rotary to linear motion. In order to show implementation of the resulting controlled systems, the following numerical values of the input reference are fixed for all simulation and tests: stroke $R = 0.1$ [m], period $T = 1$ [s].

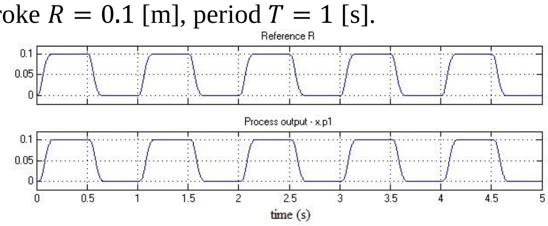

Figure 14. Simulation results in s-domain using Matlab Simulink software

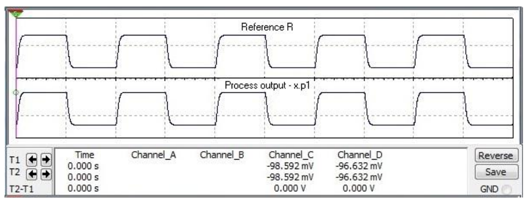

Figure 15. Simulation results for equivalent electronic circuits using Multisim software

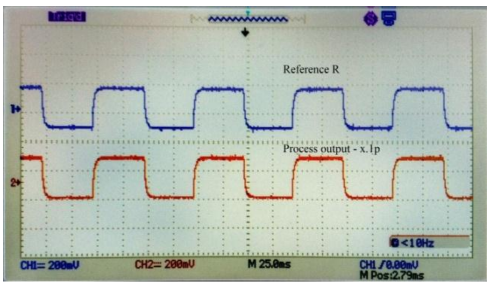

Figure 16. Experimental results in the real setup

As we expected with the same input reference the simulation results of the resulting controlled system in s-domain (see Fig. 14), also in equivalent analog electronic circuits (see Fig. 15), and experimental results in the real setup (see Fig. 16) are almost the same.

VI. Discussion

In order to design an analog controller using operational amplifiers, we propose a design procedure that has 6 following main steps:

- Modeling the system in s-domain;
- Analyzing and synthesizing the control system in s-domain;
- Simulating the controlled system in s-domain and validating the simulation results;
- Converting both the controller and the plant in s-domain to equivalent electronic circuits;
- Simulating the resulting control system in step 4 and checking the simulation results;
- Implementing the real controller and testing on-line in the real setup.

However, the above design procedure is recommended for use in the non-sophisticated systems.

VII. Conclusions

Design of the LQG controller using operational amplifiers for motion control systems has done based on a new design procedure. We propose to add two extra steps into the traditional design procedures. By implementing the controller in different domains, the performances of each domain are compared. With the same reference input, if the design steps are implemented correctly, the performances of the controlled system in s-domain, also in equivalent analog electronic circuits, and in experimental results will be almost the same. Both simulation and experimental results confirm the precise, featured by the proposed procedure.

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