

# Extending the Stable Input Range of a Single-Bit Sigma-Delta Modulator with a Saturation Element

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**Abstract**—This paper presents a simple compensation scheme for extending the stable input range of a single-bit sigma-delta ( $\Sigma$ - $\Delta$ ) modulator that is normally stabilized in sliding mode. The proposed control scheme requires a saturation element that provides additional feedback when the modulator is going to lose stability due to large input level. The proposed compensation scheme is easy to implement and is only activated when the input level is exceedingly large, so it does not affect the performance of the modulator with its input in the normal range. A third-order sigma-delta modulator with an extended input range was designed and implemented to demonstrate the practicality of the proposed design.

**Index Terms**—dynamic range, input range, over-sampling A/D converter, sigma-delta modulator, sliding mode.

## I. INTRODUCTION

A single-bit sigma-delta modulator consists of a two-level clocked quantizer and a linear filter in feedback connection, as shown in Fig. 1(a). Sigma-delta modulators have been widely applied in data conversion [1]-[3] and power conversion [4]-[6] because of its simple structure, high signal quality, and low sensitivity to circuit non-idealities. There is always a trade-off between stability and performance in designing a sigma-delta modulator. Some designs may trade a narrower stable input range for an improved signal-to-noise-plus-distortion ratio (SNDR), while others may accept less satisfactory performance for a wider input range.

Several methods have been proposed to extend the input range of sigma-delta modulators. For example, Chang *et al.* [7] presented an adaptive algorithm which automatically updated the feed-forward coefficients of the loop filter according to the first and last integrator outputs. Maghari *et al.* [8] invented a dual-quantizer modulator in place of a regular multi-bit modulator, which was capable of increasing the modulation levels when the quantizer was overloaded. Fifield and Allee [9] proposed a dynamically tuned 3rd-order sigma-delta modulator with its loop filter

automatically tuned according to the peak value of the input signal.

This study developed a compensation scheme for a single-bit sigma-delta modulator operated in sliding mode [10]. The compensation scheme is activated to stabilize the modulator when some internal signal of the loop filter goes beyond some limit prescribed by the stability condition of sliding mode, thereby widening the stable input range without sacrificing the desired performance. An illustrative design example shows the effectiveness of this easy-to-use compensation scheme.

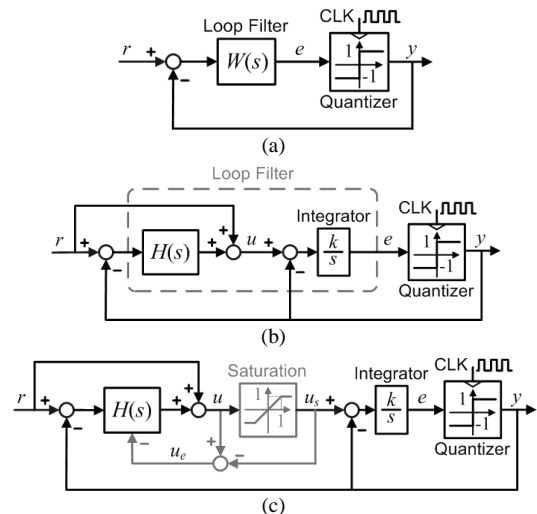


Fig. 1. (a) General single-bit sigma-delta modulator. (b) Alternative topology. (c) Sigma-delta modulator with proposed compensation scheme.

## II. $\Sigma$ - $\Delta$ MODULATION OPERATED IN SLIDING MODE

Fig. 1(a) shows the sigma-delta modulator, in which all signals are assumed to be normalized in amplitude with respect to the positive and negative supply voltages. The clocked quantizer updates the state of  $y$  (with the normalized amplitudes  $\pm 1$ ) at each rising edge of the clock and holds the state constant in between the two consecutive rising edges. The modulator output  $y$  can be derived from Fig. 1(a) in the  $s$ -domain as follows,

$$Y(s) = R(s) - W^{-1}(s)E(s) \quad (1)$$

where  $Y$ ,  $R$ , and  $E$  are the Laplace transforms of signals  $y$ ,  $r$ , and  $e$ , respectively. The modulator output  $y$  includes the

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desired signal  $R$  and the modulation noise term  $W^1E$ . The inverse of loop filter  $W$  shapes the modulation noise spectrum. Therefore, to achieve in-band noise attenuation, the loop filter  $W$  must have a high gain over the signal band of interest.

The topology in Fig. 1(b) is useful for understanding the manner in which the sigma-delta modulator is operated in sliding mode, and the loop filter in the gray dashed box is composed of the filter  $H$  and a non-inverting integrator ( $k>0$ ). Thus, the signal  $u$  can be obtained by adding the output of filter  $H$  and the input signal  $r$ . This alternative topology is equivalent to that in Fig. 1(a) when

$$H(s) = sW(s)/k - 1 \quad (2)$$

Therefore, according to [10], the existence and stability conditions for the sigma-delta modulator to operate in the sliding mode  $e=0$  are that the numerator of  $W$  is Hurwitz and the absolute value of  $u$  is smaller than 1, respectively. More detailed stability and performance analyses can be found in [10], [11].

### III. $\Sigma$ - $\Delta$ MODULATION WITH AN EXTENDED INPUT RANGE

Fig. 1(c) shows the proposed compensation scheme for extending the stable input range of the sigma-delta modulator. In comparison to the topology in Fig 1(b), this control scheme only adds a saturation element which limits its output to the upper bound 1 and the lower bound -1. Moreover, another negative feedback path is provided and the signal  $u_e$ , which is the error between the signal  $u$  and the signal  $u_s$ , is fed back into the filter  $H$ .

When the input signal  $r$  is sufficiently large to make the absolute value of  $u$  larger than 1, that is, the amplitude of the input signal  $r$  exceeds a maximum stable input level, the entire system does not operate stably in the sliding mode and the trajectory of  $u$  diverges to infinite because the existence condition did not hold. Therefore, it is crucial to decrease the amplitude of the signal  $u$  when the absolute value of  $u$  is larger than 1 to extend the stable input range. As presented in Section II, the signal  $u$  contains the input signal  $r$  and the output of filter  $H$ , therefore, signal  $u$  can be diminished only by decreasing the output signal of filter  $H$ . When the absolute value of  $u$  is larger than 1, the signal  $u_e$  is a positive value and decreases the amplitude of  $u$  through the negative feedback path to enable the modulation to be operated in sliding mode again. Furthermore, because of the additional negative feedback path, the transfer function  $W$  is changed and the shape of the quantization noise spectrum.

However, if the amplitude of  $u$  does not exceed 1, the signal  $u$  equals the signal  $u_s$  and the gray parts in Fig. 1(c) are removed. Consequently, the proposed control scheme is the same as the topology in Fig. 1(b). Therefore, the control scheme for extending the stable input range does not influence the original performance of the sigma-delta modulator (e.g., the values of SNDR and the maximum stable input level), which was designed in advance.

### IV. PRACTICAL STRUCTURE AND DESIGN CONSIDERATIONS

A possible structure to implement the proposed control scheme is shown in Fig. 2, in which a 3rd-order loop filter is used as an example. The value of the feedback gain  $d_1$  must be sufficiently large to ensure that the amplitude of  $u$  is reduced to hold the existence condition. Thus, the value of  $d_1$  in the proposed modulator must be as large as possible. However, in reality, the signal  $u_e$  contains a slight noise because of the circuit non-idealities even though the amplitude of  $u$  is less than 1 (i.e.,  $u_e=0$ ). Using such a high feedback gain amplifies this noise and feeds the amplified noise back into the filter  $H$ , which affects the original performance. The suggested feedback gain  $d_1$  must use a value which is similar to the other coefficients which are within the same integrator, for example,  $a_1$ ,  $b_1$ , and  $c_1$ . Moreover, according to [10], the sigma-delta modulator remains stable even though the absolute value of  $u$  slightly exceeds 1; therefore, the range between the upper and lower saturation values must be set to a range that exceeds  $\pm 1$ .

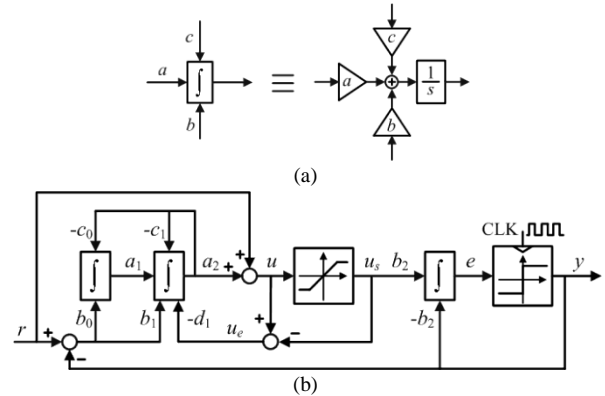


Fig. 2. One possible structure to design the proposed extended input range sigma-delta modulator with a 3rd-order loop filter. (a) Equivalent block diagram. (b) Complete structure using a 3rd-order loop filter  $W$ .

TABLE I. LISTED COEFFICIENT VALUES MULTIPLIED BY  $10^6$ .

$a_1$	$a_2$	$b_0$	$b_1$	$b_2$	$c_0$	$c_1$	$d_1$
1	1.149E-6	0.569	1	4.185	1.184E-2	0	6.6

### V. DESIGN EXAMPLE AND EXPERIMENTAL RESULTS

An illustrated example of a 3rd-order modulator with an extended input range was designed and implemented to convert signal frequencies up to 20 kHz. The frequency of the clock sent to the D-type flip-flop was 4-MHz and the oversampling rate (OSR) was 100. The loop filter  $W$  was designed as a lowpass filter, yielding

$$W(s) = \frac{4.185 \times 10^6 s^2 + 4.811 \times 10^{12} s + 2.79 \times 10^{18}}{s^3 + 1.184 \times 10^{10} s}$$

which has all zeros on the open left half of the complex  $s$ -plane for stability and all poles on the imaginary axis to achieve superior noise attenuation. The non-inverting integrator gain was chosen as  $k = 4.185 \times 10^6$ . Thus, the filter  $H$  can be derived using (2),

$$H(s) = \frac{1.149 \times 10^6 s + 6.547 \times 10^{11}}{s^2 + 1.184 \times 10^{10}}$$

Table I lists the coefficients in Fig. 2. Fig. 3 shows the entire circuit diagram of the proposed modulator, which operates from  $\pm 4V$  dual supplies. The modulator output  $y$  was scaled down using a gain of  $1/2$ . Consequently, the designed modulator has a gain of 2 in voltage amplification. The filter  $H$  and the non-inverting integrator were all implemented in the RC OP-Amp circuit [12]. The OP1 and OP2 were combined with resistors and capacitors to make  $H(s)$ . The OP3 uses two Zener diodes and resistors to construct the saturation. The upper and lower bounds of the saturation were approximately 2.8 V and -2.8 V, respectively.

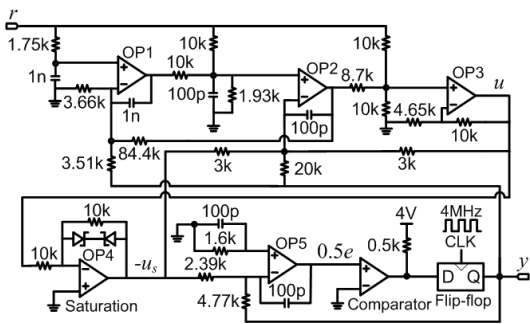


Fig. 3. Circuit diagram of the 3rd-order sigma-delta modulator with an extended input range

The performance of the designed modulator was evaluated using sinusoidal inputs of various amplitudes and frequencies. All experimental results were measured using an audio analyzer, Audio Precision SYS-2712. Fig. 4 plots the graph of the SNDR versus the normalized input level, where the black line and the gray line show the results from the sigma-delta modulator and the proposed modulator, respectively. The figure exhibits that the stable input range has been extended using the proposed control scheme. When the input level is less than 0.66 V, both curves are close and the slight variations between them are caused by the circuit non-idealities, as indicated in Section IV. The maximum SNDR values are 93.6 dB and 93.9 dB for the proposed modulator and the sigma-delta modulator, respectively. Under the performance constraint of  $SNDR \geq 80$  dB, the sigma-delta modulator has a normalized maximum input level of 0.66 V, whereas the normalized maximum input level for the proposed modulator is 0.85 V, which is a difference of 0.19 V. Fig. 5 shows the graphs of the SNDR versus the input frequency. The proposed modulator attains a consistently high SNDR above 80 dB in the entire control band for the normalized input amplitude higher than 0.1 V. Fig. 6 shows the output spectra of  $y$  for 1-kHz input signals, in which an unexpected rise at the low frequencies was caused by the inevitable dc offset in the circuit. As expected, the output noise spectrum in Fig 6(a) follows the same shape as the magnitude response of  $W^{-1}$ . In addition, as the input level is larger than 0.66 V, the transfer function  $W$  is changed by the compensation scheme, leading the shape of the output noise spectrum in Fig. 6(b) differs from that in Fig. 6(a).

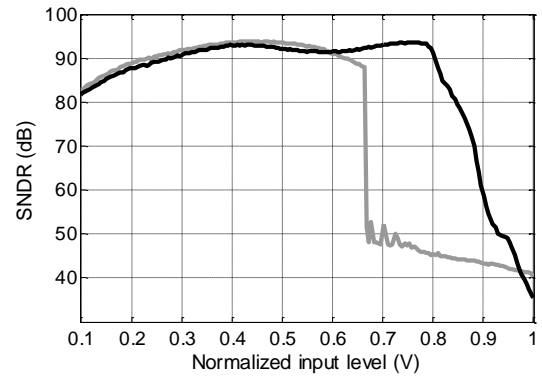


Fig. 4. SNDR versus normalized input amplitude. (Black line) Proposed modulator. (Gray line) Sigma-delta modulator.

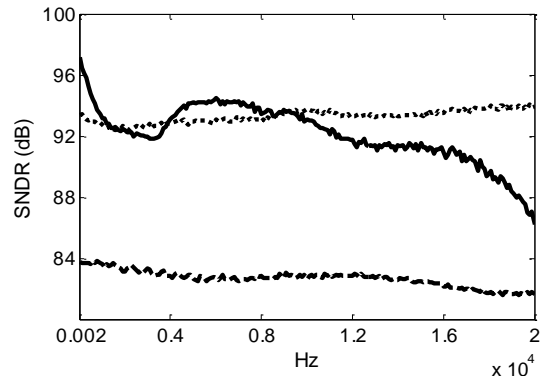


Fig. 5. SNDR versus input frequency. (Dashed line) Normalized input amplitude 0.1 V. (Dotted line) 0.4 V. (Solid line) 0.75 V.

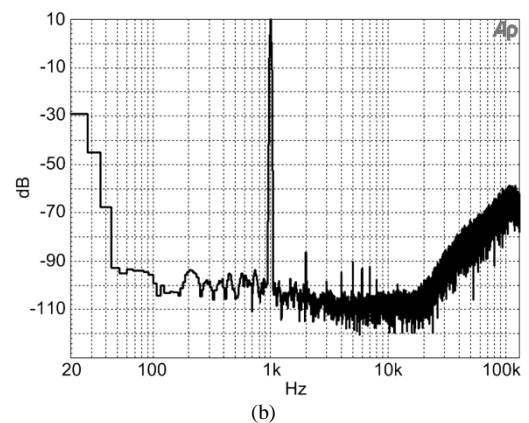
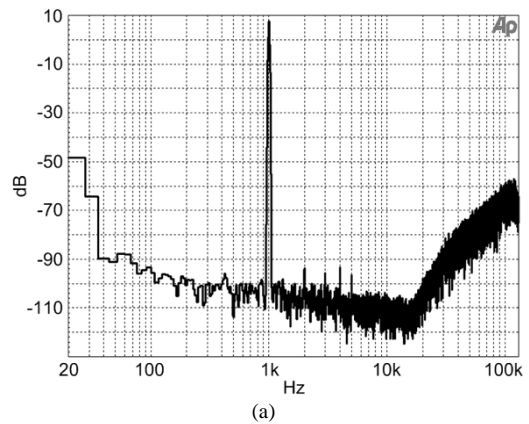


Fig. 6. Output spectrum of the proposed modulator in response to a 1-kHz sine wave input. (a) Normalized input amplitude: 0.6 V. (b) Normalized input amplitude: 0.8V.

## VI. CONCLUSIONS

This paper presents a novel compensation scheme for extending the stable input range of sigma-delta modulators based on the sliding mode theory. As shown in the illustrated example, the proposed compensation scheme is low-cost and easy to design. The experimental results verified that the proposed control scheme efficiently extended the stable input range of the 3rd-order sigma-delta modulator without compromising its original performance. Moreover, it can be designed and implemented in sigma-delta modulators of arbitrary order.

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