Parameters Analysis for the Time-Varying Automatically Adjusted LPA Based Estimators

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Abstract—**The paper provides the performance analysis of the optimal parameters selection for the weighted local polynomial approximation (LPA) estimators combined by a data-driven adaptive method used for the automatically adjusted nonparametric LPA estimator size detection. The provided examples show that the LPA estimators upgraded by the modified adaptive intersection of the confidence intervals (ICI) based method (called the RICI method) outperform those based on the original ICI rule, while the ICI rule based LPA estimators were known to outperform non-adaptive ones with fixed size window length. The method's performance is analyzed in noise environment for two signals, showing the modified ICI based LPA estimators to be superior to those based on the original ICI method in terms of denoising estimation error reduction. The method can be applied in various technical fields, including image and video filtering, beamforming for phased array radar with antenna switching, acoustic echo cancellation, instantaneous frequency estimation, etc.**

Index Terms—**adaptive filtering, signal denoising, signal reconstruction, edge preserving**

I. Introduction

The estimation goal is to detect estimated value as close as possible to the true one minimizing the estimation error. However, the estimation quality in noisy environments is often corrupted by the inescapable estimation bias and variance [1]. For biased estimators the bias usually cannot be determined in advance since it is dependent on the estimated value itself and its derivatives [1], [2]. On the other hand, noise environments, due to their stochastic nature, often cause estimation variance which is usually inversely proportional to the bias of the biased estimators [1], [2]. Hence, there exists a bias-to-variance trade-off which ensures minimal mean squared error (MSE) calculated as a sum of the squared bias and the variance [1], [2].

The estimation of the optimal parameter and corresponding bias-to-variance trade-off can be done using the non-parametric data-driven algorithm based on the ICI rule [2] or its modification called the relative intersection of the confidence intervals (RICI) [3]. This adaptive algorithm was also applied to the instantaneous frequency estimation based on time-frequency distributions [4], [5], as well as to signal and image filtering [6], the direction-of-arrival estimation [7], determination of time-frequency distributions' values [8], calculation of the Fourier transform [9], beamforming for localization of moving sources using a phased array radar with antenna switching [10], acoustic echo cancellation [11], and various other applications.

In this paper, we have analyzed parameters selection effect on the performance of the adaptive weighted LPA estimators, combined with the RICI method, in terms of the method's denoising quality when compared to the one obtained using the original ICI base method (known to outperform non-adaptive LPA estimators [2], [3]). As shown in the paper, the RICI based method performs competitively for all tested signals, with its parameters used in the paper outperforming the original ICI based method.

The paper is organized as follows. The original LPA-ICI method is briefly presented in the next section. Section III describes the modified LPA-ICI method, called the LPA-RICI method. The results are given in Section IV, while the Conclusion is presented in Section V.

II. The LPA-ICI Method

In the paper the noise-free signal estimate $\hat{x}(n)$ was extracted from a noisy signal $y(n)=x(n)+\nu(n)$ corrupted by the additive white Gaussian noise $\nu(n)$ ($\mathcal{N}\ (0,\sigma^2)$), where $x(n)$ is the original noise-free signal. There are two key requirements demanded by the LPA-ICI method in order to obtain denoised estimate from the noisy signal as close to the noise-free signal as possible, while preserving all desirable signal features (such as slope changes or jumps) [2]. The first one is to found an appropriate adaptive data-driven filter support size (calculated by the ICI rule), while the second one is to design an appropriate filter (done by the LPA used as the filter design tool) [2].

The ICI rule, used to calculate the adaptive data-driven filter support size h, results in the proper filter size minimizing the estimation error:

$$e(n,h)=x(n)-\hat{x}(n)\,, \tag{1}$$

where [1], [2]:

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$$E\left\{e^2(n,h)\right\} = std^2(n,h) + \omega^2(n,h) \tag{2}$$

$\omega(n,h)$ and $std(n,h)$ are bias and standard deviation of the estimate, respectively. As shown in [1], [2], by minimizing the mean squared risk $r(n,h)$:

$$\min(r(n,h)) = std^2(n,h^*)(1+\gamma^2) \tag{3}$$

the ICI rule the provides proper filter size h^*, where:

$$\gamma = \frac{\omega(n,h)}{std(n,h)} \tag{4}$$

As shown in [2], for the estimation error, the following inequality stands true:

$$\left|e(n,h)\right| \leq \omega(n,h) + \chi_{1-\alpha/2} \cdot std(n,h) \tag{5}$$

where $\chi_{1-\alpha/2}$ is the ($1-\alpha/2$)-th quintile of the standard Gaussian distribution.

The ICI rule introduces a sequence of growing filter support sizes $H = \left\{h_1 < h_2 < \cdots < h_N\right\}$ and selects the proper filter support size from the set H for each n by tracking the intersection of confidence intervals $D(n,h_i) = \left[L(n,h_i), U(n,h_i)\right]$, where [2]:

$$L(n,h_i) = \hat{x}(n,h_i) - \Gamma \cdot std(n,h_i) \tag{6}$$

is the lower confidence interval limit, and:

$$U(n,h_i) = \hat{x}(n,h_i) + \Gamma \cdot std(n,h_i) \tag{7}$$

is the upper confidence interval limit, Γ is the ICI rule threshold value, and $\hat{x}(n,h_i)$ is calculated using the LPA filter the size of which is h_i [12].

The proper filter support size is than calculated as the largest h_i providing the estimation bias to variance trade-off by satisfying the following condition [1]-[3]:

$$\underline{U}(n,h_i) \geq \overline{L}(n,h_i) \tag{8}$$

where $\underline{U}(n,h_i)$ is the smallest upper confidence interval limit calculated as [2]:

$$\underline{U}(n,h_i) = \min_{h_j = h_1,\cdots,h_i} U(n,h_j) \tag{9}$$

and $\overline{L}(n,h_i)$ the largest lower confidence interval limit calculated as [2]:

$$\overline{L}(n,h_i) = \max_{h_j = h_1,\cdots,h_i} L(n,h_j) \tag{10}$$

An example of confidence intervals and its intersection used for filter support size selection by the ICI based method is given in Fig. 1.

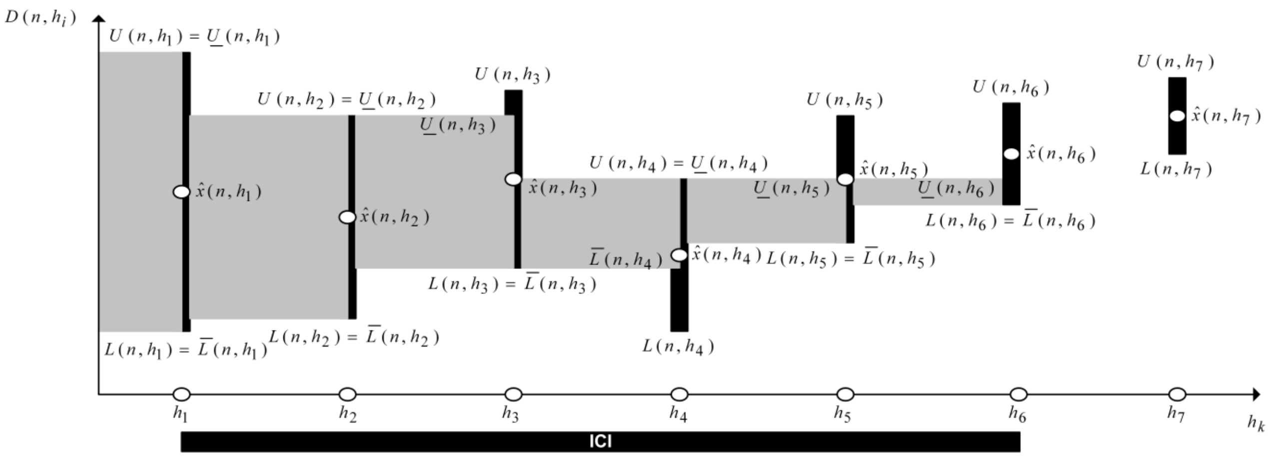

Figure 1. An example of the filter support selection using the ICI based method showing the confidence intervals $D(n,h_i)$ and their intersection.

The next section presents the modification of the LPA-ICI method resulting in the improved method's performances.

III. The Modified LPA-ICI Method

The LPA-ICI method is known to be highly dependent on the parameter Γ selection [3]. Various method for its selection were proposed, often more challenging than the denoising problem itself [2]. Imprecise selection of Γ can cause undersized filter support size in case of too small Γ value [2]. The problem of undersized filter support size can be solved by increasing the Γ value [3]. On the other hand, too large Γ values result in oversized filter support size [2]. The oversized filter support size can be solved by the modified ICI rule (RICI rule), introducing additional criteria, beside the intersection of confidence intervals, defined as [3]:

$$R(n,h_i) \geq R_c \tag{11}$$

where $R(n,h_i)$ is the ratio of the confidence interval intersection size and the confidence interval size calculated as [3]:

$$R(n,h_i) = \frac{\underline{U}(n,h_i) - \overline{L}(n,h_i)}{U(n,h_i) - L(n,h_i)} \tag{12}$$

Thus, the RICI rule considers the amount of the confidence interval intersection with regards to the confidence interval length, unlike the original ICI based method which requires only the existence of the intersection of all previous confidence intervals. An example of the filter support size selection based on the LPA-RICI method is given in Fig. 2.

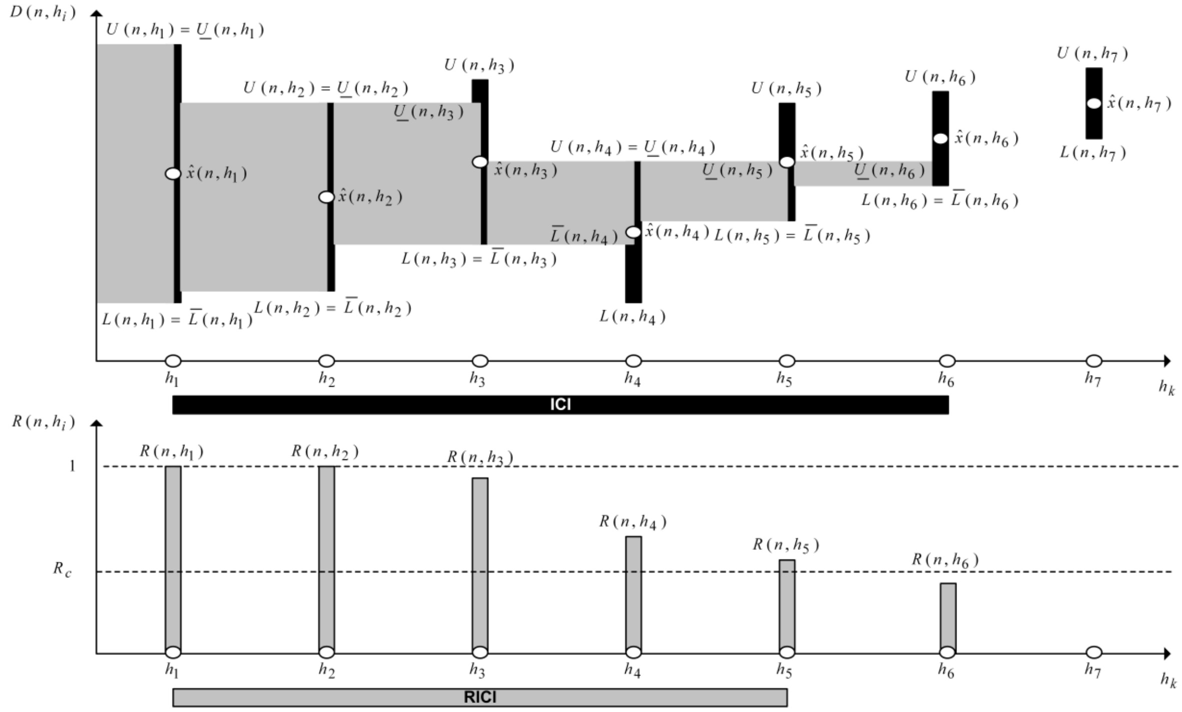

Figure 2. An example of the filter support selection using the RICI based method showing the confidence intervals $D(n,h_i)$ and their intersection (top), and relative intersection of confidence intervals $R(n,h_i)$ (bottom).

The RICI based method was shown to outperform the original ICI based method in terms of denoising quality, allowing us to use larger parameter Γ values, and at the same time avoiding the oversized filter support size due to newly introduced criteria, as explained in the next section.

The LPA-RICI method's performance depends on the proper Γ and R_c value selection, as shown in [3]. However, the LPA-RICI method's performance was shown to be significantly less sensitive to Γ and R_c value selection than the LPA-ICI method is sensitive to proper Γ value selection [13]. Furthermore, R_c was shown to belong to the finite interval $0 \le R_c \le 1$ (unlike the Γ value belonging to interval $\left[0,+\infty\right\rangle$), making it much easer than to find the proper Γ value for the ICI based method. Proper Γ and R_c values selection for the LPA-RICI method was dealt with in [13], proposing the formula for (Γ, R_c) pair.

The next section presents the performance study of the LPA-RICI method with parameters selected using the formula given in [13].

IV. Results

The LPA-RICI method is applied to the two test signals (*HeavySine* and *Cusp* signal) and its denoising performance is analyzed for various Γ and R_c values and signal lengths. The signals are corrupted by zero-mean additive white Gaussian noise with the standard deviation of the signal to the standard deviation of the noise ratio being 7, as in [12]. The estimation efficiency is measured by the root mean-square error (RMSE) of the denoised signal (averaged over $M=50$ noise realizations for each (Γ, R_c) pair), calculated as [2]:

$$RMSE = \sqrt{\frac{1}{M}\sum_{j=1}^{M}\frac{1}{N}\sum_{n=1}^{N}\left(x(n)-\hat{x}_j(n)\right)^2} \quad (13)$$

where N is the signal length (128, 256, 512, 1024 and 2048 signal lengths were analyzed) and j stands for the j-th simulation run. The LPA of order two is used for the signal estimation.

Fig. 3 shows the estimation results for the *HeavySine* signal ($N=1024$). The noise-free signal is shown in Fig. 3(a) and the noisy signal is given in Fig. 3(b). The denoised signal, using both the LPA-ICI (blue, $\Gamma=4.4$ as in [2]) and the LPA-RICI method (red, with the best (Γ, R_c) pair), is shown in Fig. 3(c), while the estimation error for both methods is given in Fig. 3(d). The estimation error energy, calculated as:

$$E_e = \sum_{n=1}^{N}\left|e(n,h_i)\right|^2 \quad (14)$$

is reduced more than four-fold, from $E_e = 241.74$ for the LPA-ICI method to the $E_e = 59.84$ for the LPA-RICI method. The adaptive filter support size for each signal sample is shown in Fig. 3(e) for both the LPA-ICI (blue) and the LPA-RICI method (red). Fig. 3(f) shows the Γ and R_c values in (Γ, R_c) region for which the RMSE does not exceed 10% of its global minimum.

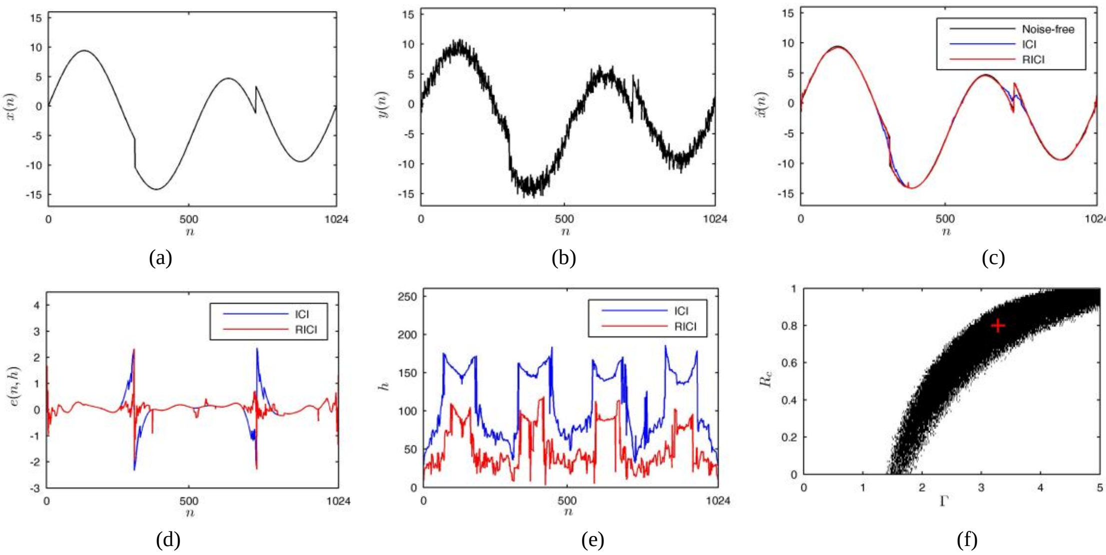

Figure 3. *HeavySine* signal, $N = 1024$, $\Gamma = 3.28$ and $R_c = 0.80$ (a) Noise-free. (b) Noisy signal. (c) Estimated signal using the LPA-RICI (red) and the LPA-ICI (blue) method. (d) Estimation error for the LPA-RICI (red, $E_e = 59.84$) and the LPA-ICI method (blue, $E_e = 254.74$). (e) Filter support lengths obtained by RICI (red) and ICI (blue) based method. (f) Region of Γ and R_c values for which the RMSE is not more than 10% larger than the minimum RMSE.

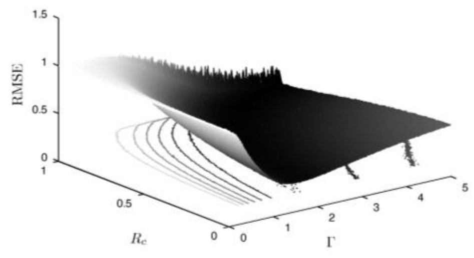

Figure 4. The RMSE as a function of parameters Γ and R_c for the *HeavySine* signal.

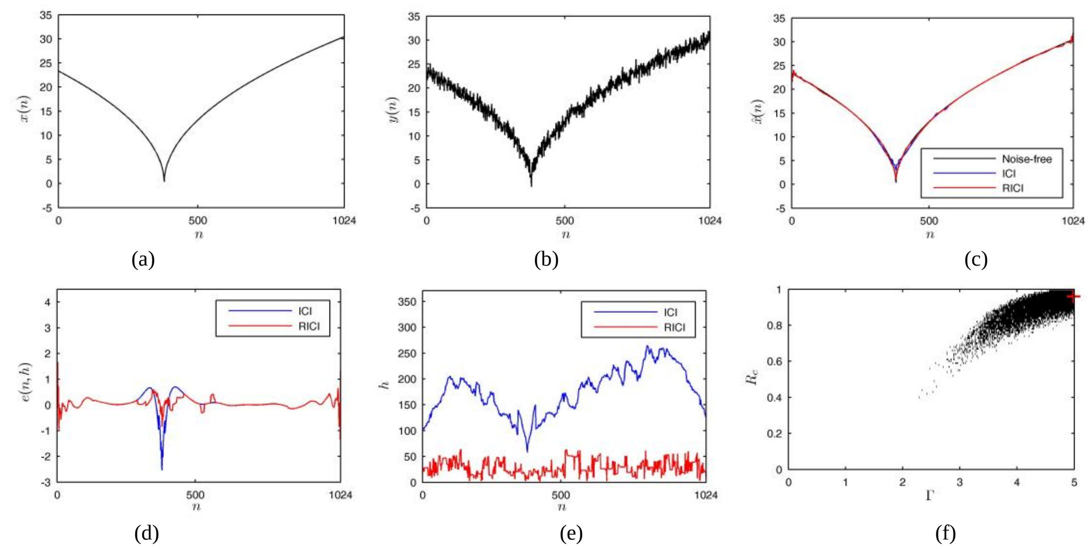

Figure 5. *Cusp* signal, $N = 1024$, $\Gamma = 4.99$ and $R_c = 0.96$ (a) Noise-free. (b) Noisy signal. (c) Estimated signal using the LPA-RICI (red) and the LPA-ICI (blue) method. (d) Estimation error for the LPA-RICI (red, $E_e = 41.96$) and the LPA-ICI method (blue, $E_e = 118.12$). (e) Filter support lengths obtained by RICI (red) and ICI (blue) based method. (f) Region of Γ and R_c values for which the RMSE is not more than 10% larger than the minimum RMSE.

TABLE I. THE RMSE OF ESTIMATED HEAVYSINE SIGNAL AS A FUNCTION OF Γ AND R_c FOR THE RANGE OF N VALUES (THE SELECTED Γ AND R_c PAIRS ARE THE BEST OBTAINED FOR EACH SIGNAL LENGTH N)

	LPA-RICI					LPA-ICI
Γ	2.41	3.37	3.65	3.28	4.32	4.4
R_c	0.77	0.90	0.89	0.80	0.89	/
$N=128$	**0.57**	0.61	0.61	0.63	0.66	0.81
$N=256$	0.50	**0.47**	0.49	0.49	0.52	0.78
$N=512$	0.40	0.39	**0.36**	0.38	0.39	0.72
$N=1024$	0.34	0.30	0.29	**0.28**	0.29	0.51
$N=2048$	0.30	0.25	0.23	0.23	**0.21**	0.37

TABLE II. THE RMSE OF ESTIMATED CUSP SIGNAL AS A FUNCTION OF Γ AND R_c FOR THE RANGE OF N VALUES (THE SELECTED Γ AND R_c PAIRS ARE THE BEST ONES OBTAINED FOR EACH SIGNAL LENGTH N)

	LPA-RICI					LPA-ICI
Γ	4.56	4.95	4.93	4.99	4.87	4.4
R_c	0.95	0.98	0.97	0.96	0.95	/
$N=128$	0.47	0.50	0.48	0.48	0.38	0.74
$N=256$	0.39	0.36	0.38	0.39	0.38	0.57
$N=512$	0.29	0.28	0.28	0.29	0.29	0.47
$N=1024$	0.22	0.22	0.22	0.21	0.22	0.33
$N=2048$	0.17	0.17	0.17	0.17	0.17	0.22

The parameters Γ and R_c, including the optimal ones for different N, for which the minimum RMSE is obtained are given in Table I. The RMSE results for the *HeavySine* signal as the function of the parameter Γ and R_c values is given in Fig. 4. As it can be seen from Fig. 4 the RMSE decreases as N grows, while the proper R_c value is $0.77 \le R_c \le 0.9$ and the proper value Γ is $2.41 \le \Gamma \le 5$ (similar to those achieved in [13] for some other signals). The method allows us to use larger Γ values, and thus the confidence intervals with larger confidence level than the LPA-ICI method.

Denoising results for the *Cusp* signal are shown in Fig. 5 ($N=1024$). The noise-free and noisy signal are given in Fig. 5(a) and Fig. 5(b), respectively. Noise-free signals estimated both by the LPA-ICI (blue, $\Gamma = 4.4$ as in [2]) and the LPA-RICI method (red, with the best (Γ, R_c) pair) are presented in Fig. 5(c). Estimation error is shown in Fig. 5(d), with the estimation error energy reduced almost three-fold, from $E_e = 118.12$ for the LPA-ICI method to the $E_e = 41.96$ for the LPA-RICI method. The adaptive automatically adjusted filter support size for each signal sample for both the LPA-ICI (blue) and the LPA-RICI method (red) are shown in Fig. 5(e). The parameters Γ and R_c in (Γ, R_c) region for which the RMSE does not exceed 10% of its global minimum are given in Fig. 5(f). As it can be seen, the region of proper Γ and R_c values minimizing estimation error is almost identical for both signals although the signals are significantly different. Furthermore, it is similar to those given in [13], proving the method for Γ and R_c value selection given in [13] robust to signal types and signal lengths.

Table II provides the *Cusp* signal RMSE results for the LPA-ICI (with $\Gamma = 4.4$ as in [2]) and the minimal RMSE values obtained by the LPA-RICI method (with best (Γ, R_c) pair also given in the table) showing the LPA-RICI method to outperform original LPA-ICI method for all signal lengths. Furthermore, the RMSE is shown to decreases as N grows, with the proper Γ value is $4.56 \le \Gamma \le 4.99$ and R_c value belonging to $0.95 \le R_c \le 0.98$ (close to the results obtained in [13] for other signals).

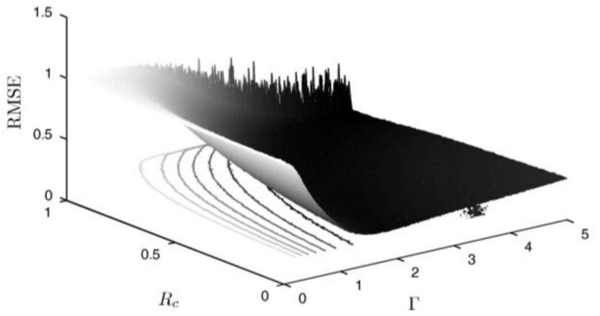

Figure 6. The RMSE as a function of parameters Γ and R_c for the *Cusp* signal.

The RMSE for the *Cusp* signal as the function of the parameter Γ and R_c values is given in Fig. 6. As it can be seen from the figure, the LPA-RICI method allows us to use larger Γ values (meaning confidence intervals with larger confidence level) than the LPA-ICI method, and at the same time provides estimation error reduction due to the additional and stricter criteria in the filter support size selection procedure for all tested signals of various signal lengths.

V. CONCLUSION

The paper has presented the performance analysis of the parameters selection procedure for the LPA-RICI method applied to two test signals. The LPA-RICI method, derived from the original LPA-ICI method, is a nonparametric data-driven method providing time-varying automatically adjusted filter supports combined with the weighted LPA based estimators and applied to the signal denoising. As shown in this paper, the LPA-RICI method outperforms the original LPA-ICI method, reducing the estimation RMSE by up to two times and reducing the estimation error energy by up to four times. Due to significant improvement of the LPA-ICI method in signal denoising by the LPA-RICI method, the similar enhancements are expected in various technical fields where the original ICI based method was shown to perform well, such as image and video filtering, time-frequency analysis, Fourier transform calculation, source tracking when sources are moving rapidly within the window and motion of other sources is weak, acoustic echo cancellation, beamforming for estimating movement parameters in noisy environment using a phased array radar with antenna switching, etc.

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