Temperature Distribution in Three Model Houses with Different Roof Geometries

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***Abstract*—Steady-state temperature distribution of three model houses, each with a roof of different shape, are presented in this paper. The governing equation used was a heat conduction equation in two dimensions. The assumption made was that the houses were of closed space with no convection nor radiation. The distributions were determined by using a finite difference method (FDM) and a finite element method (FEM). They showed that different roof geometries yielded different temperature distributions. In particular, the roof with a convex shape let in less heat than the one with a standard (triangular) shape and the one with a concave shape in that order.**

***Index Terms*—heat transfer, finite element method**

I. Introduction

Currently, the average ambient temperature in Thailand is steadily increasing due to global warming. Most buildings are equipped with an air conditioning system that consumes a large amount of energy. There are many different ways to help reduce this excessive consumption. For example, a good landscape design inside and outside of a building can make it cooler to live in. In addition, uses of reflective paint and proper insulation can cool it down further. A suitably shaped roof is also another way to prevent heat from getting into the interior of a building.

Many researchers have attempted to improve heat transfer characteristics of buildings either by doing simulation or actual experiment. In 2005, V. Cheng et.al [1] described the effects that color and thermal mass of a building had on its indoor temperature. In the same vein, Y. Ungkoon and B. Israngkura Na Ayudhya [2]-[3], built two model houses to investigate the impact of two kinds of concrete walls. Recently, S. Siriteerakul et.al [4]-[6] simulated heat diffusion through three types of roof tiles, heat conduction within a building with three different glass wall arrangement patterns, and heat transfer in buildings with different geometrical structures.

This study investigated the effect of several different shapes of roof on heat transfer in a rectangular building. The investigation was done by simulation using a finite difference method under Crank-Nicolson scheme and a finite element method under Galerkin principle in two dimensional rectangular coordinate system. The assumption made was that the building was a closed space with no convection nor radiation. The different steady-state temperature distribution at the centerline of each of the three model buildings, from the top of the roof to the floor, was computed and then all three of them were compared.

II. Governing Equation

In this paper, the heat transfer in each of our three model houses with differently shaped roof was solved by using a heat conduction equation under the assumption of no convection and radiation.

The governing equation of heat conduction in two dimensional rectangular coordinate systems can be expressed as:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{c^2}\frac{\partial T}{\partial t} \quad (1)$$

where T is the temperature and c^2 is the thermal diffusivity. At steady state, the equation becomes Laplace equation.

III. Methodology

To determine the solution, a finite difference method and a finite element method were used as follows:

A. Finite Difference Method

Finite difference method is a classical numerical method for solving differential equation with simple domain. This method (Runge [7]) was first used in 1908. Since then, many researchers have applied it to problems in elastics and fluid dynamics such as Richardson [8], Young and Wheeler [9], Richtmyer and Morton [10], Roache [11], and Crochet et al. [12].

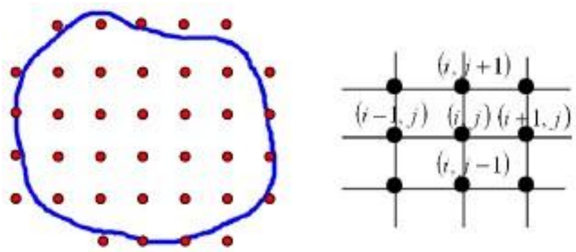

Figure 1. Generating grid points

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For a given problem, one starts by overlaying the domain with a uniform grid and assigning nodal points as shown in Fig. 1. Then, determine the finite difference that approximates the derivative of each of the nodal points. There are three approximation schemes: explicit scheme (forward difference), implicit scheme (backward difference), and Crank-Nicolson scheme (central difference). For our heat diffusion equation, the numerical solution was based on Crank-Nicolson scheme.

From our governing equation, the Laplace equation, we estimated the derivatives by using Taylor series expansion as follows:

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,y}}{h^2} \quad (2)$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{k^2} \quad (3)$$

After substitution and rearrangement, the equation for $h=k$ becomes

$$T_{i+1,j} + T_{i-1,y} - 4T_{i,j} + T_{i,j+1} + T_{i,j-1} = 0 \quad (4)$$

From equation (4), after substituting the actual value of every known term from the boundary condition, we obtained a new equation for each of the unknown points. These were simultaneous equations, a linear system of equations. This linear system of equation (Ax=b) can be solved either by a direct technique or an iterative technique. Direct techniques are such as Gauss Elimination, LU Factorization, or Cholesky decomposition LDL^t. Iterative techniques are such as Jacobi iterative method, Gauss-Seidel iterative technique, and Successive Over-Relaxation technique.

B. Finite Element Method

Finite element method is a numerical method to solve boundary value problems which the domains are various shapes. This method has been used first in 1965 by Zienkiewicz and Cheung [13]. Then, many researcher developed and applied this method to flow problems for example Oden and Wellford (1972) [14], Chung (1978) [15], Baker (1983) [16-18]. In recently, the finite element method has been solved the solution of complex problems in different field of engineering and science: heat conduction, vibration and continuum fluid dynamics.

Some group attempted create software program to solve the problems. For example, P. Dechaumpai and S.Phongthanapanich created Easy FEM software [19], which used the finite element method. This software was created under Galerkin principle.

In this study, we employed this software with the unstructured meshes. Therefore, each element was considered in linear triangular mesh and transform in ξ and η coordinates as shown in Fig. 2.

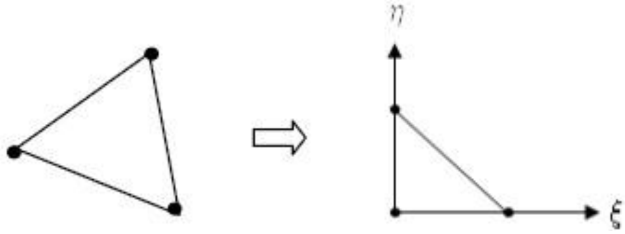


Figure 2. linear triangular mesh

Their linear shape functions are following

$$N_1 = 1 - \xi - \eta \quad (5)$$

$$N_2 = \xi \quad (6)$$

$$N_3 = \eta \quad (7)$$

And their derivative of ξ and η as follows:

$$\frac{\partial N_1}{\partial \xi} = -1, \quad (8)$$

$$\frac{\partial N_1}{\partial \eta} = -1 \quad (9)$$

$$\frac{\partial N_2}{\partial \xi} = 1, \quad (10)$$

$$\frac{\partial N_2}{\partial \eta} = 0 \quad (11)$$

$$\frac{\partial N_3}{\partial \xi} = 0, \quad (12)$$

$$\frac{\partial N_3}{\partial \eta} = 1 \quad (13)$$

IV. PROBLEM SPECIFICATIONS

The three domains of this problem are shown in Fig. 3. The high of all model houses are similar.

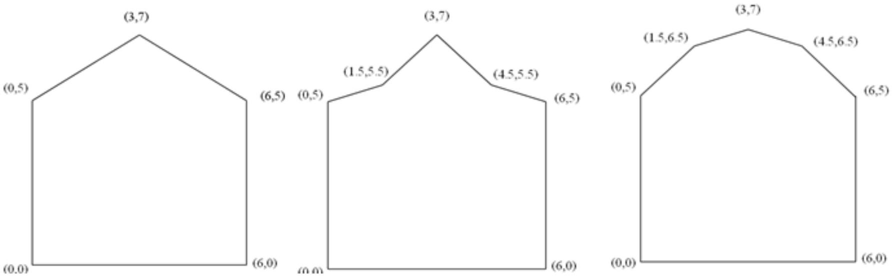


Figure 3. Domains of this problem

For computation by using finite difference method, the grid points of each domain were generated in order to solve thermal value at the points as shown in Fig. 4.

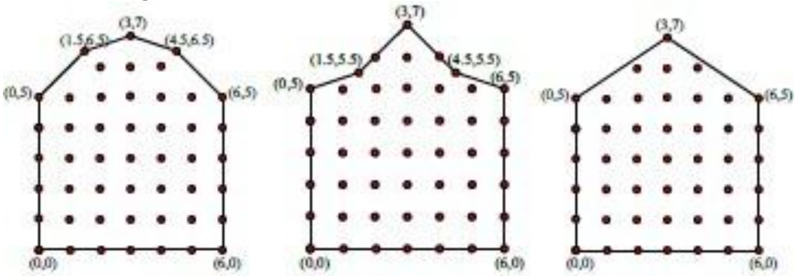
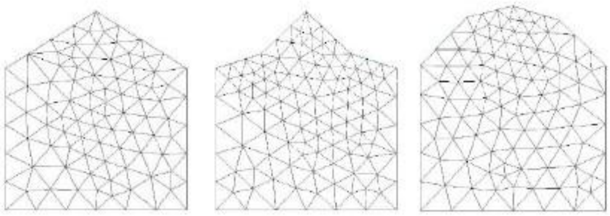


Figure 4. Grid points for finite difference method

For solutions obtained by finite element method, we employed Easy FEM software to solve the solutions. Domains were separated to partial elements by unstructured mesh as in Fig. 5.



(a) Standard (b) Concave (c) Conves

Figure 5. Domains of this problem

To compute the steady state temperature distribution of the three model houses, proper initial and boundary conditions needed to be assigned. The boundary conditions were that the temperature at (the bottom of) the roof was 30 C and the temperature at (the inside of) the wall was 25 C and the temperature at the (top of the) ground was 25 C.

V. Result

The temperatures at centerline from top to the ground which received from finite difference method are presented in Fig. 6.

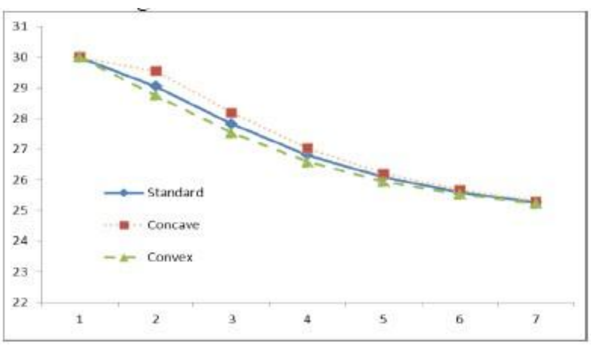
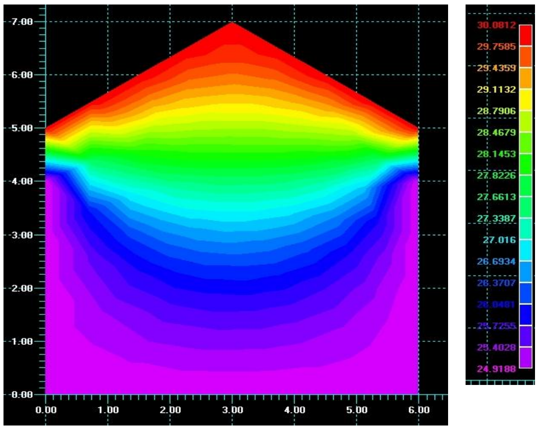


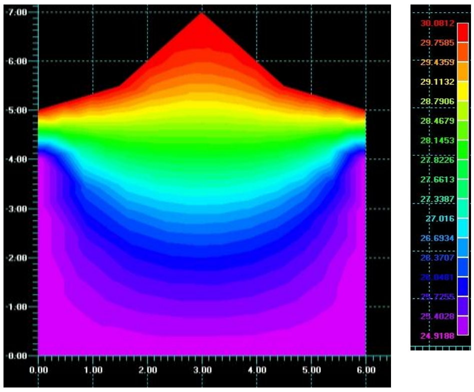
Figure 6. Temperature at centerline of each roof geometry by Finite Difference Method

From Fig. 6, we found that the temperature decreased steadily from the top of the roof to the ground.

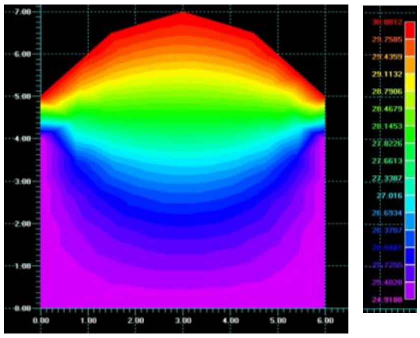
In the other hands, by using finite element method, the steady state temperature distributions of the three model houses with three different roof geometries are illustrated in color contours in Fig. 7.



(a) Standard Shape



(b) Concave Shape



(c) Convex Shape

Figure 7. Color contours of temperature distributions

The temperature (T) decreased steadily from the top of the roof to the ground. The heat transfer patterns of the three different roof geometries followed the same trend but were not identical. We found that heat was transferred more easily through the concave roof than through the standard roof and the convex roof in that order. A comparison between the temperature (at points on the centerline) of each roof geometry is shown in Fig 8.

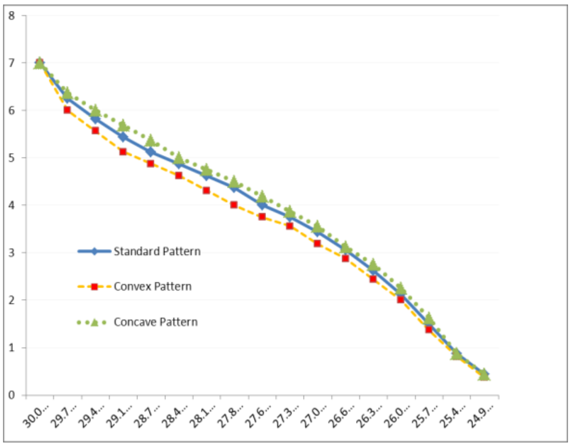


Figure 8. Temperature at centerline of each roof geometry by Finite Element Method

Both the finite difference method and finite element method methods give the solutions in the same trend. The error between two processes was evaluated by standard deviation. Standard deviation of standard pattern is 1.795355, concave pattern is 1.867539 and convex pattern 1.775234.

VI. Conclusion

Simulations of heat distribution in three model houses with different roof geometries were done. It was found that, at steady state, a convex roof let in less heat than a standard (triangular) roof and a concave roof in that order. Hence, it is clear that a proper roof design can minimize the impact of heat entering into a building.

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