On the Walking Pattern Generators of Biped Robot

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***Abstract*—In this paper, we have attempted to focus on the continuous transition of the biped mechanism from the single support phase (SSP) to the double support phase (DSP) and vice versa. Three methods have been compared for this purpose. The first two methods have exploited the notion of pendulum mode with different strategies. However, it is found that the two mentioned methods can give the same motion of center of gravity for the biped. Whereas, Method 3 has suggested to use a suitable acceleration during the double support phase (DSP) for a smooth transition. Although the Method 3 can give close results as in the former methods, the latter are more systematic in dealing with the walking parameters of the biped robot. The second issue considered is the different patterns of the foot trajectory especially during the DSP. In pattern1, the swing foot is always level with the ground during the whole walking step. While in pattern2, the swing foot leaves and strikes the ground with specified angles. A piecewise spline functions have been employed for this purpose in order to ensure zero acceleration at the ends of the foot trajectory and satisfy the constraint conditions at the break points, such that the start time in each phase is set to zero. MATLAB simulation has been performed to investigate the mentioned work. It is verified that pattern 2 can give smoother motion than the first pattern.**

***Index Terms*—Biped robot, Walking pattern generators, Gait cycle, Single Support Phase, Double Support Phase, Zero-Moment Point.**

I. Introduction

One of the important issues of the biped locomotion is the generation of the desired paths that ensure stability and avoid collision with obstacles [1]. Since biped robots are desired to operate in the same environments as humans, they should have a certain level of intelligence [2]. In addition, A high level of adaptability should be provided to cope with external environments. Lastly, In specified circumstances, optimal motion is selected to reduce the energy consumption during walking [2].

There are numerous approaches to generate the biped robot motion. These approaches can be classified according to [3]-[6], as shown in Fig. 1.

The details of these methods are explained in details in [7]. Most researchers concentrate on the control and walking patterns of the biped robot during the single support phase (SSP) due to the instability of this phase and the short time of the double support phase (DSP). It is noticed that the percentage of the DSP is about 20% during one stride of the gait cycle, while the SSP is about 80% [8]. However, the DSP is very important for smooth motion of the center of gravity (COG) trajectory [9]-[11]. To enforce the biped robot to move, one should generate stable trajectories for the hip and the feet. Accordingly, the joint motion of the biped mechanism can be obtained. Therefore, in this paper, we focus on the methods used for the generation of hip trajectory especially for the DSP. Three methods are investigated and compared. Then two walking patterns of the biped motion are considered. Consequently, the two different foot trajectories are encountered. Depending on [12], we have employed improved spline functions to ensure the zero velocity and acceleration of the end conditions. The piecewise spline functions described for each foot trajectory have been modified such that the start time in each phase is set to zero [3].

The structure of the paper is as follows. A short review of the gait cycle and the three walking patterns of the biped robot is introduced in section II. Section III investigates the hip trajectory. While section IV illustrates the generation of the foot trajectory for different walking patterns. Then the simulation results and discussion are shown in section V. The conclusion is considered in section VI.

II. Gait Cycle

The complete gait cycle of human walking consists of two main successive phases: the DSP and the SSP with intermediate sub-phases. The DSP arises when both feet contact the ground resulting in a closed chain mechanism. While the SSP starts when the rear foot swings in the air with the front foot flat on the ground. Due to the complexity of the biped mechanisms, most researchers simplify the gait cycle of the biped walking in order to understand the kinematics, biomechanics and control schemes of them. Studies have shown that there are three essential patterns used for generation of periodic biped walking. Fig. 2 illustrates the three patterns grading from simple to complex configurations.

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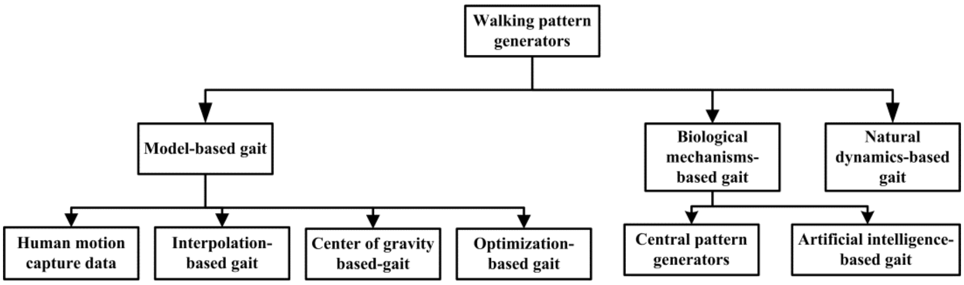

Figure 1. Classification of the approaches used for generation walking patterns of biped robot

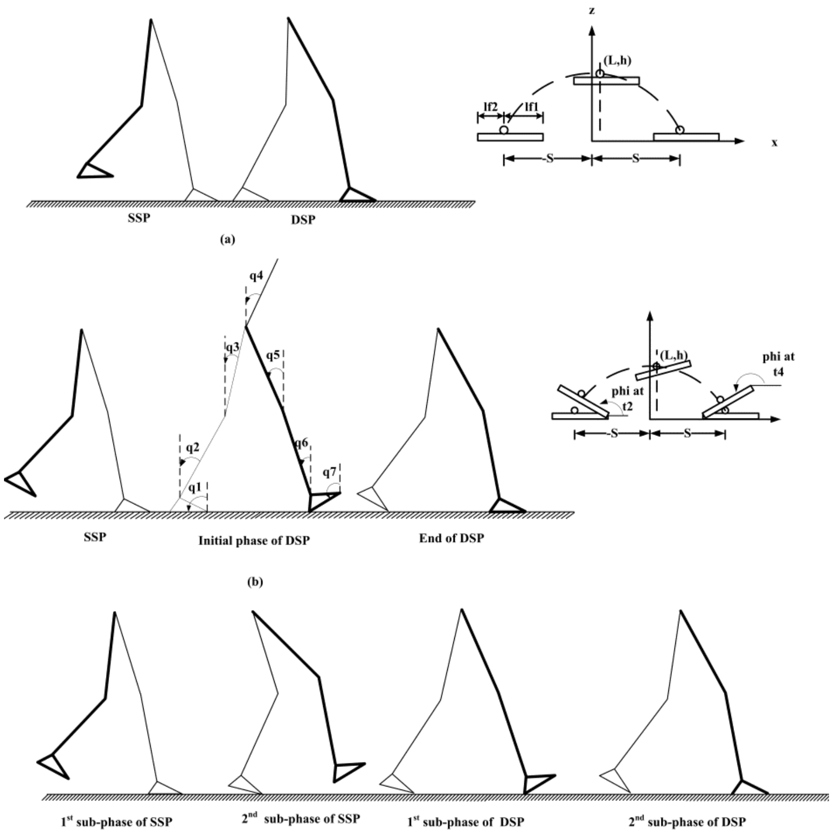

Figure 2. The walking patterns of biped robots . (a) Pattern1 with foot trajectory (b) Pattern 2 with foot trajectory (c) Pattern3

Pattern 1 [3]: It consists of successive DSP and SSP without sub-phases. The swing foot is always level with the ground during leaving and striking the ground. This type of pattern could result in unstable walking due to the sudden landing of the whole sole on the ground at the beginning of the DSP [14]. This drawback can be overcome by pattern 2.

Pattern 2: In this pattern, the swing leg leaves and lands the ground with specified angles. This results in a smooth transition of the striking foot from the heel to the whole sole at the beginning of the DSP. This pattern consists of one DSP and one SSP [4] and [14].

Pattern 3 [4]: This pattern is close to the human walking consisting of two sub-phases of DSP and two sub-phases of DSP. The first sub-phase of the DSP starts when the rear foot initiates to rotate about the front edge during small rotation of the front foot about the heel. The rear foot continues to rotate while the front foot is now flat on the ground resulting in the second sub-phase of DSP. Then the rear foot leaves the ground while the stance foot is flat on the ground resulting in the first sub-phase of the SSP. The second sub-phase of this phase starts when the stance foot rotates about its front edge. Additional DOF is added during the second sub-phase of the SSP. Thus, the system is under-actuated during this sub-phase.

Remark 1: It is possible to modify the mentioned patterns to generate the desired motion. For example, pattern2 can be performed with one SSP and two sub-phases of DSP, such that in the first sub-phase of DSP, the front foot starts to rotate about the heel until it will be level to the ground while the rear foot is in full contact to the ground. Whereas in the second sub-phase of DSP, the rear foot rotates about its front edge while the front foot is in full contact with the ground. Another modification to pattern 3 can be seen in [17].

III. CENTER OF GRAVITY TRAJECTORY

It is verified that designing a suitable hip trajectory can ensure stable dynamic motion for biped robots [14]. We can classify two essential methods regarding this topic. The first one includes designing polynomial functions (or piecewise spline functions) for the hip trajectory during the complete gait cycle satisfying the constraint and continuity conditions [14]-[16].This method selects the hip trajectory with largest stability margin represented by the zero-moment point (ZMP) stability margin. The ZMP is the point on the ground where the net moment of the inertial and gravitational forces of the entire body has zero components in the horizontal planes [18]. Whereas, the second method suggests employing a simple dynamic model for the biped robot denoted by the linear inverted pendulum mode (LIPM) [19], [3], [9] and [10]. Consequently, the notion of pendulum mode has been exploited for generation of stable hip motion. Below we will discuss three important methods used in the literature for describing the motion of the hip trajectory during the two gait phases guaranteeing stable continuous transition between the phases.

A. Method 1 [9]

S. Kudoh and T. Komura [9] have suggested a linear relationship between the ZMP and COG trajectories. In addition, they have considered the effect of the angular momentum at the COG of the biped robot. While, the classical linear inverted pendulum strategy assumes no torques are applied at this point. Thus, we will modify the authors' approach by assuming zero angular momentum and constant ZMP applied at the SSP for the sake of comparison with the next approach, as illustrated in Fig.3a. Following the authors' work, the relationship between the COG and ZMP can be computed as

$$x_{ZMP} = x_s - \frac{H}{g}\ddot{x}_s \tag{1}$$

here x_{ZMP} is the position of ZMP for the stance foot, x_s is the position of the hip at the swing phase, H is the height of the COG which is assumed fixed and g is the gravitational acceleration. Because the ZMP is assumed fixed at the center of the stance foot in this work, the left hand side of (1) will be equal to zero. Consequently, The COG trajectory motion during SSP can be denoted by

$$x_s = C_{s1}\exp(w_s t) + C_{s2}\exp(-w_s t) \tag{2}$$

where C_{s1}, C_{s2} are constants can be obtained from the boundary conditions, and

$$w_s = \sqrt{g/H} \tag{3}$$

For the DSP, similar equation can be employed

$$x_{ZMP} = x_d - \frac{H}{g}\ddot{x}_d \tag{4}$$

where x_d denotes the position of COG during DSP, and ZMP can be assumed as

$$x_{ZMP} = x_d / a_d \tag{5}$$

where a_d refers to a constant that governs the walking parameters of the biped walking. Consequently, we can get the following equation

$$x_d = C_{d1}\cos(w_d t) + C_{d2}\exp(w_d t) \tag{6}$$

where

$$w_d = \sqrt{g(1/a_d - 1)/H} \tag{7}$$

To ensure continuous acceleration at the transition moment of the two phases, it is necessary that $(\ddot{x}_d = \ddot{x}_s)$ at this moment. Thus, by substituting $x_d = -l_d, x_s = l_s$ in (1) and (4) we can obtain

$$l_s + l_d = \frac{l_d}{a_d} \tag{8}$$

If one select l_s and l_d as two independent variables, a_d can be get from (8). Moreover, the correspondent value of the time of DSP (T_d) that satisfies the constraint and continuity equation can be calculated as [10]

$$T_d = \frac{1}{w_d}\cos^{-1}\left(\frac{w_d x_d(0) x_d(T_d) + \frac{\dot{x}_d(0)\dot{x}_d(T_d)}{w_d}}{w_d x_d(0)^2 + \frac{\dot{x}_d(0)^2}{w_d}}\right) \tag{9}$$

Remark2: It is noticed that each selected value of a_d coincides with correspondent value of the time of DSP T_d as shown in (9). This means it is impossible to determine T_d arbitrary.

B. Method 2 [10]

In this method, an inverted pendulum is considered in the SSP and the same equations of the previous method we get. M.Shibuya et al [10] have suggested employing a linear pendulum mode for DSP. Additionally, the same equation (6) has been obtained to describe the COG trajectory. Then they have proven the linear relationship between ZMP and COG trajectories using this method. However, the frequency of the motion can be written as

$$w_d = \sqrt{g / H_d} \tag{10}$$

where, H_d is the distance between the COG and the tip of the pendulum mechanism as shown in Fig.3b. From Fig.3, we can compare the parameters of the two mentioned methods as follows

$$l_d = \frac{(1-k)S}{2} \tag{11}$$

where l_d is the half of the distance spanned by COG during DSP according to Method 1 while k denotes a parameter that governs the biped walking, as we will see, and S is the step length. In addition:

$$\frac{l_d}{a_d} = \frac{S}{2} \tag{12}$$

As a result, we can obtain

$$a_d = 1 - k \quad (13)$$

And the position of the ZMP can be calculated as

$$X_{ZMP} = x_d / a_d = x_d / (1-k) \quad (14)$$

which is the same equation provided by [10]. By comparing (7) and (10), and substituting (13), we can get

$$H_d = (1-k)H_s / k \quad (15)$$

which is the same equation obtained in [10]. Therefore, the two methods are equivalent and can give the same results. Up to now, we will employ the parameters displayed in the second method for our simulation purposes.

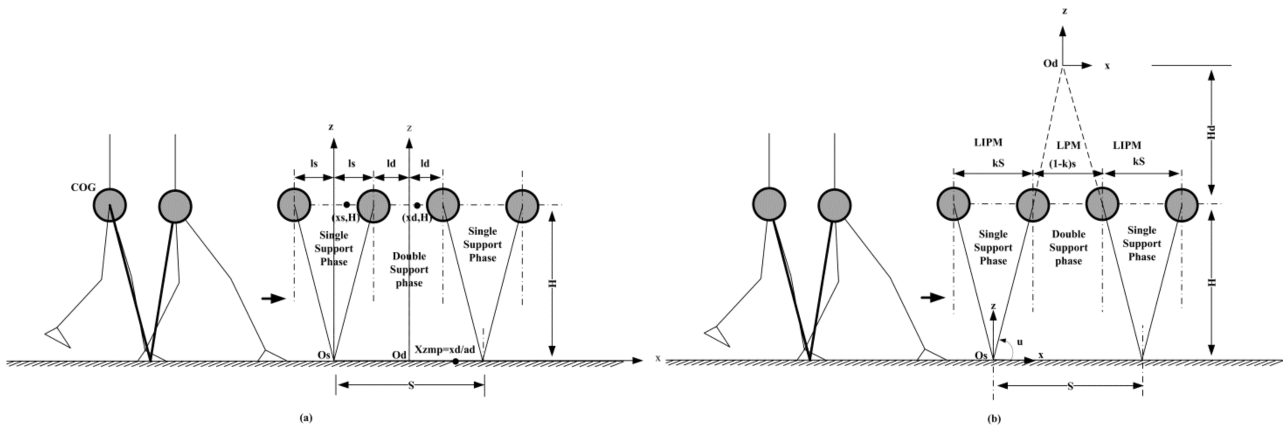

Figure 3. Methods used for generating COG trajectory according to (a) Method 1 and (b) Method 2

Remark3: From (13), we can notice the relationship between the parameter k and the parameter a_d. As a result, a relationship between the parameter k and the time of DSP (T_d) should be considered to ensure a continuous motion, which is illustrated in equation (9).

Remark 4: Following the work of [11] who considers the constraint relationship between the angle of the virtual pendulum and the coefficient of friction as follows,

$$0 \le cotu \le v \quad (16)$$

where v is the coefficient of friction between the biped feet and the ground. From Fig.3b , we can obtain

$$0 \le \frac{kS}{2H} \le v \quad (17)$$

By selecting the values of S and H, a suitable value of k that satisfies (17) can be chosen. In brief, we can summarize the procedure for determining the COG hip trajectory of the biped during the one-step walking as follows:

1. Determine the position of the COG of the biped robot. This depends on the mechanical design of the biped robot. Most researchers have tried to make the COG close to the hip position to simplify the calculations.
2. From (17), select the suitable values of k and S.
3. From (9), determine the correspondent value of T_d. Consequently the time of the SSP (T_s) can be computed as $T_d/0.2$.
4. Using (2) and (6) and their 1st and 2nd derivatives, the motion of COG of the biped robot can be generated efficiently.

C. Method 3 [3]

This method suggests describing a suitable COG acceleration during the DSP satisfying continuous conditions at the instance of the transition. B. Vanderborght [3] suggested two types of functions could be employed for this purpose. A linear acceleration at the DSP can be adopted to connect the previous SSP and the next one. However, a large computation can be arisen. Consequently, the author suggested the same acceleration of the SSP can be used but with a negative sign. We will just display the equations required for the acceleration, velocity and the position of the hip trajectory during DSP. For details, we refer to the mentioned reference. We do not mention the case of the SSP because a simplified model of the inverted pendulum can be used during this phase.

$$\ddot{x}_d(t) = -\ddot{x}_s(t) = -(C_{s1}w_s^2 \exp(-w_s t + C_{s2}w_s^2 \exp(w_s t) \quad (18)$$

$$\begin{aligned}\dot{x}_d(t) &= -(-C_{s1}w_s \exp(-w_s t) + C_{s2}w_s \exp(w_s t)) \\ &+ \dot{x}_s(T_s) + w_s(C_{s2} - C_{s1})\end{aligned} \quad (19)$$

$$\begin{aligned}x_d(t) &= -(C_{s1}\exp(-w_s t) + C_{s2}\exp(w_s t)) \\ &+ (\dot{x}_s(T_s) + w_s(C_{s2} - C_{s1}))t + C_{s1} + C_{s2} + x_s(T_s)\end{aligned} \quad (20)$$

One of the disadvantages of this method is the discontinuity in the position of the COG. This can be solved by modifying the time of the double support phase to guarantee the continuity. This can coincide with the two previous methods in the selection of suitable T_d in order to guarantee continuous COG.

Remark5: All the mentioned methods need compensation of the ZMP error related to the approximation of the biped robot to pendulum model.

IV. FOOT TRAJECTORY

It is noticed that higher order trajectory may lead to oscillation and overshoot [12]. Therefore, it is desirable to use less order polynomials represented by piecewise spline functions to get the desirable dynamic performance for the biped robot. Q. Huang et al [14] have employed piecewise cubic spline functions for interpolation of the foot trajectory. However, the authors have not assumed zero acceleration where the swing foot becomes flat on the ground (initial full contact). Therefore, Y. Guan [12] have suggested employing fourth order spline functions at the end segments with cubic spline functions for the intermediate segments to guarantee the zero constraint conditions at the end points. In this case, the impact effect should be considered at the instance of the heel strike. Table I shows the constraint conditions and the proposed piecewise spline functions for two patterns of foot walking (pattern 1 and pattern 2 only). In effect, we select the first two patterns only because pattern 3 always belongs to the periodicity-based gait rather than the ZMP-based gait [4].

TABLE I. THE CONSTRAINT CONDITIONS AND THE PROPOSED PIECEWISE SPLINE FUNCTIONS FOR THE TWO PATTERNS OF THE FOOT TRAJECTORY

Pattern	Constraint conditions	Proposed piecewise spline functions
1	• x-axis : $x_f\ (t_1)=-S,\ x_f(t_2)=L,\ x_f(t_3)=S,\ \dot{x}_f(t_1)=\dot{x}_f(t_3)=0,\ \ddot{x}_f(t_1)=\ddot{x}_f(t_3)=0.$ (21) • z-axis : $z_f\ (t_1)=0,\ z_f(t_2)=h,\ z_f(t_3)=0,\ \dot{z}_f(t_1)=\dot{z}_f(t_3)=0,\ \ddot{z}_f(t_1)=\ddot{z}_f(t_3)=0.$ (22) where x_f and z_f denote the coordinates of the ankle joint, whereas (L,h) is the coordinate of the obstacle. Additionally, $t_1=0$, $t_2=T_d+T_m$ and $t_3=T_d+T_s$. Where Tm represent the time needed to cross the obstacle.	$F_1(t)=\sum_{j=0}^{4} c_{1j}(t-t_1)^j \quad (t_1 \le t \le t_2)$ $F_2(t)=\sum_{j=0}^{4} c_{2j}(t-t_2)^j \quad (t_2 \le t \le t_3)$ (23) where F_i (.) represents x_f or z_f .
2	• x-axis : $x_f\ (t_1)=-S,\ x_f(t_2)=-S-l_{f1}cos(q_1+\pi/2),\ x_f(t_3)=L,\ x_f(t_4)=S+\ l_{f2}cos(q_7+\pi/2), x_f(t_5)=S,\ \dot{x}_f(t_1)=\dot{x}_f(t_5)=0,\ \ddot{x}_f(t_1)=\ddot{x}_f(t_5)=0.$ (24) • z-axis : $z_f\ (t_1)=0,\ z_f(t_2)=l_{f2}sin(q_1+\pi/2),\ z_f(t_3)=h,\ z_f(t_4)=-l_{f2}sin(q_7+\pi/2)\ z_f(t_5)=0,,\ \dot{z}_f(t_1)=\dot{z}_f(t_5)=0,\ \ddot{z}_f(t_1)=\ddot{z}_f(t_5)=0.$ (25) where $l_{f1}$and l_{f2} are the distance of the foot edges to the ankle joint, $q_1$and q_7 are the angles of the foot at the push-off and the heel strike respectively. Additionally, $t_1=0$, $t_2=T_d$, $t_3=T_d+T_m$ $t_4=T_d+T_s$. and $t_5=T_d+T_s+T_d$. • foot angle (phi): $phi\ (t_1)=\pi,\ phi\ (t_2)=\pi/2+q_1,\ phi\ (t_3)=\pi/2+q_7,\ phi\ (t_4)=\pi,\ \dot{phi}(t_1)=\dot{phi}(t_4)=0,\ \ddot{phi}(t_1)=\ddot{phi}(t_4)=0.$ (27)	$F_1(t)=\sum_{j=0}^{4} c_{1j}(t-t_1)^j \quad (t_1 \le t \le t_2)$ $F_2(t)=\sum_{j=0}^{3} c_{2j}(t-t_2)^j \quad (t_2 \le t \le t_3)$ $F_2(t)=\sum_{j=0}^{3} c_{3j}(t-t_3)^j \quad (t_3 \le t \le t_4)$ $F_2(t)=\sum_{j=0}^{4} c_{4j}(t-t_4)^j \quad (t_4 \le t \le t_5)$ (26) where F_i (.) represents x_f or z_f . $F_1(t)=\sum_{j=0}^{4} c_{1j}(t-t_1)^j \quad (t_1 \le t \le t_2)$ $F_2(t)=\sum_{j=0}^{3} c_{2j}(t-t_2)^j \quad (t_2 \le t \le t_3)$ $F_2(t)=\sum_{j=0}^{4} c_{3j}(t-t_3)^j \quad (t_3 \le t \le t_4)$ (28) where F_i (.) represents phi

Remark6: It is assumed that the walking step starts when the front foot strikes the ground while the rear foot in full contact, and it ends when the swing foot becomes in full contact with the ground. However, we have assumed that the start time of each phase is set to zero, consequently shifting every piecewise polynomial function is needed in order to achieve this purpose. It should be mentioned that after finding the COG and foot trajectories, the inverse kinematics is necessary to find the biped joint trajectories. For details, we refer to [3].

V. SIMULATION RESULTS AND DISCUSSIONS

A. Comparison between Method2 and Method3

Table II shows the physical parameters of the simulation biped robot [8]. Following the procedure described in section III for the generation of COG trajectory, the desired walking parameters can be obtained as shown in the same mentioned table. It is noticed that the selection of the suitable k coincides with the correspondent value of T_d as illustrated in (9). Then we have employed the mentioned parameters with Method 2 and Method 3. Consequently, the COG motion will be continuous regarding position, velocity and acceleration, as shown in Fig. 4. The two methods give similar motion. However, Method 2 is more systematic in dealing with the parameters of the biped walking and guaranteeing the constraint and continuity conditions. From Fig. 4, it is clear that the SSP encounters deceleration and acceleration sub-phases sequentially. This can be explained according to (1) where deceleration of the biped robot can occur until the middle of SSP because the COG position is behind the front stance foot. The next acceleration sub-phase can result from the progression of the COG in front of the stance foot. Another issue can be noticed is that the motion of the hip link is very close in the middle of SSP, as shown in Fig. 5 and Fig. 6. As aforementioned, the COG of the biped robot will decelerate very slowly at the middle region of the SSP, and then it accelerates slowly near this region.

B. Gait Patterns

Fig. 5 and Fig. 6 illustrate the stick diagrams of the two patterns. The two selected patterns have the same COG velocity and acceleration according to Method 2. The unique difference is represented by the generation of the foot trajectory. In pattern 1, the swing foot should decelerate at the end of SSP in order to make zero velocity and acceleration at the end of this phase. This effect can be represented by the ellipse encircling the swing knee, as shown in Fig. 5. Whereas pattern 2 strikes the ground with some velocity and acceleration and then decelerate its foot. Consequently smooth transition can be developed as depicted by the ellipse encircling the swing knee in Fig. 6. Additionally, the superiority of the second pattern on the first one can be significant in dealing with the control problem of the biped robot. For example, B. Vanderborght [3] adopted the first pattern in the generation of the gait for his biped. During DSP, the author assumed zero ankle joints. Consequently, discontinuous ankle torques can arise at the moment of the transition from SSP to DSP and vice versa. Whereas the second pattern can be exploited to ensure continuous transition for the ankle torques.

TABLE II. THE PHYSICAL AND WALKING PARAMETERS

Physical parameters	$l_{shank}=l_{thigh}=l_{trunk}=0.45m$, $l_{f1}=0.15mm$, $l_{f2}=0.1m$
Walking parameters	$v=0.1, k=0.4724$, $T_d=0.5\ s$, $T_s=2.5\ s$. $q_1=79^0$, $q_7=101^0$.

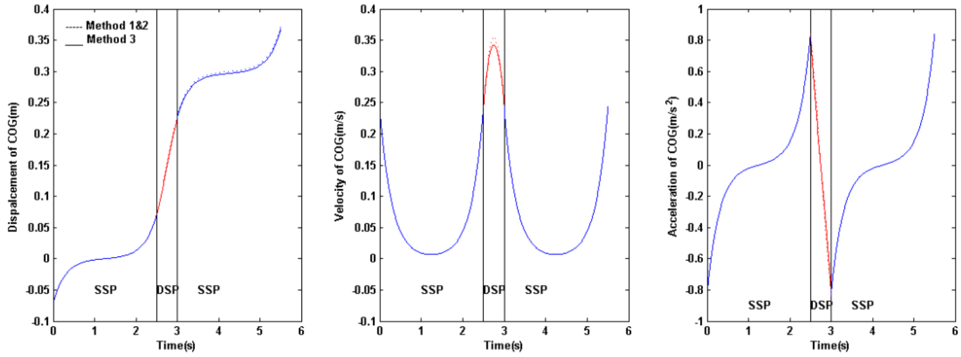

Figure 4. The position, velocity and acceleration of COG (hip)

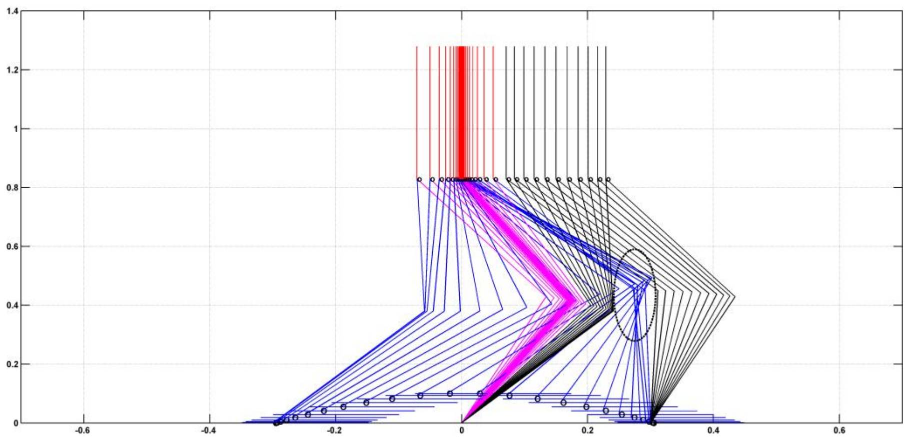

Figure 5. Stick diagram of pattern 1

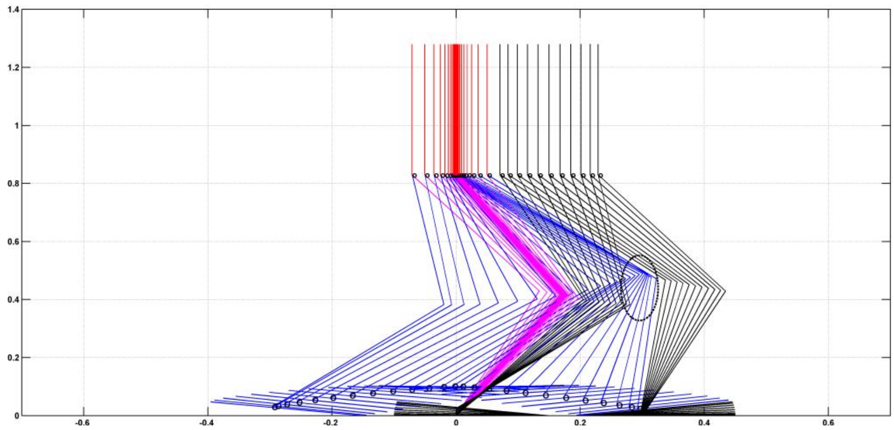

Figure 6. Stick diagram of pattern2

VI. CONCLUSIONS

In this paper, we have attempted to focus on the smooth transition from the SSP to the DSP and vice versa. Three methods have been compared for this purpose. The first two methods have exploited the notion of pendulum mode with different strategies. However, it is found that the two mentioned methods can give the same motion of center of gravity for the biped. Whereas, Method 3 has suggested to use a suitable acceleration during the double support phase (DSP) for a smooth transition. Although the Method 3 can give close results as in the former methods, the latter are more systematic in dealing with the walking parameters of the biped robot. The second issue we focus on is the different patterns of the foot trajectory especially during the DSP. A piecewise spline functions have been employed for this purpose to reduce the oscillation and overshoot resulted from higher order polynomials. The pattern2 is preferable on pattern 1 because the former can result in smoother transition. Consequently, this can ease the task of stability and control. In this paper, we do not consider the effect of variable ZMP at the SSP and its effect on the speed of the biped and its stride. This issue is left to another paper for further discussion of more walking patterns.

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