An Adaptive LQG Combined With the MRAS - Based LFFC for Motion Control Systems

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***Abstract*—The aim of this paper is to develop advanced controllers for electromechanical motion systems. A new controller is proposed to take into account the inherent non-linear disturbances, measurement noise, and variations and uncertainties in process behavior. It consists of a Linear Quadratic Gaussian (LQG) controller and a separate supplementary MRAS-based Learning Feed-Forward Controller (LFFC). Instead of design that is based on a fixed mathematical model of the process, the optimal steady-state filter gain L in the Linear Quadratic Estimator (LQE) and the feedback gain K in the Linear Quadratic Regulator (LQR) can be determined based on the parameters of the feed-forward part, which follows continuously the process at different load conditions. This will result in "an adaptive LQG combined with the MRAS-based LFFC". Simulation results demonstrate the potential benefits of the proposed method.**

***Index Terms*—model reference adaptive systems (MRAS), linear quadratic gaussian (LQG), learning feed-forward control (LFFC), motion control systems**

I. Introduction

Motion control systems can be quite complicated because many different factors have to be considered in the design [1], [4]. The following issues must typically be considered: (a) reduction of the influence of plant disturbances; (b) attenuation of the effect of measurement noise; (c) variations and uncertainties in plant behavior. It is difficult to find design methods that consider all these factors, especially for the conventional control approaches where control designs involve compromises between conflicting goals.

We start by considering a conventional PID controlled system. For this type of controller, reduction of the effect of measurement noise suggests low PID gains, but attenuation of process disturbances suggests high PID gains. Both requirements cannot be achieved simultaneously [2], [4]. This problem can be overcome by using more advanced controllers.

LQG is basically a combination of a Linear Quadratic Estimator (LQE) with a Linear Quadratic Regulation (LQR) [2], [10]. The Separation principle guarantees that if a stable LQE and a stable LQR are designed for a linear time-invariant system, then a combined LQE and LQR results in a stable LQG system. Normally, the LQG design is based on a fixed mathematical model of the process. The estimator and feedback controller may be designed independently. It enables us to compromise between regulation performance and control effort, and to take into account process and measurement noise.

The MRAS-based LFFC aims to acquire the (stable part of the) inverse dynamics of the plant [7]. The idea of LFFC is applied but without using the complex neural networks. Instead, we propose to use MRAS-based adaptive components [1], [8]. A reference model is used to generate a desired set of states. The feed-forward signal is obtained by summing the profile set-point signals multiplied by appropriate weights. On-line parameter adaptation is utilized to reduce the effect of the disturbances such as mass deviation, and friction force resulting in a dynamic inverse of the process. With feed-forward control, the state-dependent disturbances can be compensated, before they have time to affect the system. The control action for disturbance rejection is obtained from the feed-forward path output. The MRAS-based LFFC can be applied to arbitrary motion profiles.

It is clear that, the combination of LQG and MRAS based LFFC control structure is shown to be superior to the two control methods when used separately [1], [3]. This is a robust, high-performance control scheme that combines the advantages and overcomes the disadvantages of both types of techniques. However, the LQG algorithm may fail to ensure closed-loop stability if the variations or/and uncertainties are large enough [2].

In this study, design of an adaptive LQG combined with the MRAS-based LFFC is developed for motion system. The proposed control structure is based on the following observation: In Section III, as can be seen in Fig. 5, after a short time the parameters in the feed-forward part converse quickly to stationary process values ($a_m = a_p$; $b_m = b_p$; and $c_m = c_p$). They denote the characteristic of the process model and could be used for the LQG design. This will result in an adaptive LQG.

This paper is organized as follows: First, the dynamic characteristic of the setup is analyzed in Section II. In Section III, a MRAS-based LFFC is designed by applying Lyapunov's stability theory. The validity of the proposed control structure is simulated in Section IV when the system is subject to external disturbance and parameter variation. Finally, some concluding remarks are drawn in Section V.

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II. Mathematical Model of the Setup

The setup (see Fig. 1) is designed for the purpose of testing the results of the controller for linear and non-linear systems. It consists of a slider which can move back and forth over a rail. A DC motor, rail and slider are fixed on a frame. The parameters of this setup are shown in Table I [1].

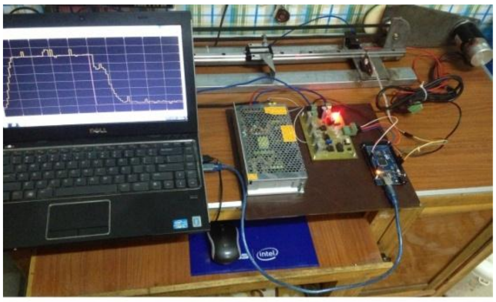

Figure 1. The configuration of the setup

The mechanical part of the setup is designed mimicking printer technology. For this process, a computer based control system has been implemented with software generated by MATLAB.

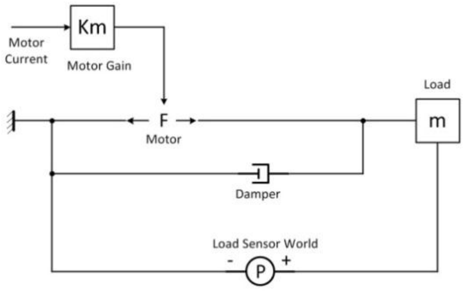

Figure 2. Second order model of the setup

TABLE I. Plant Parameters of the Setup

Element	Parameter	Value
Motor-Gain	Motor constant	8.5 N/A
Motor-Inertia	Inertia of the motor	2e-5 kg
Load	Mass of the end effector (slider)	0.35 kg
Belt-Flex	Spring constant	80 kN/m
	Damping in belt	1 Ns/m
Damper	Viscous friction	8 Ns/m
	Coulomb friction	0.75 N

The Damper component represents a viscous and Coulomb friction. Coulomb friction always opposes relative motion and is simply modeled as

$$F_c = d_c.tanh(1000.\dot{x}) \tag{1}$$

where d_c is the Coulomb parameter of the Damper element, $\dot{x}$ is the velocity of the load. Viscous friction is proportional to the velocity. It is normally described as

$$F_v = d.\dot{x} \tag{2}$$

where d is the viscous parameter of the Damper element.

The mathematical expression for the combination of viscous and Coulomb friction is

$$F = F_v + F_c = d.\dot{x} + d_c.tanh(1000.\dot{x}) \tag{3}$$

If the non-linear Coulomb friction part is disregarded, the model only contains linear components. In this case we get a linear process model. A second order approximation model is obtained with a state space description as given in (4) [1].

$$\begin{bmatrix}\dot{v}_L\\ \dot{x}_L\end{bmatrix} = \begin{bmatrix}-\frac{d}{m} & 0\\ 1 & 0\end{bmatrix}\begin{bmatrix}v_L\\ x_L\end{bmatrix} + \begin{bmatrix}\frac{k_m}{m}\\ 0\end{bmatrix}F \tag{4}$$

$$y = [0 \quad 1]\begin{bmatrix}v_L\\ x_L\end{bmatrix} + [0]F$$

where v_L is the velocity of the load; x_L is the position of the load; and F is applied force on the process. When we mention the nonlinear friction term of the Damper element then:

$$\begin{bmatrix}\dot{v}_L\\ \dot{x}_L\end{bmatrix} = \begin{bmatrix}-\frac{d}{m} & 0\\ 1 & 0\end{bmatrix}\begin{bmatrix}v_L\\ x_L\end{bmatrix} + \begin{bmatrix}-\frac{d_c}{m}sgn(v_L)\\ 0\end{bmatrix} + \begin{bmatrix}\frac{k_m}{m}\\ 0\end{bmatrix}F \tag{5}$$

The second order model of the setup is given in Fig. 2.

III. Design Of MRAS – Based LFFC

In a model reference adaptive system the reference model can play the role of a setpoint generator [1], [8]. This leads to the structure of Fig. 3, where the derivative–generating structure of the state variable filter is clearly visible. The reference model is described by

$$\frac{r_{m1}}{R} = \frac{\omega_m^2}{s^2 + 2z\omega_m s + \omega_m^2} \tag{6}$$

The process is described by

$$\frac{x_{p1}}{u} = \frac{\omega_p^2}{s^2 + 2z\omega_p s + \omega_p^2}$$

$$= \frac{1}{a_p s^2 + b_p s + c_p} \tag{7}$$

where

$$a_p = \frac{1}{\omega_p^2};\ b_p = \frac{2z}{\omega_p};\ c_p = 1.$$

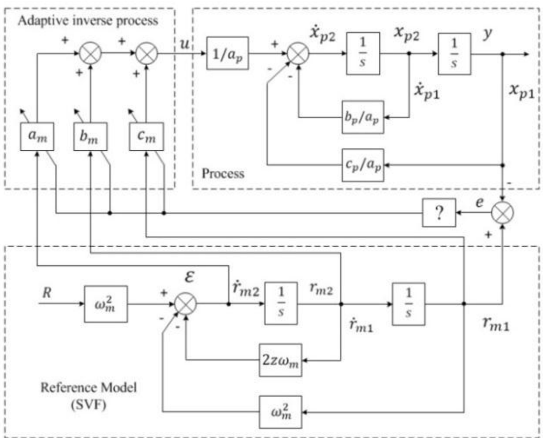

Figure 3. A process, an inverse process, and a reference model

Describe the process model in state variables

$$\dot{x}_p = \begin{bmatrix}\dot{x}_{p1}\\ \dot{x}_{p2}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p}\end{bmatrix}\begin{bmatrix}x_{p1}\\ x_{p2}\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{a_p}\end{bmatrix}u \tag{8}$$

$$= A_p x_p + B_p u$$

where

$$A_p = \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix};\ B_p = \begin{bmatrix} 0 \\ \frac{1}{a_p} \end{bmatrix}. \tag{9}$$

By means of the feed-forward controller, the SVF output signals can be used to generate an inverse model of the process [7], [8]. We should try to find a learning mechanism that, based on the errors between the output r_m of the setpoint generator and the process output x_p, adjusts the parameters a_m, b_m and c_m such that they converge to the process parameters a_p, b_p and c_p, respectively.

This suggests that we can use the well–known Liapunov approach to find stable adaptive laws for the feed–forward parameters. The design problem is thus: Find (stable) adjustment laws for the adjustable parameters a_m, b_m and c_m such that the error e between the setpoint generator and the process as well as the error in the feed–forward parameters asymptotically go to zero. The following steps are thus necessary to design an adaptive controller with the method of Liapunov [6], [11]:

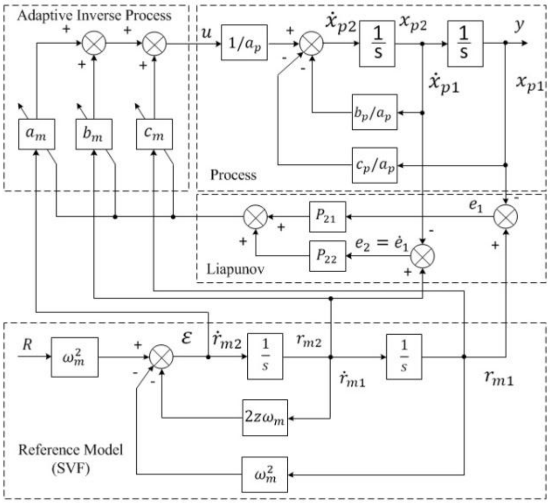

Figure 4. An adaptive inverse process designed with Liapunov

Step 1: Determine the differential equation for e

Describe the reference model in state variables:

$$\dot{r}_{m1} = r_{m2};\ \dot{r}_{m2} = \varepsilon$$

$$\dot{r}_m = \begin{bmatrix} \dot{r}_{m1} \\ \dot{r}_{m2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} r_{m1} \\ r_{m2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varepsilon \tag{10}$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} r_m + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varepsilon$$

Rewrite the process model in state variables:

$$\dot{x}_{p1} = x_{p2} \tag{11}$$

$$\dot{x}_{p2} = -\frac{c_p}{a_p} x_{p1} - \frac{b_p}{a_p} x_{p2} + \frac{1}{a_p}\left(c_m . r_{p1} + b_m . r_{p2}\right) + \frac{1}{a_p} a_m . \varepsilon \tag{12}$$

$$\dot{x}_p = \begin{bmatrix} \dot{x}_{p1} \\ \dot{x}_{p2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{c_m}{a_p} & \frac{b_m}{a_p} \end{bmatrix} \begin{bmatrix} r_{m1} \\ r_{m2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{a_m}{a_p} \end{bmatrix} \varepsilon \tag{13}$$

$$= \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix} x_p + \begin{bmatrix} 0 & 0 \\ \frac{c_m}{a_p} & \frac{b_m}{a_p} \end{bmatrix} x_m + \begin{bmatrix} 0 \\ \frac{a_m}{a_p} \end{bmatrix} \varepsilon \tag{14}$$

Here we introduce error e, which is defined in (15).

$$e = r_m - x_p; \tag{15}$$

$$\dot{e} = \dot{r}_m - \dot{x}_p \tag{16}$$

By subtracting (14) from (10), we get

$$\dot{e} = \begin{bmatrix} 0 & 1 \\ -\frac{c_m}{a_p} & -\frac{b_m}{a_p} \end{bmatrix} r_m - \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 - \frac{a_m}{a_p} \end{bmatrix} \varepsilon$$

$$= \begin{bmatrix} 0 & 1 \\ -\frac{c_m}{a_p} & -\frac{b_m}{a_p} \end{bmatrix} r_m - \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix} r_m + \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix} r_m - \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 - \frac{a_m}{a_p} \end{bmatrix} \varepsilon$$

$$= \begin{bmatrix} 0 & 0 \\ \frac{c_p}{a_p} - \frac{c_m}{a_p} & \frac{b_p}{a_p} - \frac{b_m}{a_p} \end{bmatrix} r_m + \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix} \left(r_m - x_p\right) + \begin{bmatrix} 0 \\ 1 - \frac{a_m}{a_p} \end{bmatrix} \varepsilon \tag{17}$$

$$= A_1 . r_m + A . e + B . \varepsilon \tag{18}$$

where

$$A_1 = \begin{bmatrix} 0 & 0 \\ \frac{c_p}{a_p} - \frac{c_m}{a_p} & \frac{b_p}{a_p} - \frac{b_m}{a_p} \end{bmatrix};\ A = \begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix};$$

$$B = \begin{bmatrix} 0 \\ 1 - \frac{a_m}{a_p} \end{bmatrix}. \tag{19}$$

Step 2: Choose a liapunov function $V(e)$

Simple adaptive laws are found when we use the Liapunov function

$$V(e) = e^T P e + a^T \alpha a + b^T \beta b \tag{20}$$

where P is an arbitrary definite positive symmetrical matrix; a and b are vectors which contain the non-zero elements of the A_1 and B matrices in (18); α and β are diagonal matrices with positive elements which determine the speed of adaptation.

Step 3: Determine the conditions under which $\dot{V}(e)$ is definite negative

$$\dot{V} = (A_1 . r_m + A . e + B . \varepsilon)^T . P . e + e^T . P . (A_1 . r_m + A . e + B . \varepsilon) + 2 . \dot{a} . \alpha . a^T + 2 . \dot{b} . \beta . b^T$$

$$= (A . e)^T . P . e + e^T . P . A . e. + 2 . e^T . P . A_1 . r_m + 2 . \dot{a} . \alpha . a^T + 2 . e^T . P . B . \varepsilon + 2 . \dot{b} . \beta . b^T \tag{21}$$

Let:

$$A^T P + PA = -Q \tag{22}$$

According to Liapunov's stability theory, as long as A is stable, there always exist such positive definite matries P and Q. This implies that the first part of (21):

$$e^T(A^TP + PA)e = -e^TQe \tag{23}$$

is definite negative. Such that stability of the system can be guaranteed if the two last parts of (21) get zero

$$e^T.P.A_1.r_m + \dot{a}.\alpha.a^T = 0 \tag{24}$$

$$e^T.P.B.\varepsilon + \dot{b}.\beta.b^T = 0 \tag{25}$$

where

$$a = [a_{21} \quad a_{22}];\ e = [e_1 \quad e_2];\ \alpha = \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix};$$

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix};\ A_1 = \begin{bmatrix} 0 & 0 \\ a_{21} & a_{22} \end{bmatrix};\ r_m = \begin{bmatrix} r_{m1} \\ r_{m2} \end{bmatrix}. \tag{26}$$

After some mathematical manipulations, this yields:

$$\dot{a}_{21} = -\frac{1}{\alpha_{11}}.(e_1.p_{21} + e_2.p_{22}).r_{m1} \tag{27}$$

$$\dot{a}_{22} = -\frac{1}{\alpha_{22}}.(e_1.p_{21} + e_2.p_{22}).r_{m2} \tag{28}$$

From (14) it follows that:

$$a_{21} = \frac{c_p}{a_p} - \frac{c_m}{a_p} \rightarrow \dot{a}_{21} = -\frac{1}{a_p}\dot{c}_m \tag{29}$$

It is given by the following expression to complete parameter update

$$c_m = \frac{a_p}{\alpha_{11}}\int[(e_1.p_{21} + e_2.p_{22}).r_{m1}]dt + c_m(0) \tag{30}$$

From (19) it follows that:

$$a_{22} = \frac{b_p}{a_p} - \frac{b_m}{a_p} \rightarrow \dot{a}_{22} = -\frac{1}{a_p}\dot{b}_m \tag{31}$$

There are given by the following expression to complete parameter update

$$b_m = \frac{a_p}{\alpha_{22}}\int[(e_1.p_{21} + e_2.p_{22}).r_{m2}]dt + b_m(0) \tag{32}$$

$$a_m = \frac{a_p}{\beta_{22}}\int[(e_1.p_{21} + e_2.p_{22}).\varepsilon]dt + a_m(0) \tag{33}$$

where α_{22} and β_{22} are called the adaptive gains, and e_1, e_2, ε, and r_{m2} are defined in Fig. 4; p_{21} and p_{22} are elements of the P matrix. The resulting adaptive system has been given (see Fig. 4).

Like in any MRAS-based system, adaptive disturbance compensation can be added, by realizing that the parameter d_m acts on an extra input signal 1, instead of on one of the state variables:

$$d_m = \frac{1}{\gamma}\int[(p_{21}.e_1 + p_{22}.e_2)1]dt + d_m(0) \tag{34}$$

Step 4: Solve P *from* $A_m^TP + PA_m = -Q$

Let

$$Q = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \tag{35}$$

which yields the following matrix equation:

$$\begin{bmatrix} 0 & -\frac{c_p}{a_p} \\ 1 & -\frac{b_p}{a_p} \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -\frac{c_p}{a_p} & -\frac{b_p}{a_p} \end{bmatrix} = -\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \tag{36}$$

This can be rewritten as:

$$\begin{bmatrix} -\frac{c_p}{a_p}.(p_{21} + p_{12}) & p_{11} - \frac{c_p}{a_p}.p_{22} - \frac{b_p}{a_p}.p_{12} \\ p_{11} - \frac{b_p}{a_p}.p_{21} - \frac{c_p}{a_p}.p_{22} & p_{12} + p_{21} - 2.\frac{c_p}{a_p}.p_{22} \end{bmatrix} = -\begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \tag{37}$$

This yields

$$p_{21} = p_{12} = \frac{1}{2}.q_{11}.\frac{a_p}{c_p} \tag{38}$$

$$p_{22} = \frac{1}{2}.\left(\frac{a_p^2}{c_p^2}.q_{11} + \frac{a_p}{c_p}.q_{22}\right) \tag{39}$$

Based on (30), (32), and (33) the adaptive system designed with Liapunov in Fig. 2 is redrawn as in Fig. 3. The following numerical values are chosen:

$$\omega_p = 50;\ z = 0.7;\ a_p = 0.0004;\ b_p = 0.028;\ c_p = 1;$$

$$Q = \begin{bmatrix} 4 & 8 \\ 8 & 16 \end{bmatrix};\ \frac{1}{\alpha_{21}} = 2500;\ \frac{1}{\alpha_{22}} = 10.$$

The settings result

$$p_{21} = 8 * 10^{-4};\ p_{22} = 32 * 10^{-4}.$$

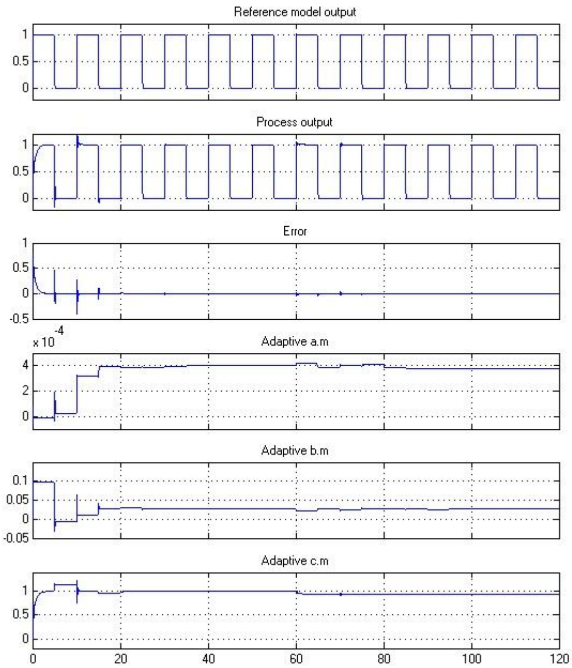

Figure 5. Simulation results (a load variation is added at $t = 60$ [s]

As it can be seen in Fig. 5 adaptive a_m, b_m and c_m automatically reach to stationary process values ($a_m = a_p = 0.0004$; $b_m = b_p = 0.028$; $c_m = c_p = 1$). Especially, when a variation of the load is switched on at $t = 60$ [s], after a few motions, the parameters a_m, b_m, and c_m quickly search to the new stationary values. They denote the characteristic of the process model and could be used for the LQG design.

IV. Design of Proposed Control Structure

Fig. 5 shows the block diagram of the proposed control structure, which combines an MRAS-based LFFC and a separate adaptive LQG controller. The model of the process to be controlled was introduced in Section II. The plant state vector is chosen such that it consists of the position and its corresponding velocity. In the feed-forward control part, the parameter adaptation is driven by the tracking error between reference output and measured process output, while in the LQG part the observer is driven by the prediction errors between measured process variables and corresponding estimated variables [2], [3].

The design of MRAS – based LFFC was shown in Section III. The A_p - matrix of the process model is used to calculate the solution P of the Liapunov equation. In the adjustment laws the derivative of the error is needed. This derivative can be obtained by means of a second-order state variable filter. For the Coulomb friction adaptive component d_m, the *sgn* of the reference velocity is used as the input (see Fig. 6).

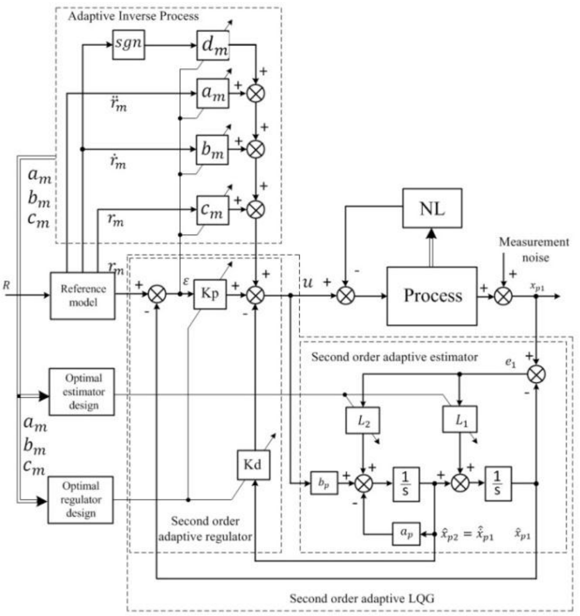

Figure 6. Adaptive LQG combined with MRAS – based LFFC

(a_m, b_m, and c_m in the feed – forward part are used for LQG design)

LQGenables us to optimize the system performance and to reduce the harmful effects of measurement noise [2]. The LQE yields the estimated states of the process. The LQR calculates the optimal gain vector and then calculates the control signal. However, in state feedback controller designs reduction of the tracking error is not automatically realized [10].

We consider the LQG design based on the 2nd order mathematical model. The optimal gain L in LQE and the feedback gain K in the LQR are determined based on the parameters a_m, b_m, and c_m of the feed-forward part, which follows continuously a_p, b_p, and c_p of the process at different load conditions, respectively.

Continuous LQR design [2]:

We consider a continuous-time linear plant described by

$$\dot{x}_p = A_p x_p + B_p u \qquad (40)$$

$$y_p = C_p x_p + D_p u \qquad (41)$$

With a performance index defined as

$$j = \int_0^\infty (e^T Q_R e + u^T R_R u) dt \qquad (42)$$

In (40), (41) and (42) A_p and B_p are continuous state matrices of the plant to be controlled, x_p denotes the state of the plant, e is the tracking error, u is the control signal, Q_R and R_R are matrices in the optimization criterion (Q_R is positive semi-definite weighting matrix and R_R is positive definite weighting). The optimal state feedback controller will be achieved by choosing a feedback vector

$$K_{LQR} = R_R^{-1} B_p^T P_R \qquad (43)$$

in which P_R is found by solving the continuous time algebraic Riccati equation

$$A_p^T P_R + P_R A_p - P_R A_p R_R^{-1} B_p^T P_R + Q_R = 0 \qquad (44)$$

The output of the state feedback controller is

$$u = -K_{LQR} \hat{x}_p \qquad (45)$$

where

$$\hat{x}_p = [\hat{x}_{p2} \quad \hat{x}_{p1}]^T,$$

$\hat{x}_{p1}$ and $\hat{x}_{p2}$ denote the state of the estimator (see Fig. 6). The following parameters are used in the simulation:

$$A_p = \begin{bmatrix} 0 & 1 \\ -\frac{c_m}{a_m} & -\frac{b_m}{a_m} \end{bmatrix}; B_p = \begin{bmatrix} 0 \\ \frac{1}{a_m} \end{bmatrix}; Q_R = \begin{bmatrix} 0.000 & 0.000 \\ 0.000 & 1000 \end{bmatrix};$$

$R_R = 0.0001$.

These values results in the following feedback controller gains

$$K_{LQR} = [K_d \quad K_p] \qquad (46)$$

Continuous LQE design [2]:

The feedback matrix L_{LQE} yielding optimal estimation of the process states is computed as

$$L_{LQE} = C_p P_E R_E^{-1} \qquad (47)$$

where$P$ is the solution of the following matrix Riccati equation

$$A_p P_E + P_E A_p^T + Q_E - P_E C_p^T R_E^{-1} C_p P_E = 0 \qquad (48)$$

in which A_p and C_p are continuous state matrices of the plant to be controlled, Q_E is the system noise covariance, and R_E the sensor noise covariance. The following settings were used:

$$A_p = \begin{bmatrix} 0 & 1 \\ -\frac{c_m}{a_m} & -\frac{b_m}{a_m} \end{bmatrix}; C_p = [0 \quad 1]; Q_E = \begin{bmatrix} 10 & 0.0 \\ 0.0 & 10 \end{bmatrix};$$

$R_E = 1000$.

These values results in the following feedback controller gains

$$L_{LQE} = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

Fig. 7 shows the corresponding responses for the system depicted in Fig. 6. In order to evaluate implementation of the adaptive controller, the sudden values of the load are added during the simulation period. As it can be seen in Fig. 7a and Fig. 7b, adaptive parameters in the feed – forward part a_m, b_m, c_m, and adaptive filter gains L_1, L_2 in the LQE automatically reach stationary values. When a mass variation of the load is switched on (at $t = 60$ [s]), after a few motions, the parameters a_m, b_m, c_m and L_1, L_2 quickly search the new stationary values.

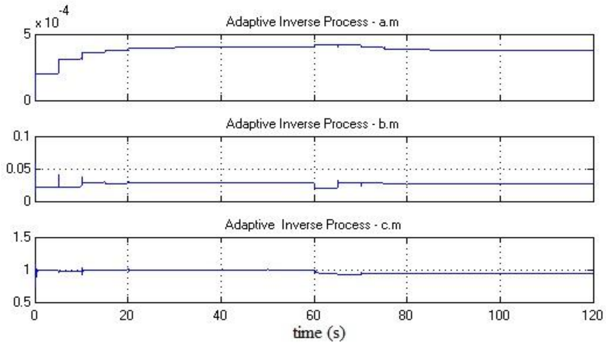

Figure 7a. Parameters in the adaptive feed – forward part

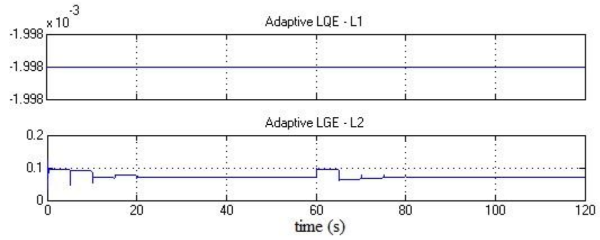

Figure 7b. Adaptive filter gain L in the LQE

As can be seen in Fig. 7c, in the beginning the maximum tracking error is large. However, when the adaptive gains K_p and K_d reach its stationary values (see Fig. 7d), it will decrease quickly to a small value. When a load disturbance is added, after a short time, the current tracking error converges rapidly to a small value. The controlled system is stable and shows convergence in the parameters.

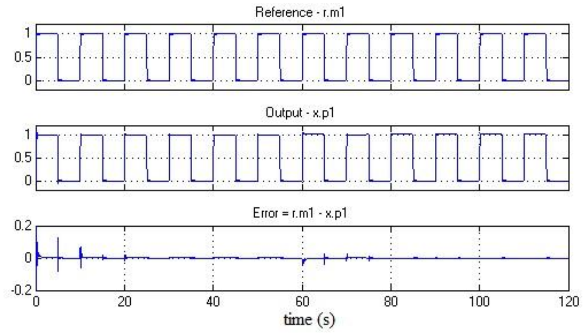

Figure 7c. Simulation results of controlled system (a load mass variation is added at $t = 60$ [s]).

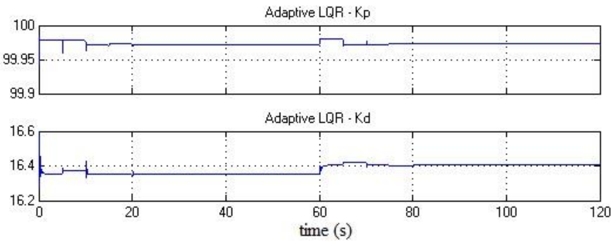

Figure 7d. Adaptive feedback gain K in the LQR

The compensation of the Coulomb friction force can be clearly observed in Fig. 8. When the adaptive Coulomb friction compensator is used, the effect of friction was compensated considerably. It can be stated that in motion control systems, Coulomb friction compensation is the key factor to obtain small tracking errors.

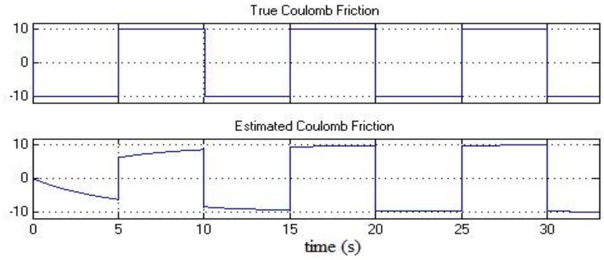

Figure 8. True and estimated Coulomb friction

With the LQG, noise on the measurements of the process has almost no influence on thesystem. This is illustrated in Fig. 9; the real position state (first line) and the position state error (third line) are corrupted by measurement noise, whereas,the estimated position state (second line) and the control signal (lowest line) are almost clean.

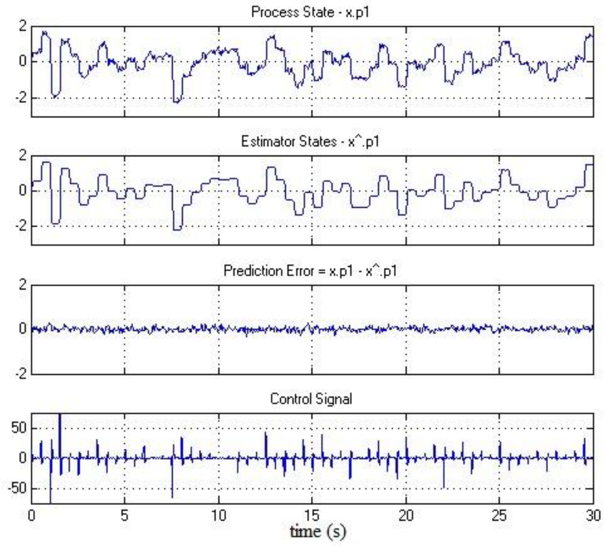

Figure 9. Control signal is insensitive for measurement noise

The LQG is designed to obtain a stable closed-loop system that is insensitive to measurement noise and variations and uncertainties in process behavior [2], [10].

V. Discussion

In this study, the design of an MRAS-based LFFC was carried out with the second-order example; however the approach can be effectively applied to higher order systems as well.

The advantages of the use of the profile setpoint signals are that they are easily accessible and noise free.

The adjustable parameter component has an integral component inside. This implies that even when the learning signal is corrupted by measurement noise the output signal is almost clean. This allows us using a large learning signal to shorten the setting time.

In case of an MRAS-based LFFC, when all disturbances can be effectively compensated for by a feed-forward signal, this allows us to reduce the values of the feedback controller gains. In this case measurement noise has almost no influence on the system.

The parameters of the LQG controller are given adaptive values that follow with the varying values of the plant. With the parameter variations of the plant considered here it appeared that the LQG was robust enough to deal with these variations and to produce good enough results for the basic feedback control system.

VI. Conclusion

Adaptive LQG combined with MRAS-based LFFC offers a potential solution to deliver more accurate and high overall performance in the presence of all the preceding issues. We investigated the effect of the controller from the simulation results. Compared to the case with LQG controller only, the proposed controller, for instance, can do the following (see Fig. 7, Fig. 8, and Fig. 9): (a) Improve the transient behavior of the system; (b) Decrease the sensitivity to plant parameter changes; (c) Eliminate steady-state errors; and (d) Decrease the influence of load disturbances and measurement noise. Strong properties achieved via the proposed method confirm that adaptive LQG combined with MRAS-based LFFC is an attractive approach for controlling electromechanical motion systems.

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