Wavelet-Based Method for Fog Signal Denoising

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***Abstract*—Fiber-Optic Gyroscope (FOG) has been widely used to measure the angle rate of vehicle in recent years. Being as unpredictable and unmeasured error, random drift generated from FOG instability create seriously bad influence on precision of FOG output, as well as Inertial Navigation System (INS). Although wavelet-based technique has made considerable progress in FOG signal denoising, almost all the achievements are based on off-line analysis that gives little contribution to practical application. This paper presents a revised plan on account of previous research on real time denoising, explaining the computational complexity reduction from theory. Through simulation and static FOG experiment using time-frequency analysis and Allan Variance as the performance evaluation standard, the denoising effectiveness compared to using traditional method has been proofed improving to a large extent.**

***Index Terms*—FOG signal, Second generation wavelet transform (SGWT), Denoising, Real time, Sliding window**

I. Introduction

Fiber-Optic Gyroscope (FOG) applied in Inertial Navigation System (INS) contains random drift generated by uncertain disturbance. However, it's hard to use proper model to represent and compensate it. Consequently, FOG signal denoise may well be the key for improving navigation precision.

When applied to FOG signal real-time denoise, the superiority of wavelet compared with self-adaptive filter has been discussed based on off-line analysis in [1]-[2]. By using enforced de-noising technique, the speed of response will be enhanced. After that, through utilizing sliding window, see for concrete algorithm [3]-[4], wavelet transform can be implemented in a fixed extent as the window moving in order to construct the real-time de-noising algorithm. Furthermore, after proposing a new principle for threshold determination, novel tactics concerning about wavelet coefficient process has been substituted in [5]-[6].

In INS based on FOG as one of chief Inertial Measurement Unit (IMU), raw signal denoising should be considered both on real-time application and satisfactory effectiveness. Although enforced denoising proposed in [1] weakens the complexity of threshold principle, it has bad influence on dynamic feature, leading to removing plenty of useful signal as well when trying to clean up the noise. The method proposed in [3] combines sliding window with traditional wavelet transform. However, the complicated approach referring to window length determination restricts it applied to real-time denoising. Moreover, threshold-based de-noising method was revised in [5]. The complexity still keeps a high level so that the computational speed will be promoted to a limited extent.

We propose to use SGWT to simplify the decomposition and reconstruction process, tremendously enhancing computational speed from theory. Specifically, combined with SGWT, several techniques stated above has been used, which mainly including sliding window, enforced denoising aimed at wavelet coefficient in first level and hard threshold principle. This new real-time de-noising plan has been proved having more notable de-noising effectiveness than traditional wavelet based method proposed in [5] from simulation. By dealing with practical FOG signal, we indicate the viability of this plan by taking statistic experiment, showing that novel plan optimize attitude information of INS.

The paper is organized as follows. In section 2, we review basic theory about SGWT including some essential concept. In section 3, we discuss the new plan for real-time denoise from three facets which are sliding window, computational complexity and simplification of threshold principle. Simulated signal experiment has been described in section 4. In section 5, FOG static experiment is taken. And we use time-frequency analysis as well as Allen Variance to judge the performance of new plan. Finally, summery of conclusions are presented in section 6.

II. Second Generation Wavelet Transform

A. Lifting Scheme

Lifting, a space-domain construction of biorthogonal wavelets developed by Sweldens [7] consists of the iteration of the following three basic operations:

Split: Divide the original data $x[n]$ into two disjoint subsets, for example, even sample $x_e[n]$ and odd sample $x_o[n]$.

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$$x_e[n] = x[2n], x_o[n] = x[2n+1] \quad (1)$$

Prediction: Design the prediction operator P to deal with $x_e[n]$. Generate wavelet coefficient $d[n]$ as the error in predicting $x_o[n]$ from $x_e[n]$.

$$d[n] = x_o[n] - P(x_e[n]) \quad (2)$$

Update: Design the upload operator U to deal with $d[n]$. Then add to $x_e[n]$ to get scaling coefficient representing coarse approximation.

$$c[n] = x_e[n] + U(d[n]) \quad (3)$$

The process of reconstruction of lifting scheme is inverted steps of decomposition. Steps are carried out by rearranging of operations stated above:

$$x_e[n] = c[n] - U(d[n]) \quad (4)$$

$$x_o[n] = d[n] + P(x_e[n]) \quad (5)$$

$$x[2n] = x_e[n], x[2n+1] = x_o[n] \quad (6)$$

B. Interpolation Subdivision

Interpolating subdivision is a kind of forecast method used in SGWT. Using prediction operator coefficient P as the example, its basic strategy is to utilize the even samples neighboring the odd samples to estimate the former during the process of decomposition or reconstruction. Here we take the [4, 4] wavelet representing four prediction and four upload coefficients applying to the next experiment. Two kinds of coefficients will be pre-computed by the method proposed in [12]. The filter coefficients are -0.0625, 0.5625, 0.5625, -0.0625 (prediction coefficient) and -0.03125, 0.28125, 0.28125, -0.03125 (upload coefficient).

As this technique does not rely on the Fourier transform [8], SGWT-based transform not only avoids signal transmitting between time and frequent domain, but only contains the simplest four fundamental operations of arithmetic. In addition to guarantee that the system will be response for input signal within a short time, refraining from elimination of signal assembled in low and medium frequency is important as well. Consequently, the next section concentrates on revised plan developed from statement above.

C. Revised Plan of Real-time Denoising

1) Sliding Window

Under normal conditions, threshold denoising is a kind of off-line signal process, which means taking entire raw signal as a set of data for de-noising manipulation. Obviously, this operation cannot satisfy the practical requisition that the output signal is needed to processed on-line. Considering the restriction of wavelet transform in real-time signal process, sliding data window has been used. The data in the first pre-specified extent of window do not cross the de-noising system. While the number of data accumulates to the length of window, the set of data begins to introduce into de-noising process. After that, as the data is received from FOG, the window keeping a fixed length slides to get the latest n^{th} data for wavelet denoising. After every times of reconstruction, the n^{th} data is taken as the denoised output transmitted to the next calculation.

2) Wavelet Computational Complexity

In a common expression, wavelet computational complexity is used to represented how complicated the wavelet computation is. It is measured by the number of multiplication and addition in a pair of coefficient (c_l, d_l). From [9], compared to normal algorithm, wavelet transform using lift scheme lower the level of complexity sharply under the same data quantity. Tab.1 shows the comparison between standard algorithm and lifting scheme. As it shown, speed increase is more than 60 percentage average.

TABLE I. COMPUTATIONAL COMPLEXITY OF STANDARD VS. LIFTING ALGORITHM

Wavelet	Standard	Lifting	Speedup
Haar	3	3	0%
Db4	14	9	56%
Db6	22	14	57%
[2,2]	10	6	67%
[4,4]	22	12	83%

3) Threshold Principal Simplification

To simplify the threshold computation, we use the common threshold:

$$T_j = \sigma_j \sqrt{2\ln(2^j)} \quad (7)$$

where T_j is threshold in j level, σ_j is standard variance of noise in j level. σ_j can be calculated from the following equation:

$$\sigma_j = \frac{median(d_j)}{0.6745} \quad (8)$$

The quantization method usually contains hard threshold, soft threshold and semisoft threshold which takes into account both hard and soft threshold's advantages. Experiment indicates that traditional wavelet using soft threshold for FOG signal denoising can get more benefits than using hard threshold. However, if using SGWT, de-noising effect is almost the same. Hence, considering the concise function, we accept hard threshold as the principle for threshold process.

Signal after decomposition from previous level will be divided into two parts: the details and approaches which represented by wavelet coefficient and scaling coefficient respectively. Since the j level decomposition is taken on account of scaling part of j+1 level. So as long as raw signal is introduced into de-noising system, noise extraction and elimination will be taken from high-frequency to low-frequency. When applied to INS installed on ship, FOG signal concentrates on low or medium frequency, especially the z-axis FOG. It is different from using in plane or some other fast vehicle. Therefore, because of this kind of slow changed signal, enforced denoising, which means that set all of wavelet coefficients to zero after the first decomposition, is reasonable. However, the principle backs to normal way that is hard-threshold after second, third, fourth… decomposition.

D. *Simulation and Analysis*

Generate a square wave with 2000s period, 1unit amplitude and 10000 samples using MATLAB. And random noise has been superimposed on it. First, we utilize db4 as the basis function in the method proposed in [5] for simulated real-time denoising. After that, we take advantage of [4,4] wavelet based revised plan for the same simulated experiment. Essential parameters are set as following: Decomposition levels: 3; sliding window length: 1024; Threshold principle: Universal method + hard-threshold.

We introduce Variance V and Signal to Noise Ratio (SNR) R_{SN} as the main standards for estimating de-nosing performance. The comparison of noise reduction methods is shown in Tab. 2. The degree that denoised signal diverges from Expectation (E) after using revised plan we proposed is smaller than using traditional plan. And SNR is improved by approximate 2dB comparing to traditional method and 6.6dB comparing to raw signal. Fig.1 and Fig.2 show the simulated result in time-domain and frequency-domain respectively.

TABLE II. COMPARISON OF VARIANCE AND SNR

	Raw signal	Traditional method	Revised method	Improved percentage
Variance (10^{-4})	5.6292	2.0786	1.2635	39%
SNR(db)	6.3527	10.8017	12.9632	20%

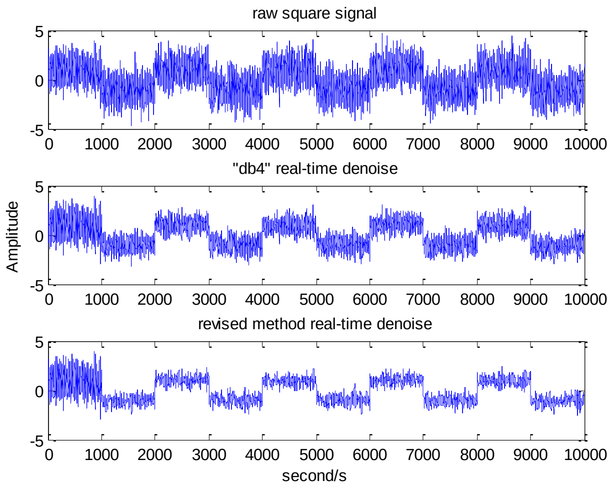

Figure 1. Time-domain diagram of simulated signal denoising

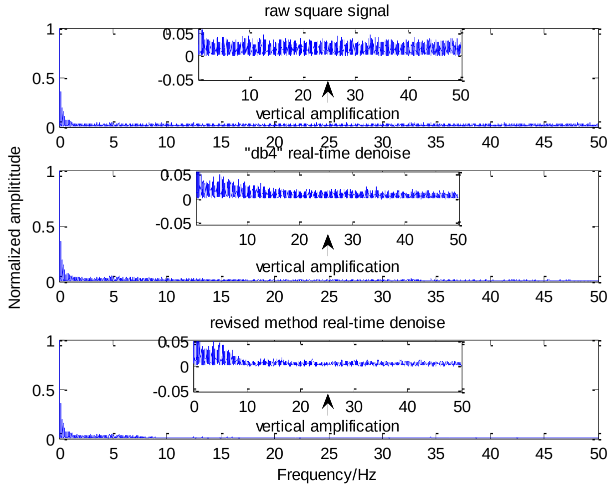

Figure 2. Frequency-domain diagram of simulated signal denoising

E. *Performance Evaluation for FOG Signal Denoising*

1) Static Experiment

Raw signal is received through serial communication with 1Hz sample frequency from a static FOG. We choose the x-axis (eastern axis) as the research axis. And cut off an extent of data containing 10000 samples. The length of sliding window equals to the length of 1024 samples. Db4 and [4, 4] wavelet have been used with decomposition level 3. Entire experiment is actually completed off-line with the help of MATLAB. Nonetheless, it has simulated the process of real-time denoising and its results are still valuable and meaningful.

Fig.3 and Fig.4 plot the original static FOG signal and denoised signal presented in time and frequency domain respectively. In fact, the point whose normalized amplitude is “1” doesn’t locate exactly in zero-frequency, but approaching to zero. The reason is that FOG may well be influenced by slow drift of its own or global self-rotation detection derived from installation error.

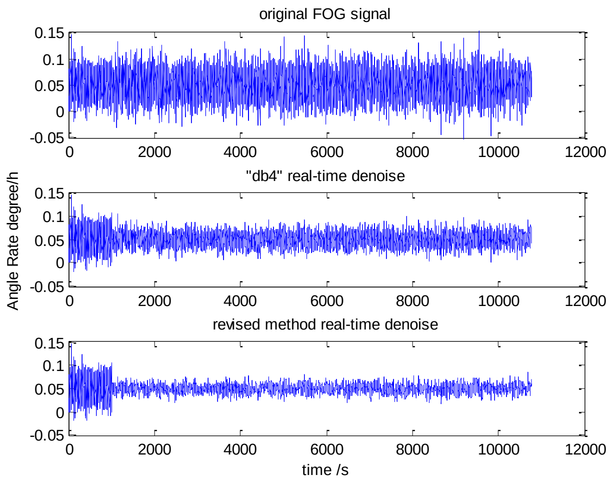

Figure 3. Time-domain diagram of FOG signal denoising

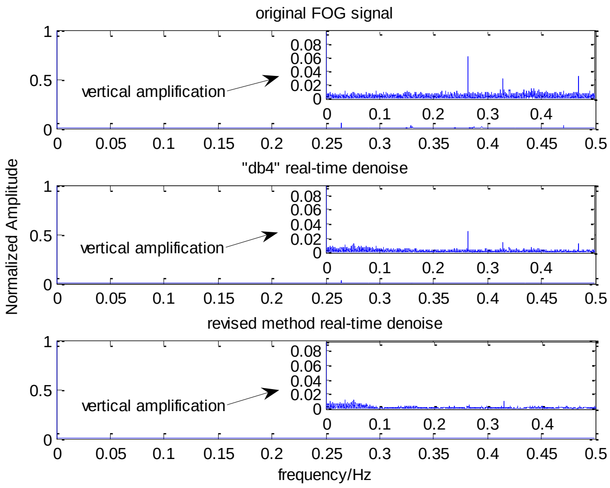

Figure 4. Frequency-domain diagram of FOG signal denoising

From comparison in frequency-domain, the noise combined with signal exists on full band. Through implementing revised plan, normalized amplitude of points whose frequency are greater than 0.1Hz decreases to a certain extent. And peaks have been almost eliminated. Experiment indicates that the noise whose frequency is greater than 0.1Hz has been repressed.

Here we introduce Allan Variance in order to evaluate de-noising effect. Allan Variance method is a time domain analysis technique originally developed to study the frequency stability of precision oscillators [10]. It is used to characterize random processes responsible for noise present in data [11]. By analyzing Allan Variance, different types of noise can be recognized in FOG signal. Table 3 lists some kinds of random error coefficients such as Bias stability, quantization noise and rate ramp.

TABLE III. RESULT OF ALLEN VARIANCE ANALYSIS

plan	Variance	Bias stability ($^\circ/h$)	Quantization noise (μrad)	Rate ramp ($^\circ/h^2$)
Raw signal	5.7056	0.2059631	1.8246057	0.5596143
Traditional method	3.8875	0.1957961	1.2364771	0.4513876
Revised method	1.2379	0.1670634	0.8127038	0.4010237

2) Attitude Error Experiment

The purpose of de-noising process is for the improvement of attitude accuracy after INS calculation. Fig.5 shows the position that de-noising process is in as a step for raw FOG signal preprocess.

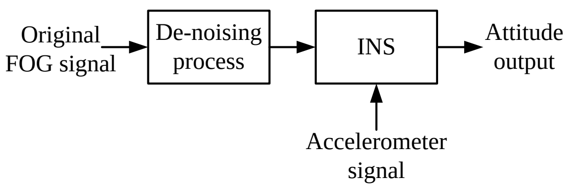

Figure 5. FOG signal preprocess before INS calculation

FOG is settled on the single-axis turn-table shown in Fig.7. The output of FOG is gained and stored. Then take it as input to the INS. At last, we get attitude information about current state including roll, pinch and yaw. Data sets as follows: 5h sampling with 98Hz frequency, cutting out a section of signal starting from 3th hour with about 100s length of signal denoised by our plan. Because of no reference position in single-axis turn-table so that it may well be affected by foundation support, we regard 0, 0, 135 as the current attitude. The result is plotted in Fig.6. Fig.6 shows the decreased amplitude of error after de-noising process using our method. It's about 6$''$ decreases from original roll and pinch angle.

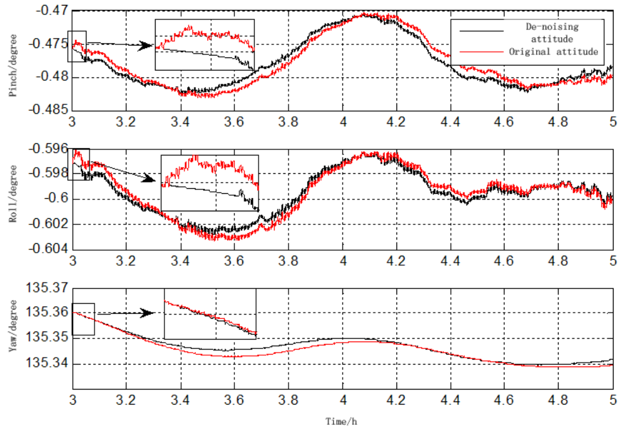

Figure 6. FOG signal preprocess before INS calculation

Figure 7. FOG and single-axis turn-table

F. Summarizes

A revised method for FOG signal real-time denoising proposed in this paper, based on SGWT avoiding time to frequency converting, leads to less computational complexity than using traditional wavelet based method.

Sliding data window, as well as enforced denoise combined with hard-threshold, are two key techniques. The former whose window length has been proofed having little influence on INS by repeated simulation, allows taking wavelet transform on-line. The later simplifies threshold principle in terms of characteristic of FOG signal, making further improvement on computation efficiency.

Simulation and static FOG experiment indicate that noise aggregates on full band. And impulse disturbance can be repressed effectively. In additional, almost every sample will be processed so that the ability for attitude error suppression can be enhanced to a large extent.

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