Predicting Maximum Traction to Improve Maneuverability for Autonomous Mobile Robots on Rough Terrain

Jayoung Kim and Jihong Lee
Mechatronics Group at Chungnam National University, Daejeon, Republic of Korea
{Jaya, jihong}@cnu.ac.kr

***Abstract*—This paper proposes a method to predict maximum traction force for autonomous mobile robots on rough terrain in order to improve maneuverability. For predicting traction force, we utilize friction-slip curves based on modified Brixius model derived empirically in terramechanics, which is a function of mobility number Bn′ and slip ratio S. Friction-slip curves include physical characteristics of various rough terrains such as firm soil, sandy soil and grass-covered soil. Also, we build prediction models for terrain parameters; maximum static friction and optimal slip ratio on friction-slip curves. Mobility number Bn′ is estimated from modified Willoughby Sinkage model which is a function of sinkage z and slip ratio S. Therefore, if sinkage z and slip ratio are measured once by sensors such as a laser sensor and a velocity sensor, then mobility number Bn′ is estimated and maximum traction force is predicted from a prediction model for terrain parameters. Estimation results for terrain parameters are shown in a driving simulation using MATLAB. Prediction performances for maximum traction of various terrains are evaluated as high accuracy through analysis of estimation errors.**

***Index Terms*—autonomous mobile Robot, brixius terrain Model, maneuverability, maximum traction, friction coefficient, rough terrain.**

I. Introduction

It is important for autonomous mobile robots of missions such as exploration, reconnaissance, and disaster relief to predict maneuverability on rough terrain in aspects of driving efficient and driving safety. Maneuverability of a mobile robot depends upon a variety of interaction between a robot wheel and terrain. In particular, among them, friction-slip characteristic of a terrain is one of crucial elements in improving maneuverability of a mobile robot. Accordingly, to ensure maximized mobility on rough terrain, it is necessary to maintain maximum traction force determined from friction-slip characteristic [1-3].

Manuscript received August 23, 2012; revised December 10, 2012.
This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2012R1A1A4A01017682).

Worldwide, a lot of studies for terrain modeling based on friction-slip characteristic on rough terrain have been performed in order to improve energy efficiency for driving and to maximize mobility of a robot. There are three main methods for terrain modeling: analytical method, empirical method, and semi-empirical method. Analytical method is an approach to model a terrain by mathematically analysis of the interaction between a wheel and terrain. Analytical method facilitates to model accurate terrain based on capability of a computer to numerically analyze minute elements in soil. There are FDM (Finite Difference Method) [4], FEM (Finite Element Method) [5], and DEM (Discrete Element Method) [6] belong to analytical method. Empirical method is an approach to model a terrain based on cone index (CI) which is acquired by penetrating a cone penetration tester into a terrain [7]-[10]. Empirical method facilitates to model a variety of terrains through experiments after driving along various rough terrains. Lastly, semi-empirical method is an approach to model a terrain using experimental data along with mathematical analysis of the interaction mechanism between a wheel and surface of a terrain [1]-[3], [9]. Although empirical method is very simple, it can be applied for modeling of a variety of terrains. Also, recently, predictions obtained by Brixius model which is one of empirical methods were shown to accord with experimental results and DEM simulations [6]. Therefore, Brixius model which is widely known as one of empirical methods is adopted to derive friction-slip curve for predicting maximum traction force in this paper.

Maximum coefficient of static friction μ_p is a parameter which means maximal reaction force generated from interaction between a wheel and surface of a terrain. Accordingly, it is a vital process to estimate maximum coefficient of static friction μ_p in predicting maximum mobility of a mobile robot on a terrain. In this paper, firstly, terrain prediction model with respect to mobility number B_n' is built to estimate maximum coefficient of static friction μ_p and optimal slip ratio S_p based on friction-slip characteristic on rough terrain. Then a method for obtaining efficient maneuverability is proposed by deriving maximum traction force on given terrains through terrain parameters estimated by the proposed modeling methods. Fig. 1 shows flow chart of the proposed algorithm estimating maximum traction force.

©2013 Engineering and Technology Publishing
doi: 10.12720/joace.1.1.1-6

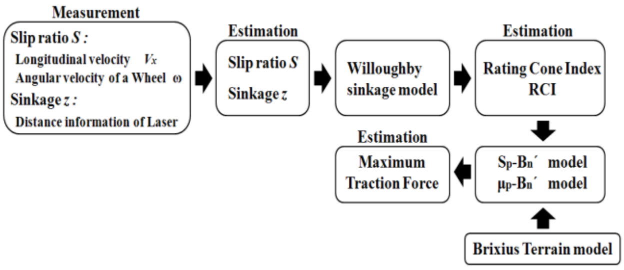


Figure 1. Flow chart of an estimation algorithm for maximum traction force

II. BRIXIUS TERRAIN MODEL

Brixius empirical model is expressed in wheel slips ratio S and mobility number $B_n^{'}$, Eq. (1) relative to wheel slip and Eq. (2) with respect to cone index CI determined by penetrating a cone into a ground, respectively. In this paper, existing cone index CI is changed into rating cone index RCI in Eq. (2). It is possible to express cone index CI as rating cone index RCI because rating cone index RCI is cone index CI multiplied by remolding index RI ($RCI= CI \times RI$)[11].

$$\textbf{slip ratio}\;\; S = \frac{r\omega - v_x}{MAX(v_x, r\omega)} \tag{1}$$

$$\textbf{mobility number}\;\; B_n^{'} = \left(\frac{(RCI)bd}{W}\right)\left(\frac{1+5\dfrac{\delta}{h}}{1+3\dfrac{b}{d}}\right) \tag{2}$$

W : vertical wheel load [kN] b : width of tire cross section [m]
d : tire diameter [m] δ : tire deflection [m]
h : height of tire corss section [m] RCI : rating cone index [kPa]

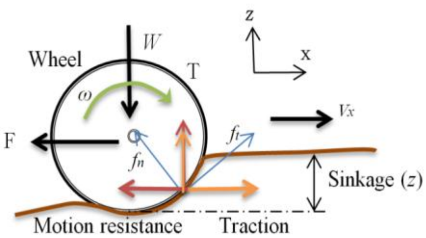


Figure 2. Longitudinal interaction force between a wheel and terrain

As shown in Fig. 2, longitudinal force between a wheel and surface of a terrain is expressed in traction force and resistance force. Brixius terrain model is derived with respect to mobility number $B_n^{'}$ and slip ratio S, defined as: $(+: S > 0, -: S < 0)$

$$\text{Gross Traction}\quad GT = \pm W\left[C_1(1-e^{-C_2 B_n^{'}})(1-e^{\mp C_3 S}) + C_4\right] \tag{3}$$

$$\text{Motion Resistance}\quad MR = W\left[\left(\frac{C_5}{B_n^{'}} + C_4 \pm \frac{C_6 S}{\sqrt{B_n^{'}}}\right)\right] \tag{4}$$

It is convenient to consider the non-dimensional coefficient of the net traction force as shown in Eq. (5).

Friction coefficien t

$$\mu = \pm\left[C_1(1-e^{-C_2 B_n^{'}})(1-e^{\mp C_3 S}) - \left(\frac{C_5}{B_n^{'}} \pm \frac{C_6 S}{\sqrt{B_n^{'}}}\right)\right] \tag{5}$$

where $C_1 \sim C_6$ are parameters which are different depending on types of tires on experiment. In this paper, these parameters are derived on the assumption that a mobile robot equips bias-ply tires. Thus, terrain model parameters are used as C_1 =0.88, C_2 =0.1, C_3 =7.5, C_4 =0.04, C_5=1, C_6 =0.5 [7].

These coefficients can be viewed as slip dependent coefficients of friction. Fig. 3 shows plots of friction coefficient for $B_n^{'}$ = 50, corresponding to a firm soil, and $B_n^{'}$ = 15, corresponding to sandy soil. As shown in Fig. 3, Brixius terrain model can consider a wide variety of rough terrains

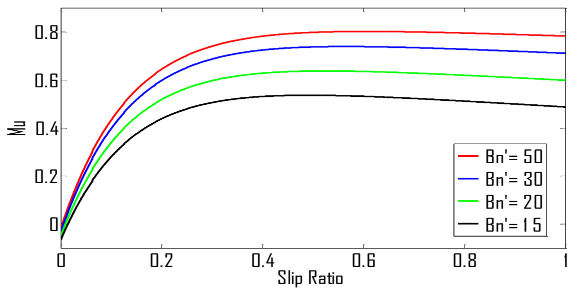


Figure 3. Brixius terrain model (S > 0)

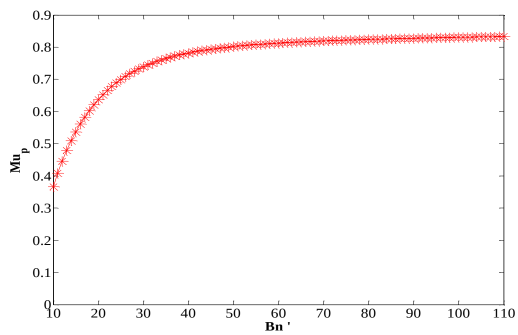
III. ESTIMATION FOR TERRAIN PARAMETER

A. Prediction Model for Terrain Parameters

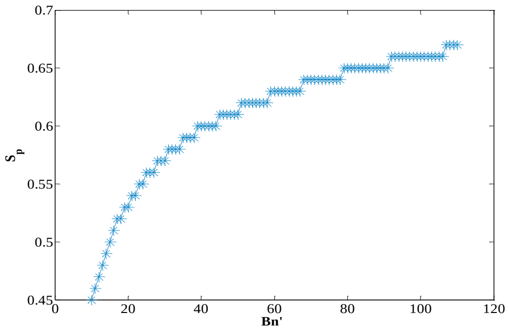
It is needed to estimate terrain parameters μ_p, S_p from friction-slip characteristic in Fig. 3 for predicting maximum traction force of a mobile robot. A terrain parameter μ_p is maximum coefficient of static friction and S_p is optimal slip ratio when μ_p is generated. From terrain models in figure 3, $B_n^{'}$ - μ_p, $B_n^{'}$ - S_p curves are drawn as shown in figure 4. And using the non-linear regression method, prediction models for terrain parameters are derived mathematically as follows Eq. (6) and (7).

$$B_n^{'} - \mu_p \;\textbf{model}\; : \quad \mu_p = \frac{0.932 B_n^{'}}{9.557 + B_n^{'}} \tag{6}$$

$$B_n^{'} - S_p \;\textbf{model}\; : \quad S_p = \frac{0.6943 B_n^{'}}{6.0487 + B_n^{'}} \tag{7}$$



(a) $B_n^{'}$ - μ_p curve



(b) B_n' - S_p curve

Figure 4. Prediction models for terrain parameters

B. Estimation for Rating Cone Index (RCI)

It is needed to estimate terrain parameters μ_p, S_p for predicting maximum traction force of a mobile robot on rough terrain. As mentioned in chap. III-A, terrain parameters μp, Sp are estimated from prediction models Eq. 6-7 for terrain parameters relative to mobility number B_n'. Therefore, first of all, it is vital to estimate mobility number B_n' for terrain parameters. In Eq. 2, since mobility number B_n' is a function of rating cone index *RCI*, Terrain parameters can be obtained by estimating rating cone index *RCI*. In this paper, Willoughby sinkage model is used to estimate rating cone index *RCI*.

Willoughby sinkage model in Eq. 8 is an empirical equation to estimate sinkage of a wheel on fine-grained soils such as silts and clays [11-12].

$$\text{Sinkage} \quad z = \frac{5d\sqrt{N}}{\left[\dfrac{RCI \times bd}{W(1-\frac{\delta}{h})^{\frac{3}{2}} S^{\frac{1}{5}}}\right]^{\frac{5}{3}}} \tag{8}$$

In case that a vehicle passes by on a terrain one time (N=1), Eq. 8 can be changed as an expression in rating cone index *RCI*, defined as:

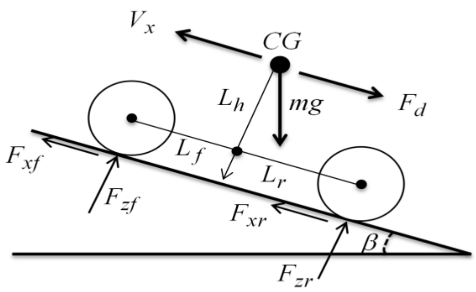
$$RCI = 2.6265 \frac{W(1-\frac{\delta}{h})^{\frac{3}{2}}}{bd^{\frac{2}{5}}} \left(\frac{S}{z^3}\right)^{\frac{1}{5}} \tag{9}$$

Accordingly, if it is able to estimate slip ratio S and sinkage z in Eq. 9, then terrain parameters μ_p, S_p from Eq. 6-7 of prediction models can be estimated through obtaining rating cone index *RCI* as well as mobility number B_n'.

IV. Simulation for Estimating Terrain Parameters

A. Longitudinal Vehicle Dynamics

In order to verify the proposed method for predicting maximum traction force in this paper, estimation simulations for terrain parameters are performed considering longitudinal vehicle dynamics of a mobile robot. Figure 5 describes longitudinal vehicle dynamics model.



F_{xf}, F_{xr} : Traction force of front and rear wheel

F_{zf}, F_{zr} : Vertical force of front and rear wheel

L_f, L_r, L_h : Distance from a middle point to front, rear wheel, and CG

β : Surface inclination F_d : Resistance force V_x : Longitudinal velocity

Figure 5. Longitudinal vehicle dynamics

From vehicle dynamics model in figure 5, longitudinal motion equations of a vehicle are follows:

$$m\dot{V}_x = F_x + F_d - mg\sin\beta \tag{10}$$

$$F_d = -\frac{1}{2} C_d \rho A V_x^2 \,\mathrm{sgn}(V_x) \tag{11}$$

$$F_{zf} = \frac{+L_h(F_d - mg\sin\beta - m\dot{V}_x) + L_r \cdot mg\cos\beta}{L_f + L_r} \tag{12}$$

$$F_{zr} = \frac{-L_h(F_d - mg\sin\beta - m\dot{V}_x) + L_f \cdot mg\cos\beta}{L_f + L_r} \tag{13}$$

$$F_{zf} + F_{zr} = mg\cos\beta \quad , \quad F_x = F_{xf} + F_{xr} \tag{14}$$

where C_d is drag coefficient, ρ is density of air, and A is a contact area of air on the driving direction of a vehicle. Also, longitudinal motion equations of a wheel are as follows:

$$F_x = \mu W \tag{15}$$

$$J\dot{\omega}_i = T_i - F_{xi} \cdot r \tag{16}$$

where μ is friction coefficient, W is a weight of a vehicle, J is inertial moment, T is wheel torque, r is radius of a wheel, and i is the number of wheels.

B. Simulation to Estimate Terrain Parameters

Fig. 6 and Fig. 7 show a diagram of simulator to estimate terrain parameters using Simulink of MATLAB. Simulation results in estimated terrain parameters using state data such as velocity of vehicle body, angular velocity of a wheel, and wheel torque measured while a mobile robot is driving on rough terrain based on velocity control. Simulator is divided into three parts: longitudinal dynamics model, terrain model, and estimator for terrain parameters.

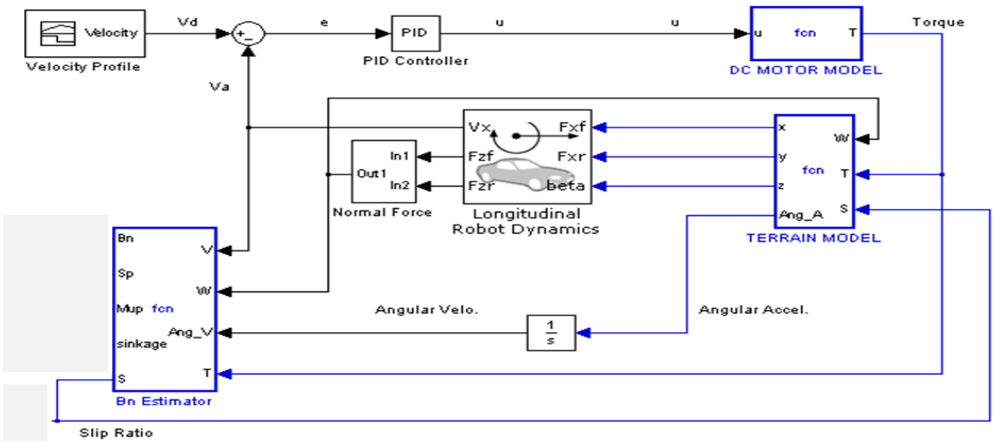


Figure 6. Simulator to estimate terrain parameters

As mentioned, longitudinal dynamics model is used to control state of a mobile robot from longitudinal motion equations. In this paper, slip ratio *S* is estimated using state variables generated from simulation as measured variables of the robot. Also, as terrain models, ranges of sinkage *z* and rating cone index *RCI* are selected through reviewing experiment data in relevant papers considering three kinds of terrains: sandy soil, firm soil, and grass-covered soil as shown in figure 8. Table 1 shows the ranges of sinkage *z* and rating cone index *RCI* on the assumption that sinkage *z* is able to be measured by a sensor such as a laser sensor and a robot is moving on a specified terrain designated by rating cone index *RCI*, respectively. Lastly, estimator for terrain parameters uses state data of a mobile robot from longitudinal motion equations as inputs, and thereby terrain parameters are estimated using the Willoughby sinkage model and the prediction model mentioned in chap. III.

TABLE I. SINKAGE AND RATING CONE INDEX OF TERRAINS [7], [8], [9], [11], [12]

Weight 1 ton	Sandy soil	Firm soil	Grass-covered soil
Sinkage *z* [m]	0.01 ~ 0.05	0.001 ~ 0.01	0.001 > z
Rating cone index *RCI* [kPa]	200 ~ 500	400 ~ 1200	1000~2000

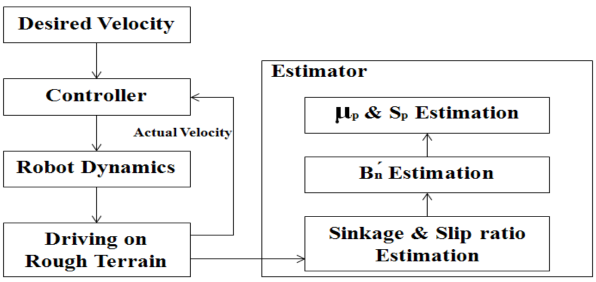


Figure 7. Flow chart of simulation to estimate terrain parameters

(a) Sandy soil

(b) Firm soil

(c) Grass-covered soil

Figure 8. Terrains for simulation

A. Simulation Results

Sampling frequency is 100 Hz for acquiring state data of a mobile robot on simulation. In order that simulation results must be quite similar to real experimental results, simulations are performed based on the prerequisite that the most important variables such as slip ratio *S* and sinkage *z* are influenced by Gaussian noise from which sensors are very susceptible.

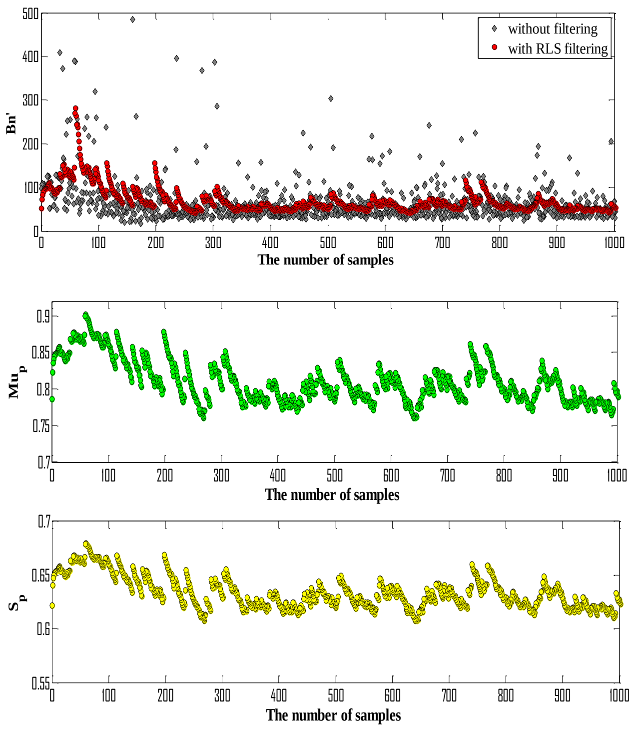


Figure 9. Estimation for terrain parameters on grass-covered soil.

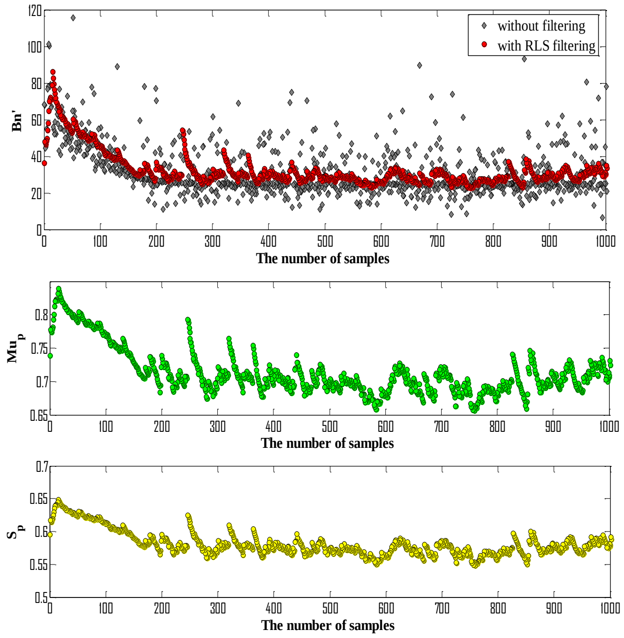


Figure 10. Estimation for terrain parameters on firm soil.

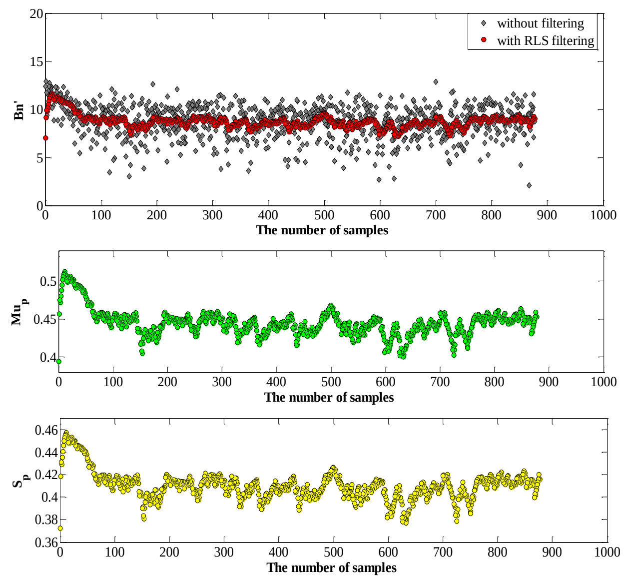


Figure 11. Estimation for terrain parameters on sandy soil.

Fig. 9-Fig. 11 show the estimation results for terrain parameters B_n', μ_p, and S_p depending on a kind of terrains. In this paper, recursive least square (RLS) filter is used to remove noise effects for improving estimation accuracy for terrain parameters. As shown in figure 9-11, the estimation graphs for mobility number B_n' result in removing noise effects by using RLS filter. Forgetting factor of RLS filter applied to simulation is 0.93.

Intuitionally, simulation results seem to be matched well to figure 3 in comparison with the graphs of terrain models in figure 3: sandy soil (B_n' = 10), firm soil (B_n' = 30), and grass-covered soil (B_n' = 100). Quantitatively, table 2 shows estimation accuracy for terrain parameters. Accuracy rates are derived by comparison between estimation values on simulation and actual values on each terrain model. Entirely, estimation performances have high accuracy as estimation accuracy of μ_p is 94% and estimation accuracy of S_p is 95%. In table 2, estimation performance on sandy soil has the worst error rate of the cases. It is shown as modeling errors of prediction models generated when B_n' - μ_p and B_n' - S_p curve are modified to mathematical equations using the non-linear regression method in chap. III-*A*. These modeling errors can be addressed by using more accurate mathematical model for remodeling.

TABLE II. ESTIMATION ERROR OF TERRAIN PARAMETERS

	Sandy soil	Firm soil	Grass-covered soil
μ_p [%]	11.43	3.51	2.45
S_p [%]	8.92	1.96	3.96

B. Estimation for Maximum Traction

Eq. 17 indicates the range of traction force which is allowable to robot wheels on rough terrain. In case of a four wheel drive robot, if front (F_{T_1}) and rear (F_{T_2}) driving motor ensure enough power for moving on rough terrain, then the allowable traction region of a robot is determined from Eq. 17 as shown in figure 12.

$$-\mu_p W \ (S = -S_p) \leq F_T \leq \mu_p W \ (S = S_p) \qquad (17)$$

Fig. 13 shows distribution of maximum traction force as changing weight of a robot based on estimation results for terrain parameters on sandy soil. Consequently, a strategy for efficient maneuverability can be built in the permissible range of traction force and/or of the maximum traction force, depending on a specified purpose of a mobile robot.

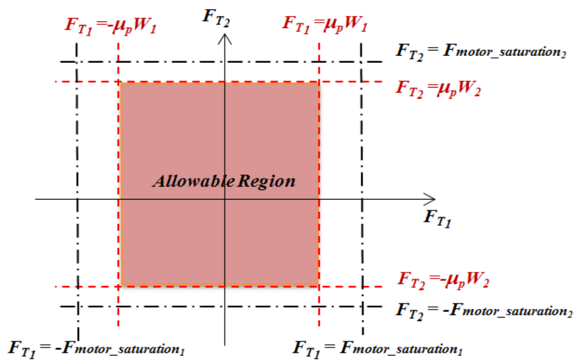


Figure 12. Allowable traction region of a mobile robot

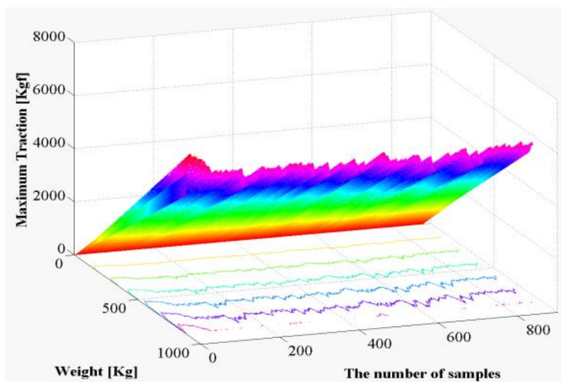


Figure 13. Distribution of maximum traction force as weight of a robot.

V. CONCLUSION

In this paper, friction-slip curves were derived using Brixius empirical model for modeling rough terrains. And prediction models for terrain parameters were built from friction-slip curves of Brixius model using the non-linear regression method. Prediction models were able to estimate terrain parameters μ_p, S_p by estimated mobility number $B_n^{'}$ because prediction models are the function of mobility number $B_n^{'}$. Mobility number $B_n^{'}$ was estimated by acquiring rating cone index RCI using Willoughby sinkage model. Estimated data were improved through the process to remove noise effects and thereby performances for estimating terrain parameters had high accuracy. Consequently, a strategy for efficient maneuverability can be built in the permissible range of traction force and/or of the maximum traction force, depending on a specified purpose of a mobile robot.

REFERENCES

[1] K. Iagnemma and S. Dubowsky, "Mobile Robots in Rough Terrain: Estimation, Motion Planning and Control with Application to Planetary Rover," *Springer tracts in advanced robotics*, vol. 12, Berlin: Springer, 2004.

[2] G. Ishigami, A. Miwa, K. Nagatani, and K. Yoshida, "Terramechanics-Based Model for Steering Maneuver of Planetary Exploration Rovers on Loose Soil," *Journal of Field Robotics*, vol. 3, no. 24, pp. 233-250, 2007.

[3] J. Y. Wong, *Terramechanics and Off-road Vehicle Engineering: Terrain behavior, Off-road vehicle, Performance and Design*, Elsevier, 2nd ed, 2010.

[4] L. K. Karafiath, and E. A. Nowaizki, *Soil Mechanics for Off Road Vehicle Engineering*, Trans Tech Publications, Germany, 1978.

[5] C. H. Liu, and J. Y. Wong, "Numerical simulations of tire-soil interaction based on critical state soil mechanics," *Journal of Terramechanics*, no. 33, vol. 5, pp. 209-221, September 1996.

[6] Z. Asaf, I. Shmulevich, and D. Rubinstein, "Predicting soil-rigid wheel performance using discrete element method," *Transactions of the ASABE*, vol. 3, no. 49, pp. 607-616, 2006.

[7] W. W. Brixius, "Traction prediction equations for bias ply tires," *ASAE*, no. 87-1622, St. Joseph, MI 49085, 1987.

[8] M. D. Evans, R.L. Clark, and G. Manor, "An Improved Traction Model for Ballast Selection," *ASAE*, vol. 3, no. 34, pp. 773-780, 1991.

[9] V. K. Tiwari, K. P. Pandey, and P. K. Pranav, "A review on traction prediction equations," *Journal of Terramechanics*, no. 47, pp. 191-199, 2010.

[10] Z. Shiller, M. P. Mann, and D. Rubinstein, "Dynamic Stability of Off-Road Vehicle Considering a Longitudinal Terramechanics Model," in *Proc. IEEE International Conference on Robotics and Automation*, Roma, 2007, pp. 1170-1175.

[11] R. T. Affleck, "Disturbance Measurements from Off-Road Vehicles on Seasonal Terrain," *Technical Report ERDC/CRREL TR-0512*,US Army Corps of Engineers, Vickburg, MISS., USA., 2005.

[12] J. S. Nam, Y. J. Park, and U. K. Kyeong, "Determination of rating cone index using wheel sinkage and slip", *Journal of Terramechanics*, vol. 47, pp. 243-248, 2010.

Jayoung Kim received the B.S degree in Mechanical Engineering from Chungbuk National University, Cheongju, Korea in 2008, and the M.S degree in the department of Mechatronics Engineering at Chungnam National University, Daejeon, Korea in 2011. Since 2011, He has been a Ph.D student at the same university. His research interests include robotics, intelligent control and outdoor mobile robot.

Jihong Lee received the B.S degree in Electronics Engineering from Seoul National University, Korea in 1983, and the M.S. and Ph.D. degrees from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Korea, in 1985 and 1991, respectively, all in Electrical and Electronics Engineering, Since 2004, he has been a Professor in the Mechatronics Engineering Department of Chungnam National University, Daejeon, Korea. His research interests include robotics, intelligent control, multi-robot localization and path planning.