Position/Force Control Using a 6-axis Compliance Device with Force/Torque Sensing Capability

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Abstract—In this paper, the position/force control algorithm using a 6-axis compliance device with force/torque sensing capability is investigated. Differently from the traditional methods using strain gage-type force/torque sensor with very small compliance, this control method uses a compliance device to provide enough compliance between robot and rigid environment. This control method is to simply control the position of a working robot’s end-effector with the total twist of compliance in (13). The position/force control algorithm and control hardware system are developed. A simple design method of a compliance device with diagonal stiffness matrix is presented. The effectiveness has been verified through position/force control experiments.

Index Terms—compliance device, force/torque sensor, position/force control

I. INTRODUCTION

Currently, most industrial robots rely on only position control capability. It is well known that accurate force control as well as position control is required to complete the tasks such as peg in hole with very small tolerance, fastening bolts, grinding, deburring, etc.

Hybrid position/force control method is usually applied for the force control tasks. However, the method is based on the assumption that force control direction (wrench of constraint) and position control direction (twist of freedom) are orthogonal to each other [1], [2]. This method can be applied only when stiffness matrix between a working robot and an environment becomes diagonalized. Another approach is to use RCC (Remote-Center-of-Compliance) mechanism [3]. However, task should be performed only at the center of compliance. In order to resolve the problems, Griffis and Duffy [4]-[6] presented a novel and general position/force control theory to decompose position/force control directions which are orthogonal with respect to stiffness matrix. A few research results on the design of compliance devices have been published [7]-[9]. However, position/force control experiment results using compliance devices have been very limited [6].

In this paper, position/force control algorithm using a 6-axis compliance device is developed based on the control theory [4]-[6]. A simple design method of a 6-axis compliance device having decoupled stiffness is presented. The control hardware system including a 6-DOF parallel robot, a 6-axis compliance device, and PC-based controller with xPC Target is developed. Finally, two kinds of position/force control experiments (1DOC-5DOF and 3DOC-3DOF) have been performed to verify the effectiveness of the developed control algorithm, where DOC and DOF mean the numbers of force and position controlled directions, respectively.

II. POSITION/FORCE CONTROL ALGORITHM

As shown in Fig. 1, the position/force control is to simultaneously control displacement and force between the tool of a robot and workpiece by using a compliance device mounted on the end-effector of a robot.

The position/force control scheme can be briefly explained as follows: The position and force controlled directions are defined here as twists of freedom, , and wrenches of constraint, , which are equivalent to the artificial constraints in the hybrid control [2]. It is noted that the twists of freedom and the wrenches of constraint should satisfy the reciprocal relation given by

\[ \delta D^T \delta w = 0 \]  \hspace{1cm} (1)

In this paper, denotes a twist in axis coordinates, where \( \delta p \) and \( \delta \theta \) are infinitesimal linear and rotational displacements. And \( w = [ f^T, n^T ]^T \) denotes a wrench in ray coordinates, where \( f \) and \( n \) are force and moment vectors.

For a given task, the wrench acting on the workpiece can be controlled by the motion of the end-effector of a robot, \( \delta D \), called twists of compliance (or force

Figure 1. Illustration of the position/force control.
controlled direction). The twists of compliance, \( \delta D_c \), required for \( \delta w \) can be obtained through the inverse of a stiffness matrix, \( K^{-1} \) (refer to Fig. 1).

\[
\delta D_c = -K^{-1}\delta w \tag{2}
\]

Since the reciprocity relation \( \delta D_j^T K \delta D_c = 0 \) obtained from Eqs. (1) and (2) holds, this position/force control is to decompose the motion of a working robot into \( \delta D_j \) and \( \delta D_c \).

Fig. 2 shows a compliance device attached at the Gough-Stewart platform parallel-kinematic working robot and the definition of frames required to derive the control algorithm. In this paper, the parallel robot is used just as a positioning source. It is noted that this compliance device can be also mounted at the end-effector of an industrial serial-kinematic robot. Even for a serial-kinematic robot, this control algorithm can be applied in the same manner.

In this paper, all the reference inputs and actual values of position and force are expressed with respect to the work frame \( \{W\} \), first. Then the position and force errors are transformed to the instantaneous frame of the working robot, \( \{C\} \) in order to control the position of the moving platform (or end-effector) of the working robot. The frame \( \{C\} \) locates at point \( B \) and has the axes parallel to those of frame \( \{A\} \) (see Fig. 2). The leading superscript denotes a frame in which a vector or matrix is expressed. The twist and wrench in frame \( \{j\} \) are transformed to those in frame \( \{i\} \) by introducing rigid body transformations,

\[
\delta D = \begin{bmatrix} R_j & p_j \times R_j \\ 0_{3x3} & 1 \end{bmatrix} \delta D_j \tag{3}
\]

with

\[
R_j = \begin{bmatrix} R_j \end{bmatrix}, \quad p_j = \begin{bmatrix} p_j \end{bmatrix}
\]

where \( R_j \) is the rotation matrix from \( \{j\} \) to \( \{i\} \) and \( p_j \times \) is the vector in frame \( \{i\} \) from origin \( \{i\} \) to origin \( \{j\} \) expressed as a 3x3 skew-symmetric matrix.

The proposed position/force control algorithm is presented in Fig. 3, where I.K., \( J_{re} \), and \( D_a \) denote the inverse kinematics, Jacobian matrix, and reference trajectory of the working robot, respectively. The procedure to calculate the twists of freedom and compliance in frame \( \{C\} \) can be summarized as follows.

\[
C \delta D_j = C[E]_w \delta D_j \tag{6}
\]

First, the twist of freedom (or position error vector) in the work frame can be calculated by

\[
w \delta D_j = \hat{w} D_d - w D_a \tag{5}
\]

Second, wrenches of constraint (or force error vector) and twists of compliance by stiffness mapping can be obtained by the following procedures. The actual (or measured) wrench is calculated by the statics relation.

\[
\hat{\tau} W_a = \hat{\tau} J \tau \tag{7}
\]

The force error vector is given by

\[
w \delta W = \hat{w} D_d - w D_a \tag{9}
\]

where \( \hat{w} D_d \) denotes a desired input wrench. Since the Jacobian and stiffness matrices are expressed in frame \( \{A\} \), the force error vector needs to be transformed to the frame,
\[ A' \delta w = A' [e]_w W \delta w \]  
(10)

where \( s[e]_w = A' [e]_w \). The twists of compliance corresponding to the wrenches of constraint (or force error vector) can be obtained from the stiffness mapping

\[ A' \delta D_C = - A' K A' \delta w \]  
(11)

Then, the twist of compliance in frame \( C \) is given by

\[ C \delta D_C = C[E] A' \delta D_c \]  
(12)

where \( C[E]_c \) is the screw transformation matrix from \( A' \) to \( C \) whose columns are expressed in axis coordinates.

Finally, the total twist of compensation is obtained by the sum of twists of freedom and compliance multiplied by scalar gains by

\[ C \delta D_f = G_f C \delta D_f + G_c C \delta D_c \]  
(13)

where \( G_f \) and \( G_c \) are the position and force gains.

III. COMPLIANCE DEVICE DESIGN

It is noted that the proposed control method does not require the center of compliance or a diagonal stiffness matrix. However, in order to reduce coupling effects among axes, it is desired to design a diagonal stiffness matrix. In order to design a 6-axis compliance device with a diagonal stiffness matrix, adjacent legs need to be perpendicular to each other as shown in Fig. 4(b). For example, the unit direction of leg 1 should satisfy the following conditions at an initial position.

\[ s_x = \frac{r_1 \cos \phi_1 - r_2 \cos (\pi / 3 - \phi_2)}{l_1} = 0 \]

\[ s_y = \frac{r_2 \sin \phi_2 - r_1 \sin (\pi / 3 - \phi_1)}{l_0} = - \frac{1}{\sqrt{2}}, \quad s_z = \frac{h_0}{l_0} = \frac{1}{\sqrt{2}} \]

When \( l_0 \), \( \phi_0 \), and \( \phi_2 \) are selected with practical considerations, \( h_0 \), \( r_2 \) and \( r_1 \) can be determined by

\[ h_0 = l_0 / \sqrt{2} \]

\[ r_2 = (l_1 / \sqrt{2}) \cos \phi_2 \sec (\phi_0 + \phi_2 + \pi / 6) \]

\[ r_1 = (l_0 / \sqrt{2}) \sin (\phi_0 + \pi / 6) \sec (\phi_0 + \phi_2 + \pi / 6) \]

Table I shows the design result for given \( l_0 \), \( \phi_0 \), and \( \phi_2 \). The Cartesian stiffness matrix at the center of compliance, \( Q (h = 17 \text{mm}) \) is given by

\[ K = k \times \text{diag} \left( \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2} \right) \]

Each leg of the compliance device has a linear spring with spring constant of \( k = 6.5 \text{ N/mm} \) mounted in the cylinder and a linear optical encoder with 5\mu m resolution. The ideal force resolution of each leg is 32.5mN and linear force resolution of the compliance device along the \( x \)- and \( y \)-axes is about 39mN and one along the \( z \)-axis is about 138mN. It is noted that for large deformation, the stiffness matrix becomes asymmetric and its derivation can be found in [10]. Based on the design results, a prototype 6-axis compliance device with linear optical encoders is shown in Fig. 5.

<table>
<thead>
<tr>
<th>TABLE I. DESIGN RESULTS OF THE 6-AXIS COMPLIANCE DEVICE.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Kinematic Parameters</strong></td>
</tr>
<tr>
<td>Initial leg length ( (l_0) )</td>
</tr>
<tr>
<td>Half angle at the fixed platform ( (\phi_0) )</td>
</tr>
<tr>
<td>Half angle at the moving plate ( (\phi_2) )</td>
</tr>
<tr>
<td>Radius of the fixed platform ( (r_2) )</td>
</tr>
<tr>
<td>Radius of the moving platform ( (r_1) )</td>
</tr>
<tr>
<td>Initial height of the moving plate ( (h_0) )</td>
</tr>
<tr>
<td>Maximum spring deflection</td>
</tr>
</tbody>
</table>

![Prototype of a 6-axis compliance device with force/torque sensing capability.](image)

**Figure 5.** Prototype of a 6-axis compliance device with force/torque sensing capability.

IV. EXPERIMENT RESULTS

A. Experimental Setup

As shown in Fig. 6, the 6-DOF Gough-Stewart platform is used as a working robot and the 6-axis compliance device is mounted under the moving platform of the working robot. Table II presents the specification of the working robot. The control system consists of a Host PC, a Target PC with DAQs, 6 AC servo drivers and interface boards. The control program is made with Simulink and xPC Target from MathWorks.

Even if several position/force control cases are possible, only two cases (1DOC-5DOF and 3DOC-
3DOF) have been experimented as shown in Fig. 7 to verify the feasibility of the proposed control algorithm and gain effects, where DOC and DOF denote degree-of-constraint and degree-of-freedom.

**TABLE II. SPECIFICATION OF THE PARALLEL WORKING ROBOT.**

<table>
<thead>
<tr>
<th>Kinematic Parameters</th>
<th>Value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of the fixed platform ($r_f$)</td>
<td>496.040 mm</td>
<td></td>
</tr>
<tr>
<td>Radius of the moving platform ($r_p$)</td>
<td>255.376 mm</td>
<td></td>
</tr>
<tr>
<td>Half angle at the fixed platform ($\phi_b$)</td>
<td>4.046°</td>
<td></td>
</tr>
<tr>
<td>Half angle at the moving platform ($\phi_f$)</td>
<td>8.223°</td>
<td></td>
</tr>
<tr>
<td>Min. length of actuators ($l_{min}$)</td>
<td>668 mm</td>
<td></td>
</tr>
<tr>
<td>Max. length of actuators ($l_{max}$)</td>
<td>979 mm</td>
<td></td>
</tr>
<tr>
<td>Stroke of actuators ($\Delta l$)</td>
<td>311 mm</td>
<td></td>
</tr>
</tbody>
</table>

**B. Experiment for 1DOC-5DOF Case**

The first experiment is to control the force along the z-axis and to control the positions, $p_x, p_y, \theta_x, \theta_y, \theta_z$. The cubic trajectory of $p_z$ is generated between ±80 mm. Three cases with different position and force gains are tested to investigate the effects of $G_p$ and $G_f$.

Desired wrench: $W_{d} = [*, *, 10; *, *, *]' [N; Nm]$

Desired position: $D_{d0} = [0, \pm 80; 0, 0, 0]' [mm; rad]$

where symbol * means that that position or force direction is not controlled. In the following figures, $f_{mea}$ and $n_{mea}$ denote the measured force and moment at the tool expressed in frame $\{W\}$, and $p_{mea}$ and $\theta_{mea}$ denote the position and orientation of the tool expressed in frame $\{W\}$, which are corresponding to position and orientation errors.

**Figure 6.** Configuration of the position/force control system.

**Figure 7.** Experimental setups.

First, the effect of $G_c$ can be seen from Fig. 8 and Fig. 9.

- **Case I:** Smaller overshoot of $f_z$, slower response, larger force error.
- **Case II:** Larger overshoot of $f_z$, larger response, smaller force error.

**Figure 8.** Case I: $G_c = 0.025$, $G_f = 0.1$.

**Figure 9.** $G_c = 0.05$, $G_f = 0.1$.

Second, the effect of $G_f$ can be seen from Fig. 9 and Fig. 10.

- **Case II:** Larger position tracking error.
- **Case III:** Smaller position tracking error.

**Figure 10.** $G_c = 0.05$, $G_f = 0.2$. 

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In this experiment, only position and force gain effects are investigated. It is also noted that $f_z$ is changing along the y-axis since unknown friction between the tool and work-piece exists.

C. Experiment for 3DOC-3DOF Case

In this experiment, force, $f_z$, and moments, $n_x, n_y, n_z$, are controlled and positions, $p_x, p_y, p_z$, and orientation, $\theta_z$, are controlled. The cubic trajectory of $p_z$ is also generated from 0 to 80 mm and from 80 to 0 mm.

Desired wrench: $w_{D_{\theta}} = [0,*,10;0,0,*]^T [N;Nm]$

Desired position: $w_{D_{pp}} = [0,80,*;*,*,0] [mm;rad]$

It is noted that $f_z$, and $n_x, n_y, n_z$, are regulated well and the friction force, $f_z$, is not negligible as shown in Fig. 11.

Further works will focus on increasing control speed and developing more applications.

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REFERENCES


Han Sung Kim Ph.D. degrees in Mechanical Engineering from Yonsei University, Seoul, Korea in 2000. He worked as a postdoctoral researcher at University of California, Riverside, USA from 2001 to 2003. He worked as a visiting professor at Georgia Tech, Atlanta, GA, USA from 2011 to 2012. Since 2004, he has been working as a professor in the School of Mechanical Engineering at Kyungnam University, Changwon, Korea. His research interests include mechanism design, kinematics, parallel robot applications, and MEMS.