New Planning Framework of Hydro-thermal System with Annual Water Consumption Constraints

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***Abstract*—This paper studies the planning problem of hydro-thermal system with annual water consumption constraints, which are motivated by the practical system in the Yellow River Basin of China. This problem is formulated as one multistage stochastic optimization problem. The high-dimensional, dynamic, nonlinear and stochastic characteristics of hydro-thermal systems are considered. Since we can hardly get the accurate estimation of natural inflows in a further future, a new framework, the-closer-the-more framework, is proposed for scenario tree construction. This framework can help to reduce the size of the problem. The solution approach is nonlinear stochastic dual dynamic programming, which is based on the approximation of the expected-cost-to-go function of stochastic dynamic programming by a piecewise linear function. Moving window method is used for planning. At each stage, annual water consumption constraints are divided two parts in optimization problem, one is for current year and the other is for next year. Numerical results are also presented for illustration. Efficiency and practicality can be seen in our simulation on several situations.**

***Index Terms*—framework, long-term planning, scenario tree, stochastic dual dynamic programming.**

I. INTRODUCTION

Hydro-thermal systems consist of reservoirs, hydropower plants and customers who need water and electric power. Long-term planning of hydro-thermal systems faces various challenges such as uncertainties of natural inflows, consumer needs and fuel price, nonlinear costs and large dimensions, etc. Long-term planning of hydro-thermal systems is usually modeled as an optimization problem to minimize total system cost subject to reservoir requirements, water balance equations, water demand constraints and electricity demand constraints.

Hydro-thermal systems have been researched for decades. The simplest model is in deterministic context [1] where future inflows are assumed known. But deterministic optimization usually results in larger cost [2].

Linear programming (LP), nonlinear programming (NLP) and dynamic programming (DP) techniques have been largely used to solve the problems of planning and management of hydro-thermal systems. Yeh provides a comprehensive survey of LP, NLP and DP [3]. Labadie gives a detailed review of different models, such as discrete dynamic programming models, stochastic dynamic programming models and so on [4]. Yakowitz presents a careful review on DP applications in water resources in both deterministic and stochastic cases [5]. Markov Decision Process (MDP) is also widely used to formulate and to solve the long-term hydro-thermal scheduling problem [6]. However, since the state space and the action space become bigger as the size of scheduling problem gets larger, the problem is intractable for both DP and MDP [7][8]. To deal with the curse of dimensionality, many algorithms are developed, like Rollout algorithm [7], aggregation methods [8] etc. Essentially, the long-term planning of hydro-thermal systems is a multistage stochastic dynamic optimization problem, which needs to consider the uncertainty of the future for sustainable water resource management. Stochastic programming (SP) is a powerful tool for optimization under uncertainty. Birge and Louveaux provide a good introduction to SP and a number of techniques [9]. Pereira and Pinto use stochastic dual dynamic programming to solve multi-stage stochastic optimization for energy planning [10]. Morton applies Benders decomposition to solve the stochastic linear reservoir scheduling problem [11]. Qi and Chen develop a nonlinear stochastic dual dynamic algorithm (NSDDP) for long-term water resources scheduling problem [12]. However, discretization of continuous random variables, e.g. natural inflows, are indispensible when implementing these algorithms. Many methods for solving stochastic optimization problems are based on scenario-tree modeling [13]. A finer discretization results in a larger size of the problem. Heitsch and Römisch develop theory-based heuristics for generating scenario trees, which are based on forward or backward algorithms consisting of recursive scenario reduction and bundling steps [14].

Regarding to the planning of hydro-thermal systems, in fact, we usually have less accurate forecast of natural inflows in a further future. On the other hand, natural inflows in a nearer future have much more impacts on

Manuscript received August 1, 2013; revised November 6, 2013.

doi: 10.12720/joace.2.3.256-264

hydro-thermal planning. As time moves on, when accurate data of natural inflows become available we can update the planning with new information. Different from the previous work on the planning of hydro-thermal systems, in this paper, we propose a new framework for multistage hydro-thermal planning in which finer discretization of continuous random variables (natural inflows) is used for nearer future. By this way, we can reduce the computation complexity for the planning problem.

The paper is organized as follows. In Section 2, the problem of long-term planning of hydro-thermal systems is formulated as a stochastic dynamic programming model. In Section 3, the-closer-the-more framework for scenario tree is discussed and the method for solving the problem is presented. In Section 4, a numerical example is given for illustration. Finally, Conclusion and future work are summarized in Section 5.

II. Formulation

In this section the long-term planning of hydro-thermal systems is formulated as a general constrained stochastic dynamic programming model. We build up a model for hydro-thermal systems which has 10 reservoirs and 6 cities. Those cities are consumers of water and electricity. The unfulfilled demands of water and/or electricity will lead to costs. The aim is to minimize total costs under various constraints, such as reservoir's capability, generator's capacity and annual water consumption quota, etc.

A. Notations

We first introduce notations used in our model.

- I number of reservoirs.
- C number of cities.
- T planning horizon. $T = 12$ months.
- t stage index, $t = 1, 2, ..., 12$.
- $\mathbf{v}_t$ reservoir storage vector at stage t; its dimension is I.
- $\mathbf{v}_0$ initial reservoir storage vector.
- $\overline{\mathbf{v}}_t, \underline{\mathbf{v}}_t$ upper and lower bounds of $\mathbf{v}_t$.
- ξ_t natural inflow vector at stage t; it's a random vector.
- $\mathbf{r}_t$ water used to generate electricity at stage t.
- $\overline{\mathbf{r}}_t$ upper bound of $\mathbf{r}_t$.
- $\mathbf{s}_t$ water spillage vector at stage t.
- $\mathbf{w}_t$ water consumption at stage t.
- $\overline{w}_{annual}$ annual water consumption quota vector.
- $\eta_{i,t}$ coefficient of hydro efficiency of reservoir i at stage t.
- p_t electricity shortage at stage t.
- de_t electricity demand at stage t.
- $\mathbf{o}_t$ water shortage vector at stage t.
- $\mathbf{dw}_t$ water demand vector at stage t.

B. Objective Function

Fig. 1 depicts a hydro-thermal system which consists of 10 reservoirs and 6 cities.

The objective function is

$$\min \underset{\xi_1,\xi_2,\dots,\xi_T}{E}\left\{\sum_{t=1}^{T} c_t(p_t,\mathbf{o}_t;\xi_t)\right\} \tag{1}$$

where $c_t(p_t,\mathbf{o}_t;\xi_t)$ represents the cost at stage t which is a function of electricity shortage and water shortage.

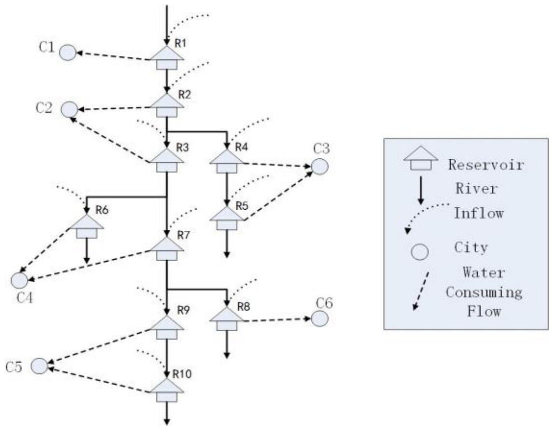

Figure 1. A hydro-thermal system with 10 reservoirs and 6 cities

C. Constraints

1) Water balance

$$\mathbf{v}_t = \mathbf{v}_{t-1} + \mathbf{A}(\mathbf{r}_t + \mathbf{s}_t) - \mathbf{w}_t + \xi_t + \mathbf{h}_t \tag{2}$$

where $\mathbf{h}_t = \begin{bmatrix} h_{t1} & 0 & \cdots & 0\end{bmatrix}^T$ and h_{t1} is the amount of water that flows into reservoir 1 from upstream river at stage t. $\mathbf{A} = \{a_{ij}\}^{I\times I}$ is the connection matrix of reservoirs and rivers. The components of $\mathbf{A}$ are defined as follows:

$$a_{ij} = \begin{cases} -1 & i = j \\ \tau_{ij} & \tau_{ij} \in [0,1], i \neq j, \end{cases} \tag{3}$$

where $\tau_{i,j}$ is the percentage of reservoir j's downstream water going into reservoir i. If reservoir j and reservoir i have no connection, then $\tau_{i,j} = 0$. If reservoir j has downstream reservoirs, then

$$\sum_{i,i\neq j} \tau_{i,j} = 1 \tag{4}$$

For the system shown in Fig. 1, the matrix of $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tau_{2,1} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau_{3,2} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau_{4,2} & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_{5,4} & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_{6,3} & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_{7,3} & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{8,7} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{9,7} & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau_{10,9} & -1 \end{bmatrix} \tag{5}$$

2) Shortage constraints

At each stage, water shortage vector $\mathbf{o}_t$ is determined by the water consumption vector $\mathbf{w}_t$, that is

$$\mathbf{o}_t = \max(\mathbf{0}, \mathbf{dw}_t - \mathbf{Uw}_t) \tag{6}$$

where $\mathbf{U} = \{u_{ij}\}^{C\times I}$ describes the relations between cities and reservoirs. If reservoir i supplies water to city j, then $u_{ij} = 1$; otherwise, $u_{ij} = 0$. For the system shown in Fig. 1, it is

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{7}$$

For the sake of simplicity, we write it into inequality constraints:

$$\mathbf{Uw}_t + \mathbf{o}_t \ge \mathbf{dw}_t \tag{8}$$

$$\mathbf{o}_t \ge 0. \tag{9}$$

Since electricity transmission is easier than water, we no longer count its shortage separately. As de_t represents the summation of all electricity demands and p_t is the shortage at stage t, p_t satisfies the following constraints:

$$\boldsymbol{\eta}^T \mathbf{r}_t + p_t \ge de_t \tag{10}$$

$$p_t \ge 0. \tag{11}$$

3) Annual water consumption quota

Total water consumption of all cities in a year cannot exceed the quota according to state regulations:

$$\sum_{t=1}^{T}\sum_{i=1}^{I} w_{ti} \le \bar{w}_{annual} \tag{12}$$

D. Summary

Put all these constraints together, the optimization problem has a form like this:

$$\begin{aligned} &\min \quad \underset{\xi_1,\xi_2,\dots,\xi_T}{E}\left\{\sum_{t=1}^{T} c_t(p_t, \mathbf{o}_t; \xi_t)\right\} \\ &s.t. \\ &\mathbf{v}_t = \mathbf{v}_{t-1} + \mathbf{A}(\mathbf{r}_t + \mathbf{s}_t) - \mathbf{w}_t + \xi_t + \mathbf{h}_t \\ &\mathbf{Uw}_t + \mathbf{o}_t \ge \mathbf{dw}_t \\ &\boldsymbol{\eta}^T \mathbf{r}_t + p_t \ge de_t \\ &\sum_{t=1}^{T}\sum_{i=1}^{I} w_{ti} \le \bar{w}_{annual} \\ &\mathbf{v}_t, \mathbf{r}_t, \mathbf{s}_t, \mathbf{w}_t, \mathbf{o}_t, p_t \ge 0 \quad t = 1, 2, \cdots, T. \end{aligned} \tag{13}$$

To make it simpler we define a new variable $\mathbf{aw}_t$:

$$\mathbf{aw}_t = \sum_{k=1}^{t} \mathbf{w}_k \tag{14}$$

Then (13) can be written as:

$$\begin{aligned} &\min \quad \underset{\xi_1,\xi_2,\dots,\xi_T}{E}\left\{\sum_{t=1}^{T} c_t(p_t, \mathbf{o}_t; \xi_t)\right\} \\ &s.t. \\ &\mathbf{v}_t = \mathbf{v}_{t-1} + \mathbf{A}(\mathbf{r}_t + \mathbf{s}_t) - \mathbf{w}_t + \xi_t + \mathbf{h}_t \\ &\mathbf{Uw}_t + \mathbf{o}_t \ge \mathbf{dw}_t \\ &\boldsymbol{\eta}^T \mathbf{r}_t + p_t \ge de_t \\ &\mathbf{aw}_{t-1} + \mathbf{w}_t = \mathbf{aw}_t \\ &\mathbf{1}^T \cdot \mathbf{aw}_T \le \bar{w}_{annual} \\ &\mathbf{v}_t, \mathbf{r}_t, \mathbf{s}_t, \mathbf{w}_t, \mathbf{o}_t, p_t \ge 0 \\ &t = 1, 2, \cdots, T, \end{aligned} \tag{15}$$

where $\mathbf{1}^T$ is a row vector that all elements are 1.

We denote $\mathbf{x}_t = (\mathbf{v}_t, \mathbf{r}_t, \mathbf{s}_t, \mathbf{w}_t, \mathbf{aw}_t, \mathbf{o}_t, p_t)$, then problem (15) can be described in the following form:

$$\begin{aligned} &\min \quad \underset{\xi_1,\xi_2,\dots,\xi_T}{E}\{\sum_{t=1}^{T} c_t(\mathbf{x}_t; \xi_t)\} \\ &s.t. \\ &\qquad \mathbf{M}_1\mathbf{x}_1 \le f_1(\xi_1) \\ &\qquad \mathbf{L}_{t-1}\mathbf{x}_{t-1} + \mathbf{M}_t\mathbf{x}_t \le f_t(\xi_t) \\ &\qquad t = 2, \dots, T. \end{aligned} \tag{16}$$

Note that only variables in adjacent stages appear in any inequality constraint. ξ_t represents the amount of natural inflow which is a continuous random variable. Its probability distribution can be estimated with historical data. The problem described with (16) is a multistage stochastic dynamic optimization problem.

III. Methodology

A. Moving Window for Dynamic Planning

Weather usually exhibits remarkable seasonal variation which has a period of one year. Naturally, we use a 12-month moving window for planning in each month. Suppose at the end of each year the annual water consumption quota will be checked. Hence we need to calculate the water consumption within one year.

For example, when current month moves to May, total water consumption in the first 4 months (from January to April), denoted as $w_{consumed}$, becomes known. Therefore, for the remaining 8 months (from May to December), the water consumption quota is

$$\tilde{\bar{w}} = \bar{w}_{annual} - w_{consumed}.$$

To update the planning for May, we simply set the water consumption quota as $w_{consumed}$ for the first 4 months in the next year as shown in Fig. 2.

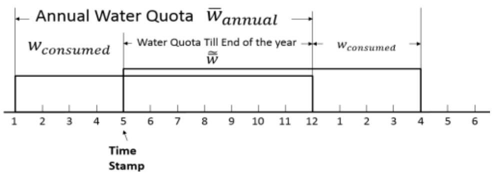

Figure. 2. Moving window and water consumption quota

By this way, the annual water consumption constraint, $\mathbf{1}^T \cdot \mathbf{aw}_T \le \bar{w}$, in model (15) can be written into two parts. The first part is about quota from current stage to December:

$$\mathbf{1}^T \cdot \mathbf{aw}_t \le \tilde{\bar{w}} \quad (t = 1, 2, \cdots, \tilde{T}),$$

where $t = 1, 2, \cdots, \tilde{T}$. For example, $\tilde{T}$ =8 when current stage is in May. The second part is about quota in next year:

$$\tilde{\bar{w}} \le \mathbf{1}^T \cdot \mathbf{aw}_t \le \bar{w} \quad (t = \tilde{T}+1, \cdots, T).$$

B. Scenarios based optimization

Usually, scenarios based multistage stochastic programs are used to solve problems as formulated in (16). For example, we consider only two months. The model (16) reduces to

$$\begin{aligned} \min \quad & c_1(\mathbf{x}_1; \xi_1) + E\{c_2(\mathbf{x}_2; \xi_2)\} \\ s.t. \quad & \\ & \mathbf{M}_1\mathbf{x}_1 \le f_1(\xi_1) \\ & \mathbf{L}_1\mathbf{x}_1 + \mathbf{M}_2\mathbf{x}_2 \le f_2(\xi_2) \end{aligned} \tag{17}$$

It is hard to evaluate the expectation of cost in the second month. Usually the probability density function of ξ_2 is considered to be discretized at several possible values with specific probabilities. For instance, we get the discrete distribution of ξ_2 from samples as, $P(\xi_2 = \xi_{21}) = p_1$ and $P(\xi_2 = \xi_{22}) = p_2$. Thus the inflow scenario tree which represents the probabilistic nature of the model is shown in Fig. 3.

Each node in the scenario tree is a realization of inflow at that stage. Thus a scenario is a path that begins at a node in first stage and ends at a second-stage node. With the scenario tree, the problem (17) can be described as a deterministic optimization problem:

$$\begin{aligned} \min \quad & c_1(\mathbf{x}_1; \xi_1) + p_1 \cdot c_2(\mathbf{x}_{21}; \xi_{21}) + p_2 \cdot c_2(\mathbf{x}_{22}; \xi_{22}) \\ s.t. \quad & \\ & \mathbf{M}_1\mathbf{x}_1 \le f_1(\xi_1) \\ & \mathbf{L}_1\mathbf{x}_1 + \mathbf{M}_2\mathbf{x}_{21} \le f_2(\xi_{21}) \\ & \mathbf{L}_1\mathbf{x}_1 + \mathbf{M}_2\mathbf{x}_{22} \le f_2(\xi_{22}) \end{aligned} \tag{18}$$

It is obvious that the scale of problem (18) is proportional to the number of nodes in the scenario tree shown in Fig. 3. From this example, we see that the scenario tree represents the random data and affects the accuracy of the optimization results. Detailed methods of generating scenarios from sampling data or given distributions can be found in [13].

In order to explain our idea clearly, we impose the following assumptions.

- The distribution of ξ_t , $t = 2, ..., T$ is independent for different t . Hence, conditional probability $P(\xi_t \mid \xi_1, \xi_2, ..., \xi_{t-1})$ simply equals to $P(\xi_t)$.
- The distribution of ξ_t is discrete and concentrated on finite points.

For example, a 3-stage problem of which the number of possible values of discrete probability distribution of $\xi_t (t = 1, 2, 3)$ are $1^2 2^4 3^3$ (t^n means ξ_t has n discrete possible values). The scenario tree of it is shown in Fig. 4.

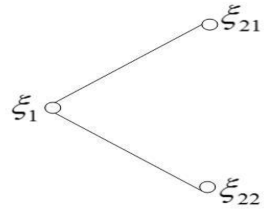

Figure 3. Scenario tree of a simple example

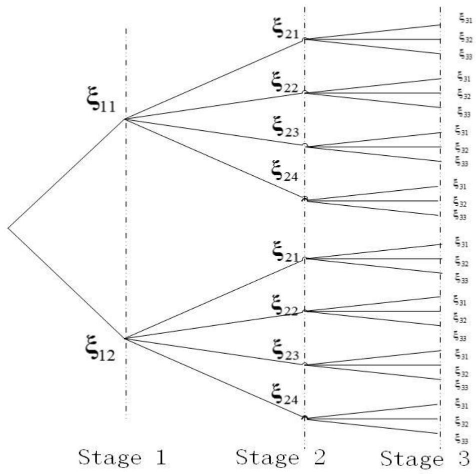

Figure 4. Scenario tree of a multistage example

C. NSDDP Algorithm

We have shown that, with scenario tree, a stochastic optimization problem can be written into a deterministic optimization problem and its scale is proportional to the number of nodes of the tree. Hence the scale of the problem increase as the number of stages increases. With nonlinear stochastic dual dynamic programming (NSDDP) algorithm we can get the optimal solutions for larger scale problems [12].

Under the dynamic programming formulation of the problem, when given $\mathbf{x}_{T-1}$, for the terminal stage, we have

$$\begin{aligned} Q_T(\mathbf{x}_{T-1}; \xi_T) = \min_{\mathbf{x}_T} \quad & c_T(\mathbf{x}_T; \xi_T) \\ s.t. \quad & \mathbf{L}_{T-1}\mathbf{x}_{T-1} + \mathbf{M}_T\mathbf{x}_T \le f_T(\xi_T). \end{aligned} \tag{19}$$

Let

$$\mathcal{Q}_{t+1}(\mathbf{x}_t) = E_{\xi_{t+1}}\left[Q_{t+1}(\mathbf{x}_t; \xi_{t+1})\right], \; t = 1, ..., T-1 \tag{20}$$

and

$$\begin{aligned} Q_t(\mathbf{x}_{t-1}; \xi_t) = \min_{\mathbf{x}_t} \quad & c_t(\mathbf{x}_t; \xi_t) + \mathcal{Q}_{t+1}(\mathbf{x}_t) \\ s.t. \quad & \mathbf{L}_{t-1}\mathbf{x}_{t-1} + \mathbf{M}_t\mathbf{x}_t \le f_t(\xi_t). \end{aligned} \tag{21}$$

Therefore, problem (15) can be rewritten in the following form:

$$\begin{aligned} Q_1(\mathbf{x}_0;\xi_1) = \min_{\mathbf{x}_0} \quad & c_1(\mathbf{x}_1;\xi_1) + \mathcal{Q}_2(\mathbf{x}_1) \\ s.t. \quad & \mathbf{L}_0\mathbf{x}_0 + \mathbf{M}_1\mathbf{x}_1 \le f_1(\xi_1). \end{aligned} \tag{22}$$

Since closed form expression of $\mathcal{Q}_2(\mathbf{x}_1)$ is not available, (22) cannot be solved directly.

It has been proved that $Q_t(\mathbf{x}_{t-1};\xi_t)$ and $\mathcal{Q}_t(\mathbf{x}_{t-1})$ are convex function of $\mathbf{x}_{t-1}$ when $c_t(\mathbf{x}_t;\xi_t)$ is convex. Thus we can create piecewise linear functions of $\mathbf{x}_{t-1}$ to approximate $Q_t(\mathbf{x}_{t-1};\xi_t)$ and $\mathcal{Q}_t(\mathbf{x}_{t-1})$.

Given a feasible solution $\tilde{\mathbf{x}}_{T-1}^l$ of stage $T-1$, solving problem (19) and its dual problem

$$\begin{aligned} Q_{T,dual}(\tilde{\mathbf{x}}_{T-1}^l;\xi_T) = \sup_{u\ge 0}[\ \inf_{\mathbf{x}_T} c_T(\mathbf{x}_T;\xi_T) + \\ u^T(\mathbf{L}_{T-1}\tilde{\mathbf{x}}_{T-1}^l + \mathbf{M}_T\mathbf{x}_T - f_T(\xi_T))] \end{aligned} \tag{23}$$

we can get the Lagrange multiplier $\tilde{\mathbf{u}}(\tilde{\mathbf{x}}_{T-1}^l;\xi_T)$.

Lemma 1[12]*:* Consider problem

$$\begin{aligned} G(\mathbf{x}) = \min_{\mathbf{y}} \quad & g(\mathbf{y}) \\ s.t. \quad & \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} \le \mathbf{f}. \end{aligned} \tag{24}$$

If $g(\mathbf{y})$ is a convex function of $\mathbf{y}$, problem (24) is feasible, and $G(\mathbf{x}) > -\infty$, then there is no duality gap between problem (24) and its dual problem, i.e.,

$$G(\mathbf{x}) = \sup_{\mathbf{u}\ge 0} q(\mathbf{u}), \tag{25}$$

where

$$q(\mathbf{u}) = \inf_{\mathbf{y}}\ g(\mathbf{y}) + \mathbf{u}^T(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} - \mathbf{f}), \quad u \ge 0,$$

and there exits at least one Lagrange multiplier $\tilde{\mathbf{u}}$, such that $G(\mathbf{x}) = q(\tilde{\mathbf{u}})$. Moreover, $G(\mathbf{x}') \ge G(\mathbf{x}) + \tilde{\mathbf{u}}^T\mathbf{A}(\mathbf{x}' - \mathbf{x})$, i.e., $\tilde{\mathbf{u}}^T\mathbf{A} \in \partial G(\mathbf{x})$, where $\partial G(\mathbf{x})$ means the subgradients of function $G(\mathbf{x})$ at $\mathbf{x}$.

With Lemma 1, we have

$$\begin{aligned} Q_T(\mathbf{x}_{T-1};\xi_T) \ge Q_T(\tilde{\mathbf{x}}_{T-1}^l;\xi_T) + \\ [\tilde{\mathbf{u}}(\tilde{\mathbf{x}}_{T-1}^l;\xi_T)]^T\mathbf{L}_{T-1}(\mathbf{x}_{T-1} - \tilde{\mathbf{x}}_{T-1}^l). \end{aligned} \tag{26}$$

Using (26), we can construct a piecewise linear approximation (27) for $\mathcal{Q}_T(\mathbf{x}_{T-1})$:

$$\begin{aligned} \hat{\mathcal{Q}}_T(\mathbf{x}_{T-1}) = \min_{\theta} \quad & \theta \\ s.t. \quad & \theta \ge g^l{}_T + \pi^l{}_T(\mathbf{x}_{T-1} - \tilde{\mathbf{x}}_{T-1}^l), \quad \forall l, \end{aligned} \tag{27}$$

where

$g^l{}_T = E_{\xi_T}[Q_T(\tilde{\mathbf{x}}_{T-1}^l;\xi_T)]$ and $\pi^l{}_T = E_{\xi_T}\left\{[\tilde{\mathbf{u}}(\tilde{\mathbf{x}}_{T-1}^l;\xi_T)]^T\mathbf{L}_{T-1}\right\}$.

Similarly, for $t = T-1, T-2, ..., 2$, given some feasible solution $\tilde{\mathbf{x}}_{t-1}^l$ of stage $t-1$, we can solve the following approximate problem:

$$\begin{aligned} \hat{Q}_t(\mathbf{x}_{t-1};\xi_t) = \min_{\mathbf{x}_t} \quad & c_t(\mathbf{x}_t;\xi_t) + \theta \\ s.t. \quad & \mathbf{L}_{t-1}\mathbf{x}_{t-1} + \mathbf{M}_t\mathbf{x}_t \le f_t(\xi_t) \\ & \theta \ge g^l{}_{t+1} + \pi^l{}_{t+1}(\mathbf{x}_t - \tilde{\mathbf{x}}_t^l), \quad \forall l, \end{aligned} \tag{28}$$

and get Lagrange multiplier $\mathbf{u}^*(\tilde{\mathbf{x}}_{t-1}^l;\xi_t)$. Approximation function $\hat{\mathcal{Q}}_t(\mathbf{x}_{t-1})$ can be constructed in almost the same way:

$$\begin{aligned} \hat{\mathcal{Q}}_t(\mathbf{x}_{t-1}) = \min \quad & \theta \\ s.t. \quad & \theta \ge g^l{}_t + \pi^l{}_t(\mathbf{x}_{t-1} - \tilde{\mathbf{x}}_{t-1}^l), \quad \forall l, \end{aligned} \tag{29}$$

where

$$g^l{}_t = E_{\xi_t}[\hat{Q}_t(\tilde{\mathbf{x}}_{t-1}^l;\xi_t)] \text{ and } \pi^l{}_t = E_{\xi_t}\left\{[\mathbf{u}^*(\tilde{\mathbf{x}}_{t-1}^l;\xi_t)]^T\mathbf{L}_{t-1}\right\}.$$

After constructing a piecewise linear approximation of $\mathcal{Q}_2(\mathbf{x}_1)$, we can solve the following problem (30), which is an approximate problem of (22):

$$\begin{aligned} Q_1(\mathbf{x}_0;\xi_1) = \min_{\mathbf{x}_0} \quad & c_1(\mathbf{x}_1;\xi_1) + \hat{\mathcal{Q}}_2(\mathbf{x}_1) \\ s.t. \quad & \mathbf{L}_0\mathbf{x}_0 + \mathbf{M}_1\mathbf{x}_1 \le f_1(\xi_1). \end{aligned} \tag{30}$$

The solution of (30) is an approximated optimal solution of (22).

In fact, to make the approximated optimal solution closer to the real one, more and more cuts should be added iteratively. The detailed steps of NSDDP algorithm can be found in [12].

The NSDDP algorithm decomposes large-scale problem generated from the scenario tree into numbers of small-scale problems. It sacrifices spaces to make large-scale problems be solved but computational time cost does not decrease like the scale. There is an exponential increase in the time consumption and in needed memory along with the increase of time horizon. The reason is that NSDDP algorithm divides the total optimization problem into very small but interconnected sub-problems and the number of sub-problems equals to the number of edges in scenario tree.

For a scheduling problem with 12 stages, even if the number of possible values of ξ in each stage is 4 (corresponding to critical, dry, normal, and above normal situations), there are $(4^{12}-1)/3 = 5592405$ sub-problems.

Four possible values of each random variable are not enough to describe a continuous probability distribution. But with more values, the curse of dimension difficulty needs to be overcome. There is a trade-off between the desired precision of the results and a manageable size.

D. The-closer-the-more Framework

In fact, our estimation of natural inflows which happens in a far future can hardly achieve satisfactory precision. Only for near future, we have relative accurate distribution of natural inflows. From this perspective, we propose the-closer-the-more framework for constructing scenario trees, as shown in Fig. 5. In this framework, each of the random variables in the first n_{near} stages has multiple possible values while each one in the rest stages takes its expectation as its unique possible value. This framework is more in line with reality. Moreover, with n_{near} fixed, this framework can help to avoid the curse of dimensionality as T increasing.

Denote n_{ξ_i} $(i = 1, 2, ..., n_{near})$ as the number of possible values for ξ_i respectively. Then the number of sub-problems (also the number of nodes on scenario trees) is

$$1 + n_{\xi_1} + n_{\xi_1} n_{\xi_2} + \cdots + n_{\xi_1} n_{\xi_2} \cdots n_{\xi_{n_{near}}} (1 + (T - n_{near})) \quad (31)$$

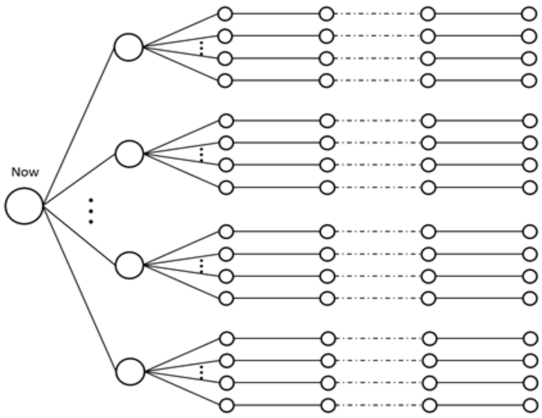

Figure. 5. Framework for scenario tree (n_{near} = 2 in example)

This number increases linearly with the time horizon T. For example, if $n_{\xi_i} = 4$ for each of T stages, then the total number of sub-problems is $(4^{12} - 1)/3 = 5592405$. Using the-closer-the-more framework with $n_{near} = 4$, the total number is 2133.

We see that, with the-closer-the-more framework, smaller n_{near} can save much more computation. But too small n_{near} may cause information losses which at last will affect the quality of planning.

IV. Case Study

In this part, we present a numerical example to illustrate how to solve the planning problems using scenario-based NSDDP algorithm under the-closer-the-more framework. All these programs are run in MATLAB on a PC with Intel® Core™2 Duo CPU E7500 @2.93GHz and 3.21G RAM.

A. Numerical Results of the Decomposition Method

The hydro-thermal system is as illustrated in Fig. 1. $T = 12$, representing 12 months in one year. We set $n_{near} = 4$. Electricity demand and water demand of cities are assumed to be known at each stage. The state variables are continuous and natural inflows, as random variables, are assumed discrete. The given data of the case is shown in Appendix.

The cost function at every stage is

$$c_t(p_t, \mathbf{o}_t; \xi_t) = (0.01 p_t + 0.001 p_t^2) + \sum_{i=1}^{6} (9 o_{ti} + 4 o_{ti}^2). \quad (32)$$

It's a quadratic function. Here, water demand is of higher priority than power demand. So we set the coefficients of water demands much larger than that of power demand. We need to meet the water demand as much as possible and then try to generate more electricity.

We simulate the decision-making process for each month of one year. In the process, we assume the decision is executed at the end of each month. Also, we assume, at the end of stage t $(t = 1, ..., 12)$, natural inflows in stage t become known. Hence, random variables ξ_t has been realized at the end of stage t. Each of $\xi_{t+1}, ..., \xi_{t+n_{near}}$ takes 4 realizations, and $\xi_{t+n_{near}+1}, ..., \xi_T$ are assigned with their own expectations respectively.

The 4 realizations of random variables represent 4 levels of water resources: critical, dry, normal, above normal. The twelve months include wet season, normal season and dry season. May. Jun. Jul. Aug. Sep. belong to wet season. Mar. Apr. Oct. Nov. belong to normal season. Jan. Feb. Dec. belong to dry season. In wet season the average natural inflow is the largest and that of dry season is the smallest.

We compute three kinds of realization of whole year natural inflows, namely best case, worst case and random case. For the random case, when time stamp moves to stage t, the natural inflow of that stage is generated from the probability distribution. For the worst (best) case, realization of natural inflow at every stage is the critical (above normal) one.

In the first stage, the optimal solution for the random case is obtained within 8 iterations as shown in Fig. 6.

The total costs of all stages for the three different cases are in Table I.

TABLE I. Total Cost in Four Cases

	Best case	Random case	Worst case
Total cost	60351.87	70664.33	89054.63

TABLE II. Water Shortage of Each City

Stage	City1	City2	City3	City4	City5	City6
1	39.42	39.15	39.4	39.34	39.37	39.41
2	38.97	38.78	38.59	38.8	38.76	38.65
3	41.75	41.52	41.1	41.49	41.28	41.55
4	43.19	43.04	42.68	42.87	42.88	42.89
5	40.38	40.57	40.39	40.38	40.6	40.3
6	43.18	43.02	43.08	43	43.04	43.01
7	39	40.79	36.54	39.08	38.16	41.45
8	42.13	42.76	42.5	42.9	43.02	42.57
9	43.17	42.95	43.01	42.97	42.95	43.04
10	37.93	38.17	37.72	37.85	37.93	38.23
11	39.77	39.4	38.59	39.33	39.25	41.29
12	44.49	44.46	43.77	44.49	43.77	50.74

Actually, if the shortages of all cities are almost the same, the total cost can be minimized according to the convex property of quadratic functions.

Water shortage of each city in every stages in the worst case are shown in Table II.

Even though the water demands of cities are quite different from each other, their water shortages have small differences.

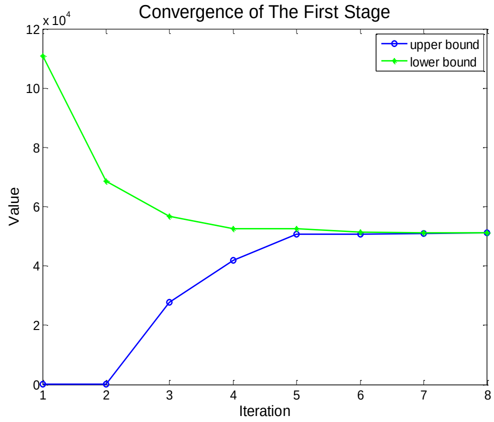

Figure. 6. Convergence of the first stage optimization problem

If the cost function is in a linear form, for example,

$$c_t(p_t, \mathbf{o}_t; \xi_t) = 0.01p_t + \sum_{i=1}^{6} 9o_{ti}.$$

Then we obtain the water shortage as below:

TABLE III. WATER SHORTAGE OF EACH CITY

Stage	City1	City2	City3	City4	City5	City6
1	107.00	200.00	102.95	130.07	146.78	91.85
2	102.00	194.00	0.00	196.78	0.34	90.28
3	93.00	208.00	97.69	1.48	99.12	0.11
4	93.00	204.00	100.96	155.30	154.77	0.14
5	73.16	34.34	0.00	182.82	0.00	98.29
6	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	7.10
11	0.00	0.00	0.00	0.00	0.00	88.85
12	92.00	54.04	34.67	0.00	0.00	34.09

We can see that, linear cost function cannot help to strike a balance in water consumption among cities over the planning horizon. Moreover, the total water shortage is larger with linear cost function as shown in Table IV.

TABLE IV. TOTAL WATER SHORTAGE UNDER DIFFERENT COST FUNCTION

	quadratic	linear
total water shortage	2962	3269

B. *Influence of the Annual Water Consumption (AWC) Constraint*

We compare the solutions of the optimal scheduling problems with and without the constraint over the planning horizon under worst case. The water shortages without AWC constraint are in Table V.

Without water quota, water shortage is relatively less. Only downstream city, city 6, suffers bad water shortage. Since there is no limit in water consumption, upstream cities will use water to meet their demands and less water flows downstream.

TABLE V. WATER SHORTAGE OF EACH CITY

Stage	City1	City2	City3	City4	City5	City6
1	0.0	0.0	0.0	0.0	0.0	28.5
2	3.6	3.6	0.0	6.5	1.4	51.9
3	0.0	0.0	0.0	0.0	0.0	47.8
4	0.0	0.0	0.0	0.0	0.0	54.4
5	0.0	0.0	0.0	0.0	0.0	30.3
6	0.0	0.0	0.0	0.0	0.0	27.7
7	0.0	0.0	0.0	0.0	0.0	15.5
8	0.0	0.0	0.0	0.0	0.0	22.1
9	0.0	0.0	0.0	0.0	0.0	35.8
10	0.0	0.0	0.0	0.0	0.0	52.9
11	2.7	2.7	0.0	4.0	0.0	57.4
12	46.6	46.6	15.8	50.0	12.8	77.6

Annual water quantity that goes into downstream regions are as follows.

TABLE VI. WATER FLOW TO DOWNSTREAM REGIONS

	with AWC	without AWC
downstream water	3686.42	1993.03

Without AWC constraint, downstream regions get much less water. This comparison shows that AWC can help to allocate water resource fairly among upstream and downstream regions.

V. CONCLUSION

In this paper we propose the-closer-the-more framework for constructing the scenario tree for long-term planning problem of hydro-thermal systems. The idea of this framework, the closer the more, comes from the innate character of long-term planning problems. Then we apply stochastic dual dynamic programming to get the optimal planning and update the planning when new accurate data of natural inflows become available. The-closer-the-more framework can help reduce the computation complexity of the planning problem. Analysis and case study show the efficiency and practicability of our method.

APPENDIX

We list out the data of the testing case in section IV. Table VII presents parameters of reservoirs. Table VIII presents water demand and electric power demand. Inflows from upstream river and natural inflow in different situations are shown in Table IX, X, XI, XII.

TABLE VII. RESERVOIR PARAMETERS

Reservoir	v_0	v_{max}	v_{min}	v_{final}	r_{max}	η
1	1050	1300	1040	1000	600	0.9
2	950	1100	880	900	500	0.85
3	900	1050	840	850	500	0.93
4	850	1000	800	800	600	0.87
5	800	950	760	750	600	0.9
6	800	950	760	750	500	0.85
7	750	900	720	700	500	0.93
8	750	900	720	700	600	0.87
9	700	850	680	650	600	0.93
10	650	800	640	600	500	0.87

TABLE VIII. WATER DEMAND AND ELECTRIC POWER DEMAND EVERY MONTH

month	dw_1	dw_2	dw_3	dw_4	dw_5	dw_6	de
1	107	200	103	197	147	92	3200
2	102	194	91	200	144	94	4400
3	93	208	100	209	140	101	3600
4	93	204	101	198	157	109	3600
5	96	196	100	197	152	107	4400
6	90	207	91	197	146	104	4400
7	91	197	105	203	152	92	4800
8	107	202	92	200	142	97	5200
9	97	194	109	195	154	103	4000
10	100	196	103	192	157	104	3600
11	108	202	94	208	146	101	3200
12	92	203	103	208	155	91	3200

TABLE IX. WATER FROM UPSTREAM INFLOW

	above normal	normal	dry	critical
Wet Season	1500	1200	900	600
Normal Season	1000	800	600	400
Dry Season	200	160	120	80

TABLE X. NATURAL INFLOW DISTRIBUTION DRY SEASON

Reservoir	above normal	normal	dry	critical
1	51	37	38	17
2	49	35	33	26
3	52	30	23	16
4	52	31	38	20
5	44	48	32	24
6	49	30	37	13
7	50	46	36	13
8	58	30	38	11
9	54	38	25	29
10	51	45	38	17
P	0.35	0.4	0.15	0.1

TABLE XI. NATURAL INFLOW DISTRIBUTION IN WET SEASON

Reservoir	above normal	normal	dry	critical
1	85	61	59	44
2	86	66	59	45
3	75	69	65	46
4	86	69	69	46
5	79	77	53	41
6	84	60	69	49
7	80	72	63	52
8	71	67	54	46
9	85	72	50	51
10	89	79	58	57
P	0.35	0.4	0.15	0.1

TABLE XI. NORMAL NATURAL INFLOW DISTRIBUTION

Reservoir	above normal	normal	dry	critical
1	66	47	40	38
2	61	48	48	40
3	69	62	45	48
4	55	61	49	33
5	61	61	40	36
6	70	64	52	44
7	63	64	40	30
8	57	53	38	46
9	63	52	40	44
10	59	46	48	41
P	0.2	0.3	0.3	0.2

ACKNOWLEDGMENT

This work was supported in part by the 111 International Collaboration Project of China.

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