

Integrated Optimization of Airport Taxiway and Runway Scheduling

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Abstract—Capacity limitation of runway and taxiway in an airport is often the major limiting factor of air traffic operation. The congestions on the apron of an airport cause severe delay on aircraft schedule. This paper proposes an integrated algorithm to solve problems of runway scheduling and taxiway routing simultaneously for both departure and arrival aircrafts. A set partitioning model with side constraint is proposed in which each possible aircraft route in the taxiway and runway is regarded as a decision variable. Beside traditional set partitioning constraints, two types of constraints are proposed to maintain a minimum separation distance between aircrafts in the taxiway and runway. The preliminary results show that our integrated algorithm outperforms the sequential method.

Index Terms—runway scheduling, taxiway routing, set partitioning, optimization

I. INTRODUCTION

Due to the rapid growth of air traffic, major airport around the world have become extremely busy, especially at those that service as regional hubs. All the aircrafts tend to move at the same time in an airport [1]. Many delays occur or aggravated because of ground traffic congestion as well as the inefficient utilization of the runway. Therefore, an integrated approach to optimize the use of both runway and taxiway can improve airport operation and minimize the delay of aircraft schedule.

A. Taxiway Scheduling

The taxiway is a set of paths or area connecting the gates and runway and other facilities in airport. Usually the path of taxiway for a departure aircraft leaving from each gate is pre-defined and if there exists conflict among aircrafts traveling on the same taxiway, one of them will stop and wait. This will cause delay on departure, and possible downstream delay in the arrival to its destination airport, which may cause the further network delay.

Previous studies related to taxiway scheduling concentrate on two aspects: routing problem and timing problem. The routing problem is to find a taxiway route for each aircraft and usually assumes that an aircraft taxis at a constant speed. When conflict occurs between

two or more aircrafts on the taxiway, some aircrafts will have to wait. This problem is similar to the classical of vehicle routing problem (VRP) and shortest path problem (SPP). Current researches include development of algorithms for solving taxiway routing problem [1], [2]. These studies consider the timing problem for determining the time for each aircraft to cross an arc (or the travelling speed of an aircraft on an arc, or holding time at the nodes) based on a pre-defined path. As such variable speed on an arc is modeled. Represented works such as Sivakumar Rathinam et al. (2008) proposed a mixed integer linear programming model for this problem [3].

In a different perspective, the taxiway routing and scheduling problem considered in this paper aims to find the routes and starting time for each departure and arrival aircraft with a view to minimize the total taxiing time on the taxiway with the major constraints being conflict avoidance at each node where nodes are intersections on the taxiway.

B. Runway Scheduling

The runway being one of the most important resources of an airport is used by aircrafts to take off and land. The runway capacity is often a major bottleneck in an airport as many of the airports have only one or two runways. The efficiency of runway utilization depends largely on the sequence of landing and departure. This is because a small aircraft will suffer instability from the wake-vortex generated by a large aircraft before it. In contrast, a large aircraft will suffer relative low turbulence from air flow caused by a small aircraft before it. Therefore, if re-arranging the landing/takeoff sequence by switching the order of different type of aircraft, delay may be minimized or alleviated in many situations.

Objectives related to runway research include reducing deviation between the real schedule time and target scheduled time of all the considered aircrafts, minimizing the average landing/ takeoff time, and minimizing the makespan of a fixed number of aircraft (or the time when the last aircraft lands/takes off). Constraints being considered include the separation time between every two aircrafts to eliminate the risk caused by wake-vortex. For the important literature related to runway scheduling, one can refer to [4]-[6].

The objective being considered in this research is to arrange the aircraft sequence using the runway to

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minimize the deviation of runway take-off and gate-arrival from the original scheduled time.

C. Integrated Runway and Taxiway Scheduling

In practice, taxiway routing and runway scheduling operations are inter-related. An optimized taxiway schedule without considering the sequence of aircraft using the runway may not provide a globally optimal solution. This may also be the case if we only consider the optimized sequence of using the runway and ignoring the routing of taxiway. Very often, the sequential scheduling of runway and taxiway results in sub-optimal solution. Therefore, an integrated method that considers both the runway and taxiway simultaneously should provide better airport operation decisions of utilizing both resources. Current studies that focusing on an integrated approach to the problem include research by Clare and Richards[7], [8] and Lee and Balakrishnan[9]. Clare and Richards considered the taxiway routing and timing problem, and considered the runway departure separation constraints at the same time. They adopted a receding horizon-based approach to formulate the problem and solve it with CPLEX. However, the authors took only the departure of aircrafts on the runway into account, while fixed the arrival aircraft time as a constant input. Moreover, the optimization focused only on taxiway scheduling, that is to say, the runway performance was not considered in the objectives but only served as constraints. Lastly, the receding-horizon approach applied in this problem is quite restricted as it cannot predict the aircraft sequence on the runway when the method is used to schedule the period that an aircraft taxis on the taxiway. Lee and Balakrishnan on the other hand proposed a mathematical optimization model for the integrated problem, which is based on [3]. They also proposed a sequential approach to solve the problem and the comparisons between the integrated and sequential methods are made by analyzing several airport performance metrics at Detroit airport. The objectives in the problem integrated the runway performance, which is measured by the runway delay for departure aircraft with taxi-out and taxi-in time. Nonetheless, the study did not try to optimize the scheduling of arrival aircrafts. In addition an assumption was made in the study that the taxiway path for each aircraft is pre-determined, which may not be able to provide a true picture of reality.

This paper aims to find an integrated model for simultaneous optimizing runway and taxiway schedules. Both departure and arrival aircraft are considered in the proposed model with a solution method for solving the problem.

II. PROBLEM DESCRIPTION

A. Assumptions

- In order to simplify the problem, we assume the speed of the aircraft travelling on the taxiway is constant, independent of aircraft types, weight classes and taxiway passages. Therefore, for the

aircraft with common nodes (traversing the same taxiway intersections), we only need to check potential conflict on their first common nodes.

- The earliest and latest arrival times of a landing aircraft are defined as hard constraints. The earliest time is limited due to the aircraft flying speed in the air, while the latest time is dependent on the fuel capacity limitation. Similarly, the gate release time and clear time are considered as the earliest and latest time of a departure aircraft.
- The target arrival time at the runway for a departure aircraft and the at-gate time for an arrival aircraft are known and the deviation from these target time will be penalized.

B. Objectives

Our objectives include minimizing the deviation of runway departure time and target time, deviation of actual gate arrival time and target gate arrival time, and taxi time of both departure and arrival aircraft.

C. Notation

A set of arrival flight

D set of departure flight

$m = |A \cap D|$, the number of total flight

N the set of all nodes including gates, runway and intersections on taxiway, $|N| = n$

E the arc which denote the taxiway arc connecting the nodes

e_i the earliest time of aircraft $i \in A \cup D$

l_i the latest time of aircraft $i \in A \cup D$

t_i^t the target time of an aircraft at the gate for $i \in A$, on the runway for $i \in D$

t_{jk} the arrival of an aircraft with schedule j arrival at node $k \in N$

D. Constraints

- Starting time of the aircraft must be in the time window.
- The aircraft type sequence-dependent separation time should be satisfied on the runway.
- The taxiway separation time should be satisfied at the common nodes and arcs of the aircraft.

III. SOLUTION METHODOLOGY

In the literature, sequencing and timing model with side constraints are usually adopted for taxiway and runway scheduling. However, there are two limitations about the sequencing model, first the number of decision variable grows exponentially to the number of nodes in the network. Secondly, it is hard to simultaneously consider departure and arrival flights. In our study, we propose a set partitioning model for integrating taxiway routing and taxiway scheduling. One routing and schedule decision for a certain flight is represented as a decision variable. Time window of each flight is discretized and for each discretized time node, an individual aircraft route and schedule is generated.

The solution process is composed of four main parts and illustrated in Fig. 1.

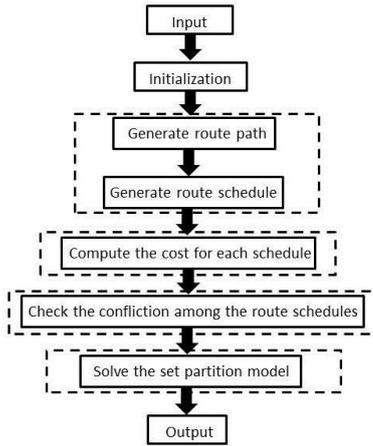


Figure 1. Flowchart of solution method process

A. The Route Paths and Route Schedules Generation

We have two different concepts here: route path and route schedule. The route path is the node sequence taken by an aircraft before reaching the runway. The route schedule is composed of a sequence of nodes and the time passing each of them. That is, for one route path, there would be many different route schedules.

The initial route schedule pool, which is candidates for later use in set partition model, is generated as follows:

Firstly, for each aircraft $i \in A \cup D$, we obtained its K shortest route paths by solving the K shortest path problem based on the network of path taken by the aircraft, with nodes including gates, runway, and the intersections of the taxiway.

We discretize the continuous time by assuming each small time slot takes δ seconds, therefore, according to the time window $[e_i, l_i]$ which limits the starting time of the i_{th} aircraft at its first node, we obtain $L_i = \lfloor \frac{l_i - e_i}{\delta} \rfloor + 1$ route schedule for each route path.

Finally, we could get total number of $L_i K$ route schedules for each aircraft i . The route schedule set is denoted as RS.

The process is shown in Fig. 2.

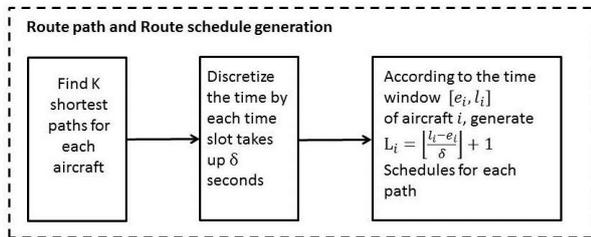


Figure 2. The route path and route schedule generation

B. Compute the Cost for Each Route Schedule

Our objectives include reducing the departure time deviation and taxiing time for a departing aircraft, as well as reducing taxiing time and gate arrival time for an arriving aircraft, respectively. Therefore, for $i \in D$, the cost of route schedule j is calculated by the equation:

$$c_j^i = \alpha_D(t_{jr} - t_{jg}) + [a_{de} \max(T_i - t_{jr}, 0) + a_{al} \max(t_{jr} - T_i, 0)] \quad (1)$$

And for $i \in A$, the cost of route schedule j is calculated by the equation:

$$c_j^i = \alpha_A(t_{jg} - t_{rj}) + a_{al} \max(t_{jg} - T_i, 0) \quad (2)$$

In which t_{jr} and t_{jg} are the arrival time of schedule j on runway and at gate respectively. T_i is the target runway takeoff time for $i \in D$ and the target gate arrival time for $i \in A$. a_{de} , a_{al} and a_{al} are the weight coefficients of earliness of departure, lateness of departure, and lateness of arrival. α_D and α_A are the coefficients to weight the taxiway time and target deviation of departure and arrival respectively.

The procedure for route schedule cost calculation is illustrated in Fig. 3.

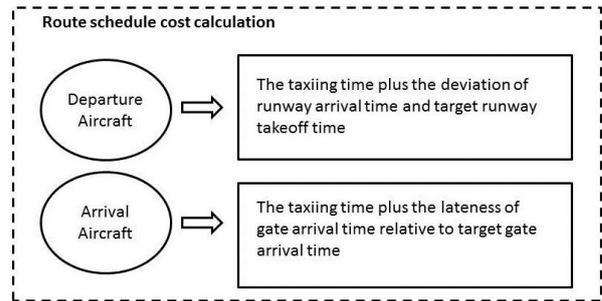


Figure 3. The route schedule cost calculation

C. Check Conflict among the Route Schedules

For each aircraft i , each route schedule j is checked with all route schedules of other aircraft. In practice, the safety distance on the taxiway is 60m. By dividing the speed, we could obtain the separation time, denoted as s_T . As the speed of taxiing aircraft is assumed to be constant, we only need to check the conflict on the nodes. The schedule set $CN(j)$ contains all the schedules having some common nodes with schedule j . For $j' \in CN(j)$, the $N(j, j')$ is the node set containing the common nodes of j and j' . We use $\beta_{jj'}$ to denote the relationship between two schedules j and j' . The schedule set $CA(j)$ contains all the schedules going opposite to some arcs with schedule j . For $j' \in CA(j)$, the $A(j, j')$ is the corresponding arc set. $\beta_{jj'} = 1$ if there are conflicts between them, 0 otherwise.

1) Rear-end collision

The schedule $j' \in CN(j)$ with the schedule time within the time period $[t_{jk} - s_T, t_{jk} + s_T]$ at the node $k \in N(j, j')$ will result in a conflict schedule with schedule j , i.e., $\beta_{jj'} = 1$.

$$t_{jk} - s_T \leq t_{j'k} \leq t_{jk} + s_T \quad (3)$$

2) Head-on collision

The schedule $j' \in CA(j)$ with the schedule time on the arc $(k, k') \in A(j, j')$ satisfying either one of the following conditions is regarded as a conflicting schedule with schedule j .

$$(t_{jk'} - t_{j'k'}) > 0 \&\& (t_{jk} - t_{j'k} - s_T) \leq 0 \quad (4)$$

$$(t_{j'k} - t_{jk}) > 0 \&\& (t_{j'k'} - t_{j'k} - s_T) \leq 0 \quad (5)$$

In which j is supposed to traverse node k before node k' , j' is goes from the opposite direction.

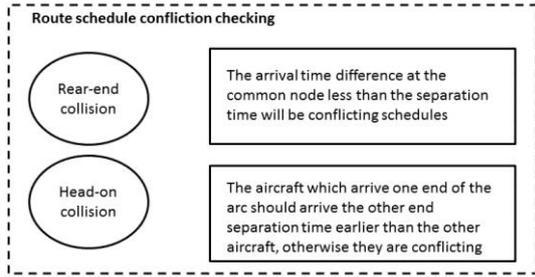


Figure 4. The route schedule conflict checking

The process of route schedule conflict checking is shown in Fig. 4.

However, this process will generate enormous constraints on the taxiway nodes and arcs, and thus the computation will become intractable. Therefore, we put forward another criterion to check for conflict on the taxiway. It is an iteratively node conflict checking process describes as follows:

For each node k , calculate the period when there are schedules passing through it. Then divide the period into θ ($=50$ in our experiment) time nodes. For each of the time nodes t_k , ensure there is only one schedule passing through it within $[t_k - s_T, t_k + s_T]$.

D. Set Partitioning Model

From part B, we obtain the cost for each route schedule, denoted by c_j . The set partitioning model is formulated as follows:

The objective is to minimize the total cost of all selected schedules.

$$\text{Min } \sum_{j \in RS} c_j x_j \tag{6}$$

The main constraints include two parts. The first is to ensure each aircraft has one and only one schedule to be selected. The total aircraft number is $m = |A \cup D|$

$$\sum_{j \in RS} \alpha_{ij} x_j = 1, \forall i = 1 \dots m \tag{7}$$

The second is to ensure no conflict occur among these selected schedules.

$$x_j + x_{j'} \leq 1, \forall j, j' \in \{j, j' | \beta_{jj'} = 1\} \tag{8}$$

The constraint (8) could be substituted with a bundle constraints (9) and (10) for better efficiency when solving the integer solutions.

$$x_j + \sum_{j' \in RS} \alpha_{ij'} x_{j'} \leq 1, \forall j, j' \in \{j, j' | \beta_{jj'} = 1\} \tag{9}$$

$$x_{j'} + \sum_{j \in RS} \alpha_{ij} x_j \leq 1, \forall j, j' \in \{j, j' | \beta_{jj'} = 1\} \tag{10}$$

The final procedure is shown in Fig. 5.

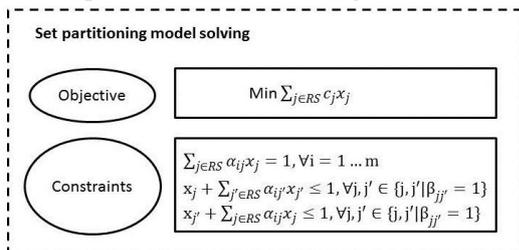


Figure 5. The Set partitioning model

IV. COMPUTATIONAL RESULT

We simulate the data as follows: the taxiway is composed of 36 nodes (including gate as part of the taxiway nodes) and one runway node. There are 6 aircraft, of which three aircraft are waiting for arrival and the other three are waiting for departure. The problem is illustrated in Fig. 6 and Table I.

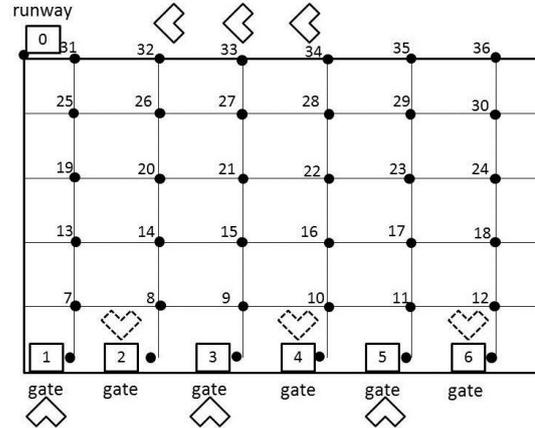


Figure 6. Illustration of the problem considered

The sequential method used for comparison works as follows:

- Sorting all the aircraft according to their target time, the aircraft ranking front have higher precedence when assign them schedules
- Find the shortest path for each aircraft one by one
- The scheduling time is calculated without violating the aircraft schedule with higher precedence.

TABLE I. PROBLEM DATA

Dep	No	Ei(s)	Li(s)	Dep	Target time	Start node	End node
Dep	1	5	100	1	185	1	0
Arr	2	45	125	0	255	0	2
Dep	3	105	150	1	405	3	0
Arr	4	15	135	0	345	0	4
Dep	5	25	95	1	325	5	0
Arr	6	35	200	0	410	0	6

The problem parameters are set as follows: $\alpha_D = \alpha_{de} = \alpha_{dl} = \alpha_A = \alpha_{al} = 1, s_T = 10$

The separation time on runway is shown in Table II. Aircrafts 1 and 2 belong to large aircraft type, aircrafts 3 and 4 belong to middle aircraft type, aircrafts 5 and 6 belong to small aircraft type.

TABLE II. RUNWAY SEPARATION REQUIREMENT

$s_R(s)$	large	mid	small
large	30	60	90
mid	30	30	60
small	30	30	30

The solution obtained from the sequential method is given as follows:

Aircraft 1:

1(5)-7(35)-13(65)-19(95)-25(125)-31(155)-0(185)

Aircraft 2:

0(45)-31(75)-32(105)-26(135)-20(165)-14(195)-8(225)-2(255)

Aircraft 3:

3(105)-9(135)-15(165)-21(195)-27(225)-33(255)-32(285)-31(315)-0(355)

Aircraft 4:

0(15)-31(45)-32(75)-33(105)-34(135)-28(165)-22(195)-16(225)-10(255)-4(285)

Aircraft 5:

5(25)-11(55)-17(85)-23(115)-29(145)-35(175)-34(205)-33(235)-32(265)-31(295)-0(325)

Aircraft 6:

0(365)-31(395)-32(425)-33(455)-34(485)-35(515)-36(545)-30(575)-24(605)-18(635)-12(665)-6(695)

The number outside the bracket represents the node, and the number in the bracket denotes the actual time in seconds when aircraft passing the node. The total cost is therefore 1915.

The proposed approach is tested using optimization software CPLEX and implemented in Java. The problem is solved on a 2.7-GHz dual-core PC with 4 GB of RAM.

The solution obtained from the proposed integrated approach is given as follows:

Aircraft 1:

1(5)-2(35)-8(65)-14(95)-20(125)-26(155)-32(185)-31(215)-0(245)

Aircraft 2:

0(95)-31(125)-25(155)-19(185)-13(215)-7(245)-8(275)-2(305)

Aircraft 3:

3(145)-9(175)-15(205)-21(235)-27(265)-26(295)-25(325)-31(355)-0(385)

Aircraft 4:

0(65)-31(95)-25(125)-19(155)-20(185)-21(215)-22(245)-16(275)-10(305)-4(335)

Aircraft 5:

5(35)-4(65)-10(95)-9(125)-15(155)-14(185)-20(215)-19(245)-25(275)-31(305)-0(335)

Aircraft 6:

0(35)-31(65)-25(95)-19(125)-13(155)-7(185)-8(215)-2(245)-3(275)-4(305)-5(335)-6(365)

The total cost is 1730.

We could see that the proposed method outperform the sequential method in terms of the total cost.

V. CONCLUSION

In this paper, we have built a set partitioning model to solve the integrated runway scheduling and taxiway routing problem. Moreover, we consider the simultaneous optimization on delay on the runway, at the gate and taxiing time of arrival and departure aircrafts, which is different from most previous works that only consider departure aircrafts. In addition, we formulate the

problem using a set partitioning model to largely reduce the number of constraints and make the problem more manageable. Our preliminary results based on simulated data prove the feasibility and efficiency of the proposed methodology. Future analysis will include the improvement on this methodology on the computation efficiency.

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